

## A MODEL OF STEADY PROPAGATION OF A PLANAR FIRE FRONT THROUGH POROUS FUEL BED

Zhu Zhonghua, Liu Xiande

(Institute of Mechanics, Chinese Academy of Sciences,  
Beijing 100080, China)

### ABSTRACT

A physical model is proposed to describe the steady propagation of a planar fire front through a uniform, horizontal, porous fuel bed. The model assumes that there is no ambient wind and the bed is thermally-thin. In the present model, the mechanisms of preheating of unburned fuel contain radiation from the flame, radiation and conduction through the fuel bed due to the non-uniform temperature distribution of unburned fuel. The cooling mechanisms including radiative cooling from the upper surface of fuel bed and convective cooling induced by the flame are also considered. A simple formula is obtained to relate the rate of spread (ROS) with the major influential factors. The model is tested against laboratory fires. If the convective cooling coefficient is constant in variable situation, the predicted ROS is about 1.5 times the measured value and the predicted temperature distribution of unburned fuel differs from the observed values, too. Adjusting the convective cooling coefficient properly would give better predictions for both ROS and the temperature distribution.

Key words: forest fire, fire spread, physical model, heat transfer

### INTRODUCTION

Fire-prediction models of a forest or grassland fire have been proposed by a number of authors.<sup>1</sup> These models can be classified into three types: pure empirical model which is based on relationships developed from observations of wildfires or prescribed burns, such as McArthur<sup>2</sup>; semi-empirical model which uses physical parameters and arguments to produce a formula for the rate of spread (ROS) with data from laboratory fires, such as Rothermel<sup>3</sup>; and physical model which invokes the laws of physics and chemistry of fire in vegetation fuels, such as Albini<sup>4,5</sup>, de Mestre<sup>6</sup>.

Ideally a physical model should be able to predict the fire behaviour such as rate of spread, flame height etc, from the information about the fuel and combustion condition. Unfortunately no such a model is given. Most of the previous physical models assume that heat source in terms of flame height, flame angle of inclination and flame temperature is known, and use the laws associated with the physics of heat transfer. All these models are very helpful to understand the process of fire spread although none of them is completely satisfactory.

Herein we propose a physical model for steady propagation of a planar fire front through a uniform, porous fuel bed on a horizontal base. It is assumed that there is no ambient wind (except induced by the flame), the fuel bed is thermally thin, and heat source is known. With the help of computer, the curves of the relationships between ROS and major influential factors are obtained. Using curve fitting technique with computational data, a simple formula for ROS relating with these factors is obtained. All these expressions are tested against laboratory fires.

### MATHEMATICAL MODEL

In order to simplify the problem, the bush or grass in wildland is usually modelled as a porous fuel bed on a horizontal insulated base, and the combustion region as a known heat source in terms of flame height, flame angle and flame temperature. Besides, the following assumptions are made:

- (1) the combustible is isotropic, and homogeneously arranged with an equal depth  $\delta$
- (2) the fuel bed is thermally thin, the temperature distribution along the depth of fuel bed is uniformized in a short duration.
- (3) the combustible is opaque
- (4) both the combustible and the flame are regarded as black bodies
- (5) the flame front is planar and vertical, the width of flame is infinite and the depth of flame is neglected
- (6) the interface between burnt and unburnt fuel is modelled as a plane normal to the fuel bed surface
- (7) the fuel adjoining the flame reaches the ignition temperature  $T_i$  and begins to burn after absorbed sufficient heat for pyrolysis
- (8) the air is transparent
- (9) there is no ambient wind (except induced by the flame), and the flame spreads in a free, steady state

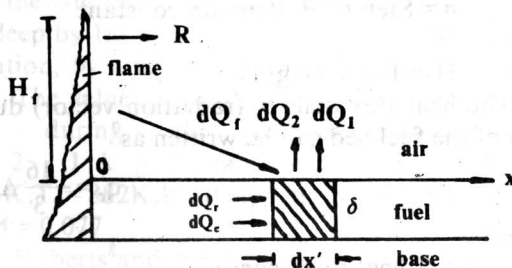


Fig.1 Scheme of fire spreading

Consider a rectangular coordinate system whose origin is fixed onto the flame front and a differential volume with a unit width  $\delta dx'$ , as showed in fig.1. Within the time interval  $dt$ , the differential volume absorbs heat  $dQ_f$  from flame radiation,  $dQ_c$  from the adjacent fuel radiation and  $dQ_c$  from the adjacent fuel conduction. The volume loses heat  $dQ_1$  and  $dQ_2$  due to upper surface radiation and convection respectively. The internal energy of the volume increases by  $dE$ . Because of conservation of energy, the following equation can be obtained.

$$dE = dQ_f + dQ_c + dQ_c - dQ_1 - dQ_2 \quad (1)$$

The internal energy increment  $dE$  is equal to the product of the specific heat  $C_f$  of the fuel, mass  $\rho_b \delta dx'$  and temperature increment  $dT$ , i.e.

$$dE = C_f \rho_b dT \delta dx'$$

where

$\rho_b$  = fuel bulk density = mass of fuel per unit volume

$$C_f = C_d + \frac{M}{1+M} (C_w(373-T_a) + L) / (T_i - T_a)$$

and

$C_d$  = specific heat of dry fuel

$C_w$  = specific heat of water

$T_a$  = ambient temperature

$L$  = latent heat of vaporization of water at 373 K

$M$  = fractional mass moisture content = mass of water / mass of dry fuel

The heat from flame radiation  $dQ_f$  is written by Stefan-Boltzmann law as

$$dQ_f = \sigma(T_f^4 - T_a^4)F(x)dx'dt$$

where

$$F(x) = \text{view factor} = \frac{1}{2} \left( 1 - \frac{\frac{x}{H_f}}{\sqrt{1 + \left(\frac{x}{H_f}\right)^2}} \right)$$

$T_f$  = flame radiation temperature

$T$  = fuel temperature

$\sigma$  = Stefan-Boltzmann constant

and

$H_f$  = flame height

The heat flux density (radiation vector) due to the non-uniform temperature distribution of the fuel bed can be written as<sup>7</sup>

$$q_r = -\frac{16}{3} \Lambda_r \sigma T^3 \frac{dT}{dx}$$

where

$q_r$  = heat flux density

$\Lambda_r$  = mean free path length for radiation

Therefore, one can obtain that

$$dQ_r = \frac{16}{3} \Lambda_r \sigma \frac{d}{dx} \left( T^3 \frac{dT}{dx} \right) \delta dx' dt$$

The heat of conduction  $dQ_c$  is written by Fourier's law as

$$dQ_c = k \frac{d^2 T}{dx^2} \delta dx' dt$$

where  $k$  = fuel heat conductivity

The loss of heat from upper surface radiation is written by Stefan-Boltzmann law as

$$dQ_1 = \sigma(T_a^4 - T_f^4)dx'dt$$

The loss of heat due to convection is written by Newton cooling law as

$$dQ_2 = h(T - T_a)dx'dt$$

where  $h$  = convective cooling coefficient

= heat transfer coefficient of convection induced by flame

In the following analysis,  $h$  is assumed a constant for a particular case.

According to the derivation in ref.10, the boundary conditions are obtained as

$$T(0) = T_i$$

$$T(\infty) = T_a$$

$$\lambda \rho_b R = \frac{16}{3} \Lambda_r \sigma T_i^3 \left( \frac{dT}{dx} \right)_{x=0} + k \left( \frac{dT}{dx} \right)_{x=0} + \sigma(T_f^4 - T_i^4) + h'(T_f - T_i)$$

where  $\lambda$  = effective fuel latent heat at  $T_i$

$h'$  = heat transfer coefficient of convection, flame diffusion, etc

Summarizing the preceding considerations and noticing the equation for ROS

$R = -dx/dt$ , we obtain the following mathematical model of steady propagation of a planar fire front through the porous fuel bed.

$$-\delta\rho_b C_f R \frac{dT}{dx} = \sigma(T_f^4 - T_a^4)F(x) + \frac{16}{3}\delta\Lambda_r \sigma \frac{d}{dx}(T^3 \frac{dT}{dx}) + k\delta \frac{d^2 T}{dx^2} - \sigma(T_f^4 - T_a^4) - h(T - T_a) \quad 0 \leq x < \infty \quad (2)$$

$$T(0) = T_i, \quad T(\infty) = T_a \quad (3)$$

$$\lambda\rho_b R = \frac{16}{3}\Lambda_r \sigma T_i^3 \left(\frac{dT}{dx}\right)_{x=0} + k\left(\frac{dT}{dx}\right)_{x=0} + \sigma(T_f^4 - T_i^4) + h'(T_f - T_i) \quad (4)$$

Eq.(2) combined with eqs.(3,4) can be solved for  $R$  and  $T(x)$  by use of the finite difference method.

## RESULTS AND DISCUSSIONS

To demonstrate the usefulness of the model, it is necessary to compare the numerical results with experimental data. Let us consider the laboratory experiment described in ref.6. A bed of dead Ponderosa Pine needles, 8cm deep by 1m wide by 3m long, is carefully characterized and then burnt, in a no-wind situation, by a line fire lit at one end. The flame front spreads at a steady rate of 0.0049m/s. The values of most of the parameters in the model are known or measured during the experiment, they are  $\rho_f = 509 \text{ kg/m}^3$ ,  $\rho_b = 32.1 \text{ kg/m}^3$ ,  $\sigma = 5.68 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$ ,  $H_f = 0.70\text{m}$ ,  $T_a = 296\text{K}$ ,  $T_i = 593\text{K}$ ,  $T_f = 882\text{K}$ ,  $k = 0.05 \text{ Wm}^{-1}\text{K}^{-1}$ ,  $C_d = 1370 \text{ Jkg}^{-1}\text{K}^{-1}$ ,  $C_w = 4187 \text{ Jkg}^{-1}\text{K}^{-1}$ ,  $L = 2.254 \times 10^6 \text{ Jkg}^{-1}$ ,  $M = 0.077$

The values of  $\lambda$  and  $h'$  are quoted from Roberts<sup>8</sup> and Emmons<sup>9</sup> respectively,  $\lambda = 2.5 \times 10^5 \text{ J/kg}$ ,  $h' = 240 \text{ Wm}^{-2}\text{K}^{-1}$ .

Up to now, only two parameters,  $\Lambda_r$  and  $h$ , are unknown. Fortunately ROS and temperature distribution can be easily measured in the experiment and used to check how large the values of  $\Lambda_r$  and  $h$  should be. Similar to solving two equations for two unknowns,  $\Lambda_r$  and  $h$  can be suitably adjusted and determined. If  $\Lambda_r = 0.29\text{m}$ ,  $h = 80 \text{ Wm}^{-2}\text{K}^{-1}$ , the computational results of  $R$  and temperature distribution are in agreement with the experimental data excellently. In fig.2, the temperature distribution is showed.

The dependences of ROS on the major influential parameters are studied by numerical method. Fig.3-9 shows ROS as a function of moisture, ambient temperature, fuel depth, packing ratio, flame temperature, flame height, convective cooling coefficient respectively. The packing ratio is defined by  $\beta = \rho_b / \rho_f$ . Using curve fitting technique with computational data in fig.3-9, a formula relating ROS with these parameters is obtained.

$$R = C e^{-0.0047h} (1 + 1.53 e^{-3.51M}) \beta^{-1} T_a^{0.85} (T_f - 693.5)^{1.13} H_f^{0.12} \delta^{-0.073} \quad (5)$$

where  $C$  is a constant associated with the fuel.

Formula (5) is tested against an experiment by us. The experiment is performed as that in ref.6, but some parameters are changed. They are  $M = 0.150$ ,  $\beta = 4.8\%$ ,  $T_a = 286\text{K}$ ,  $T_f = 823\text{K}$ ,  $H_f = 0.30\text{m}$ ,  $\delta = 0.04\text{m}$ ,  $R = 0.003\text{m/s}$ . At first, the convective cooling coefficient  $h$  was considered unvaried ( $h = 80 \text{ Wm}^{-2}\text{K}^{-1}$ ). The predicted ROS by formula (5) was 0.0043m/s, larger than the measured value by 43%. Afterward  $h$  is replaced by  $145 \text{ W/m}^2\text{K}$ , it gives a excellent result. The prediction by eqs.(2)-(4) is in good agreement with the experiment, and the predicted ROS by formula (5) is 0.0032m/s, which differs from the measured value by 7%. Fig.10 shows the comparison of temperature between the prediction and the experiment with  $h = 145 \text{ W/m}^2\text{K}$ .

Ref.6 gives a value of  $\Lambda_r = 0.011\text{m}$ , smaller than that in the present model. The reason is

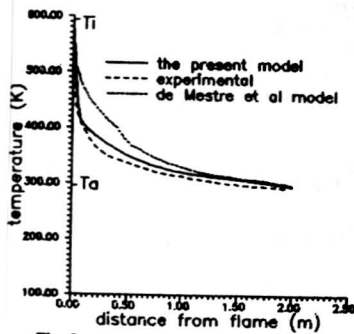


Fig.2 temperature distribution of unburnt fuel

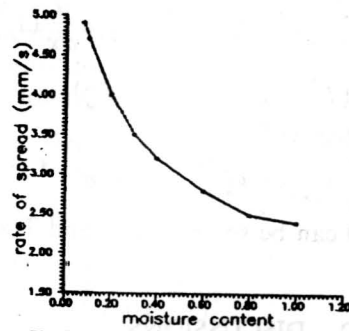


Fig.3 ROS as a function of moisture content

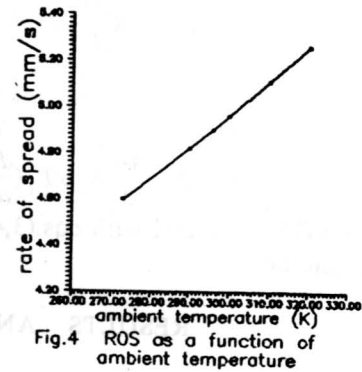


Fig.4 ROS as a function of ambient temperature

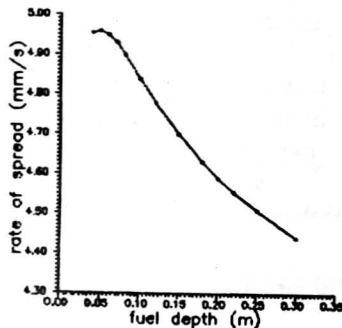


Fig.5 ROS as a function of fuel depth

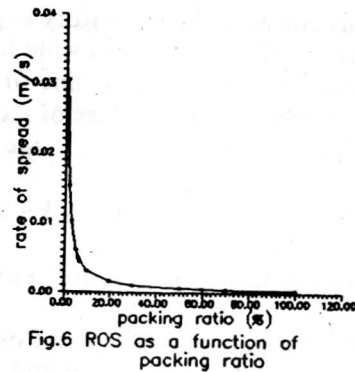


Fig.6 ROS as a function of packing ratio

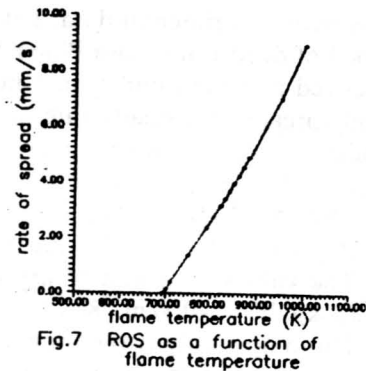


Fig.7 ROS as a function of flame temperature

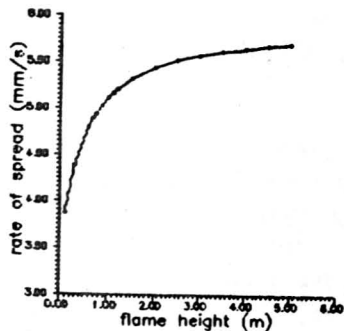


Fig.8 ROS as a function of flame height

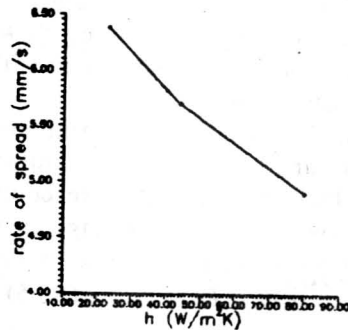


Fig.9 ROS as a function of convective cooling coefficient

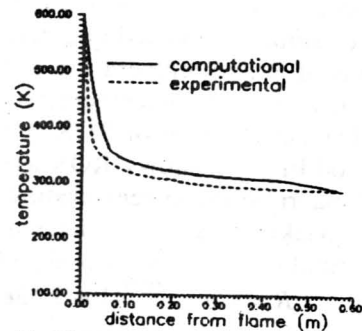


Fig.10 comparison of computation with experiment

possibly that the short-range heating mechanism is included in the term of  $dQ_r$  in the present model. In ref.6, the value of convective cooling coefficient is given as  $23.2 \text{ W/m}^2\text{K}$ , because only the natural convection is considered. In fact, the forced convection induced by flame should be an important factor. The heat transfer coefficient of such a convection ranges over  $25 \sim 300 \text{ W/m}^2\text{K}$ . The values of  $h$  in the present model are in that range.

Table 1 shows the prediction of ROS by formula (5). The experimental data are from ref.11. Suppose that  $h = \text{constant}$  and  $T_f = \text{constant}$ , formula (5) provides a good prediction for ROS.

Table 1. comparison of predicted with experimental ROS

fire	Ta(K)	M	$\beta$	$\delta$ (cm)	Hf(cm)	R1(mpm)	R2(mpm)	error(%)
1	304.7	0.025	0.010	5.08	45.94	0.790	*	*
2	302.4	0.025	0.010	5.08	47.35	0.824	0.788	-4.4
3	302.4	0.027	0.010	5.08	46.45	0.802	0.783	-2.4
4	300.2	0.056	0.010	5.08	34.92	0.554	0.709	28.0
5	300.2	0.059	0.010	5.08	38.90	0.634	0.714	12.6
6	302.4	0.061	0.010	5.08	33.88	0.677	0.725	7.1
7	294.1	0.101	0.010	5.08	38.94	0.683	0.648	-5.1
8	300.2	0.131	0.010	5.08	31.50	0.542	0.611	12.7
9	300.2	0.200	0.010	5.08	29.83	0.416	0.542	30.3
10	303.0	0.285	0.010	5.08	20.40	0.409	0.464	13.4
11	300.2	0.315	0.010	5.08	23.40	0.437	0.451	3.2

+ the constant C is calculated from fire 1

++ R1:experimental ROS, R2:predicted ROS, the unit mpm means meter per minute

## CONCLUSIONS

To conclude this paper, the following conclusions are drawn :

- (1) The present model is essentially proper. Formula (5) is simple and useful for predicting ROS
- (2) The short-range heating mechanism may play an important role in fire spreading.
- (3) The convective cooling coefficient varies in different situations.
- (4) It is valuable to determine the values of  $\Lambda_r$  and  $h$  physically in the next work.

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## REFERENCES

1. R.O. Weber, *Prog. Energy Combust. Sci.*, 17(1991), 67
2. A.G. McArthur, *Aust. For. and Timber Bureau Leaflet No.100* (1966)
3. R.C. Rothermel, *Forest Service USDA Res. Pap. INT-115* (1972)
4. F.A. Albin, *Combust. Sci. Technol.* 42, (1985) 229
5. F.A. Albin, *Combust. Sci. Technol.* 45, (1986) 101
6. N.J. de Mestre, E.A. Catchpole, D.H. Anderson and R.C. Rothermel, *Combust. Sci. Technol.* 65, (1989) 231
7. V.P. Isachenko et al, *Heat Transfer* (translated by WANG Feng) Higher Education Press, Beijing (1987), (in Chinese)
8. A.F. Roberts, *13th Symp. (Int.) on Combust.* (1971), 893
9. H.W. Emmons and T. Shen, *13th Symp. (Int.) on Combust.* (1971), 917
10. N. Fujii, J. Hasegawa, L. Pallop and Y. Sakawa, *Appl. Math. Modelling* 4(1980), 176
11. R.A. Wilson, Jr., *Forest Service USDA Res. Pap. INT-289* (1982)