On the Nonlinear Fire Dynamics

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ABSTRACT

As a result of the development of the nonlinear theory, the complicated fire dynamics can be investigated by means of nonlinear methods, rather than the usually adopted linear approximation. In this paper the current practical nonlinear theory usually used in fire dynamics was described, and the topics and corresponding achievements in nonlinear fire dynamics were reviewed. The nonlinear methods described here mainly involve stability analysis and the characterization of chaotic attractors. Examples of stability analysis, power spectrum analysis and fractal dimension analysis used in nonlinear fire dynamics were addressed.

KEYWORDS: nonlinear fire dynamics, bifurcation, stability analysis, chaos, fractal, power spectrum, characterization of chaotic attractors

INTRODUCTION

Physical phenomena or processes (e.g., fire processes) concern the interrelationship of a set of physical variables (e.g., the temperature, the pressure, etc.) which are deterministic (within some accuracy). ***Nonlinear phenomena*** concern physical processes involving variables which are governed by nonlinear equations. In other words, an initial change of one variable in nonlinear phenomena does not produce a proportional change in the behavior of that variable, or some other variable. Different kinds of mathematical models are constructed to describe various physical phenomena. From a traditional point of view, in order to grasp the essence of a nonlinear phenomenon, one has to put aside all the secondary factors and construct simple, yet nontrivial, mathematical models. Since few models can be solved rigorously, it is often necessary to resort to further approximations. Thereby linear models are usually obtained. For nonlinear ones, we can simulate its behavior on computer by means of numerical techniques. While in numerical experiments, linearization is usually applied to seek convergent solutions. Linearity is beautiful, and it is very valuable in understanding many features of the nonlinear processes. While as described by Lakshmikantham *et al.* [1], *linear models are honest and a bit sad and depressing: proportional efforts and results.* Many other important properties of the original complex nonlinear processes may be abandoned just because of linearization. In a word, the situation is far less satisfactory in regard to the dynamical essence of nonlinear phenomena. To seek the dynamics or in other words, the physical essence of a nonlinear model, nonlinearity of the model must be investigated directly.

To date it is clear for fire researchers that extremely complex behaviors including chaos as the results of nonlinearity exist very commonly in a wide variety of fire systems. Indeed, complex nonlinear dynamics has by now been shown to be of potential importance in many different fields of fire science including explosion and flashover [2], premixed combustion [3], kinetics of soot aggregates [4], ignition of flame [5], transition from one state to the other state [6], gasless or condensed phase combustion [7], etc.

The aim of the present review is to describe the current practical nonlinear theory and to present a number of representative developments in nonlinear fire dynamics. Despite widespread interest and broad application, nonlinear science is a young field and consequently, the standardization of methods is less well developed. We begin with the introduction of the present nonlinear dynamical concepts and theories, and then we address the developments of stability and bifurcation analyses in fire dynamics. Linear, nonlinear and bifurcation stability analyses are of great importance in understanding many complex fire behaviors governed by nonlinear relations. In the last section the developments in characterizing chaotic fire dynamics are presented. For reasons of length we only address the power spectrum analysis and the fractal dimension analysis.

THEORY OF NONLINEAR DYNAMICS

In recent years, the ideas of nonlinear dynamics have considerably improved our understanding of irregular behavior of nonlinear physical phenomena, including that of many complicated nonlinear processes involved in fire. The dynamics of a mathematical model described by a set of differential equations is visualized to take place in a phase space constructed from the state variables of the system. The behavior of the system can then be characterized by looking at the ***attractors*** that the phase space orbits settle onto (these may be point, periodic, or even chaotic) and their ***basins of attraction***, that is, the area surrounding a given attractor in phase space that will fall onto that attractor.

In general any model will depend on one or more parameters or controls. As a given parameter is varied the behavior of the system will also vary. Most of the time a small change in parameter will produce only a slight change in the response of the system (its structure is qualitatively the same as before). However, at certain critical parameter values the behavior of the system can change dramatically, and its structure becomes qualitatively different (for example, the number of attractors could change or the behavior type of a given attractor could go from point to periodic or chaotic). These critical parameter values are called ***bifurcation points***, and the changes in system behavior are termed ***catastrophes*** or ***bifurcations*** (it is more usual to use the term catastrophe when referring to a jump in system state). For special sequences of these bifurcations a lot is known, and even quantitative features can be predicted, as in the case of the period-doubling cascades. We do not, however, possess a complete classification of the possible transitions to more complicated behavior.

For a general nonlinear model we would like to map out its structure with respect to parameter variation. For this reason, the bifurcation points, which divide the parameter space into regions of different behavior, are of great importance. Two techniques, which are termed as ***linear*** and ***nonlinear stability analysis*** respectively, can be used to reveal how the solutions of the nonlinear equation evolve with the variation of the parameter. In contrast to a linear stability analysis, which merely determines whether or not the basic solution of the equation is stable to infinitesimal disturbances, the nonlinear analysis which we employ determines the new solution to which disturbances of the basic solution evolve. In this way we gain specific information about the transition from the basic solution to the bifurcated solution. For further details of the basis of the bifurcation theory see, for example, Chow and Hale [8], Keller and Antman [9]. Representative monographs of the catastrophe theory are those of Arnold [10], Thom [11] and Gilmore [12]. Different types of bifurcations or catastrophes existed commonly in fire processes. Flashover, for instance, which is characterized by a sharp increase in both the burning rate of the fire itself and the temperature of the hot gas layer which forms above [2], involves an obvious catastrophe process.

It is well known that deterministic problems, governed only by some few nonlinear ordinary differential equations or by iterations of maps in finite-dimensional spaces, could give rise to chaotic turbulent-like behavior, highly sensitive to initial conditions and unpredictable after long times of many iterations. For the time being, there is still no generally accepted definition of ***chaos***. Hao [13] suggested a working or operational definition for chaos, that is, *if seemingly random motion occurs in a system, without applying any external stochastic forces, and the individual output depends on the initial conditions sensitively, but at the same time, some global characteristics, (e.g., a positive Lyapunov exponent or entropy, fractal attractor dimension, etc.) turn out to be quite independent of the initial conditions, then one may well be dealing with chaos.* In general an attractor with sensitive dependence on initial conditions is defined as ***a strange or chaotic attractor.***

For a dynamical system, a first bifurcation may be followed by further bifurcations, and when a certain sequence of bifurcations has been encountered, chaos may be achieved. As an example, Ruelle and Takens [14] proved that if a system undergoes three Hopf bifurcations, starting from a stationary solution, as a parameter is varied, then it is likely that the system possesses a strange attractor with sensitivity to initial conditions after the third bifurcation. Bayliss and Matkowsky [7] investigated the models of gasless combustion which exhibit pulsating solutions. Two routes to chaos by means of different bifurcations in condensed phase combustion were examined.

Either in experiments or in numerical calculations, we cannot judge chaos only from the seemingly randomness of the dynamical data. On a computer it is impossible to distinguish a very long periodic orbit from a quasiperiodic or chaotic one by merely looking at the trajectories. In addition, inevitable round-off errors place doubts on any claim that the observed erratic motion is deterministic chaos. Similarly, in experimental nonlinear science a principal difficulty is the distinction between nonperiodic dynamics caused by deterministic processes (chaos) and that caused by stochastic processes (noise). The former is intrinsic to the system and the latter is extrinsic to the system. For these reasons, ***characterization of chaotic attractors*** is of great importance. As suggested by Hao [13], a correct strategy for computer experiments and, to some extent, also for laboratory experiments, involves two consecutive steps: identification of periodic solutions and characterization of chaotic attractors.

Characterization of chaotic attractors employs such notions as Lyapunov exponents, various dimensions and power spectrum. For a dynamical system, if the initial state of a time evolution is slightly perturbed, the exponential rate at which the perturbation increases (or decreases) with time is called ***a Lyapunov exponent***. An n-th order system of ordinary differential equations has n Lyapunov exponents. The practical calculation of the Lyapunov exponents from evolution equations can be found in Parker and Chua [15], Rasband [16], and Jackson [17]. From measurements of a time series Lyapunov exponents can be obtained by many methods (see, for example, Eckmann *et al.* [18], Sano and Sawada [19], and Bryant, Brown and Abarbanel [20]). In any case, positive exponents are generally regarded as equivalent to the presence of real dynamical chaos, and the Lyapunov exponents are classifiers of the dynamics since they are characteristic of the attractor and independent of any given orbit or initial condition. As far as we know, however, it seems that few studies have adopted Lyapunov exponents to characterize the dynamical behavior in fire processes. The reasons may in part lie in two aspects. Firstly, the exponents calculations often require large accurate data sets which are difficult to obtain for general fire processes because of variations in the parameters. Secondly, the present algorithms of Lyapunov exponents have some limitations. For instance, in the method put forward by Eckmann *et al.* an embedding dimension and another so-called matrix dimension were defined and used to reconstruct the dynamics representing the time evolution of the system by the time-delay method. The choice of these two dimensions is of great importance to achieve satisfactory results. However, the authors admitted that there exists no clear and definitive principle of how to choose these two dimensions. In addition, many methods can only be used to obtain nonnegative exponents, and the number of obtainable exponents is one or two.

The ***power spectrum*** of a scalar signal is defined as the square of its Fourier amplitude per unit time. In a sense, ***power spectrum analysis*** is the immediate task one has to carry out after drawing the trajectories and the Poincare maps, because it provides us with the simplest method for distinguishing between chaotic and quasiperiodic motion, the spectra of the former being noisy broad bands and the latter discrete lines without simple frequency interrelationship. Even for periodic regimes, the fine structure of power spectra can furnish useful information on our whereabouts in the parameter space. In fire science, el-Hamdi, Gorman, and Robbins [21] succeeded in classifying the periodic and chaotic modes of pulsating and cellular premixed flames by means of power spectrum analysis. Their work will be reviewed in detail in the later sections.

Fractal theory provides a method of characterizing geometries that cannot be described by conventional methods of Euclidean geometry. The dimension of a set is roughly the amount of information needed to specify points on it accurately. Mandelbrot [22] was the first to point out and popularize the notion for fractal geometry as a useful and accurate representation for naturally occurring phenomena. It is particularly useful in characterizing naturally occurring geometries that display a wide range of self-similar shapes and forms. This similarity between different size scales implies that the dynamic processes operating at each scale are similar. An important and useful characteristic of fractal geometries is that the measured size of a fractal object varies with measurement scale by a power law relationship. In fire science many developments have been achieved by the method of fractal especially in determining the premixed flame speed (e.g. Gouldin [23]).

To conclude, the nonlinear dynamical theory of physical systems is a rather mathematical subject, in the sense that it appeals to difficult mathematical theories and results. On the other hand, these mathematical theories still have many loose ends. In fire science many complex phenomena remain unresolved, and perhaps more clarified mathematical theories are in demand. Fortunately, based on the interplay between present dynamical theory and fire physics, plenty of fruitful developments of fire dynamics have been achieved, which will be reviewed in detail in the following sections.

STABILITY AND BIFURCATION ANALYSES

Dynamics of Flashover

Flashover is an important phenomenon in building fires whereby a relatively small, localized fire can suddenly undergo a rapid increase in its rate of growth and intensity. Such a transition is characterized by a sharp increase in both the temperature of the upper layer (which forms under the ceiling) and the burning rate of the fire itself. Apart from flashover, the regime of a quasi-steady low-intensity fire is possible with relatively little heating of the upper layer.

The nature of the flashover jump suggests a nonlinear dynamical process is at work. Initial attempts to understand this phenomenon have already been made by Thomas *et al.* [24] to model flashover as a jump phenomenon. In the work by Thomas *et al.* a zone model was constructed which assumes that the compartment is divisible into two homogeneous regions: a hot/smoke zone and a cool/lower zone. A quasi-steady state assumption was made. Thermal radiative feedback is seen as the significant factor in this model. In general the existence of radiative heat transfer ensures that the equations governing the behavior are highly nonlinear and likely to present a rich field of study for nonlinear instabilities. Bishop *et al.* [2,25] used the approach of Thomas *et al.* and conducted a qualitative analysis of the occurrence of flashover. They considered an idealized energy balance of the hot layer in a compartment fire which has the following form

$$\frac{\mathrm{d}E}{\mathrm{d}t} = G(T,\ t) - L(T,\ t) \tag{1}$$

where $G(T, t)$ and $L(T, t)$ are, respectively, the rate of gain and loss of energy, E, of the layer. The energy of the smoke layer and, hence, its temperature are therefore governed by the forms of gain and loss that are functions of gas layer temperature, T, and time, t. The gain rate of energy of the hot layer is chiefly determined by the burning rate of the fire. This in turn is governed by two factors. The first of these is whether the fire is fuel or ventilation controlled. As we know, a fire with a perfect ratio (balance) of air to fuel is termed "stoichiometric". In the case of excess air the ratio of air to fuel flow is greater than stochiometric and the fire is fuel controlled (i.e., limited by the amount of fuel flow). Conversely, in the case of insufficient air the ratio of air to fuel flow is less than stoichiometric and the fire is ventilation controlled (i.e., limited by the amount of air flow). This switch-over mechanism constitutes the first important nonlinear behavior of the model.

An increase in temperature of the layer will result in it emitting more radiation to the base of the fire. This will increase the burning rate of the fire and hence the amount of energy released into the layer. The energy increase of the layer will in turn increase its temperature, and a positive feedback loop is created. The total radiation flux to the fire base is given by

$$q = q_{\mathrm{fl}} + \kappa\sigma(T^4 - T_{\mathrm{b}}^{\ 4}) \tag{2}$$

where T_{b} is the temperature of the fire base, σ is the Stefan-Boltzmann constant, and q_{fl} is the free-burn term. The fourth-power temperature dependence of radiation feedback is the source of the second major nonlinear behavior of the model.

The rate of energy loss of the layer is given by a combination of energy losses from the hot gas flowing out of the vent and heat being conducted through the walls of the room. Both of these processes are driven by the temperature difference between the layer and the surroundings. Having outlined the form of the governing factors we can now sketch the gain and loss rates as two functions of temperature as shown in Figure 1. It can be seen that depending on the relative positions of the gain and loss curves there can be up to three intersections, one corresponding to small fire (e.g. point C_1 or C_2), one to a large fire (e.g. point A_1 or A_2) and one being unstable (e.g. point B). The gain and loss rate curves shown in Fig. 1 are a snapshot at one particular instant of time. In general the two curves can be considered to shift relative to one another as time passes. Thus quasi-steady approximation was used. A fire represented by intersection C_1 or C_2 would then move up the gain curve until it eventually encountered intersection B, resulting in a fold catastrophe of saddle node bifurcation. This would lead to a sudden jump in temperature of the layer, to the large ventilation controlled fire represented by state A. Bishop *et al* identified this jump in layer temperature and burning rate of the fire with the phenomenon of flashover.

Two ordinary differential equations describing the temperature of the smoke layer filling the compartment above a fire, *T*, and the radius of the fire itself, *R*, were used to model the growth of fire and reflect the qualitative behavior outlined above. The two equations are

$$\frac{dT}{dt} = \frac{G - L}{c_p m} \tag{3}$$

and

$$\frac{dR}{dt} = V_f[1 - \exp(\frac{[R - R_{max}]}{R_{edge}})] \tag{4}$$

In equation (3) T is the hot gas layer temperature, G and L are respectively energy gain rate and loss rate of the layer, c_p is the specific heat at constant pressure of the gas and m is the mass of the gas layer. In equation (4) V_f is the flame spread rate, R_{edge} is the distance over which the edge of the fuel affects the spread rate and R_{max} is the maximum radius, representing the size of the fuel sample.

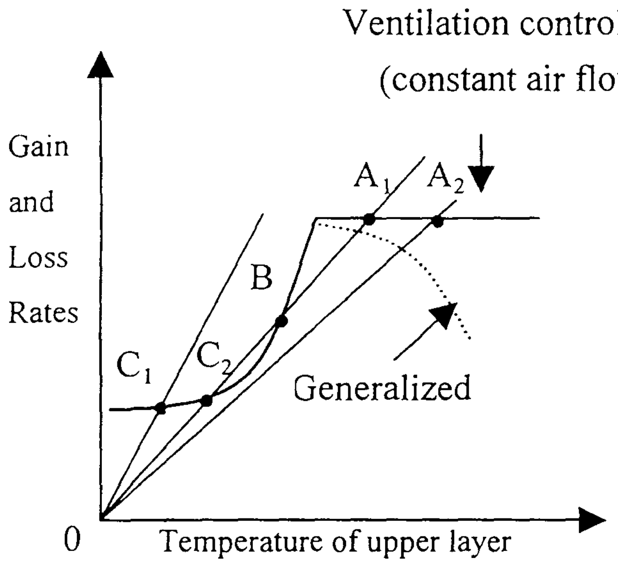

Figure 1. Gain rate and three loss curves illustrating possible types of intersections

Bishop *et al.* considered the fire development process to be made up of a "fast" variable representing the temperature of the gas and a "slow" variable in the form of the fire radius. To get a measure of the effective stability of the fire's evolution the eigenvalue of the differential equation for temperature was used which has the form of

$$\lambda = \frac{F(T+\delta, R) - F(T, R)}{\delta} \tag{5}$$

where δ is small. This is found numerically and the eigenvalue may then be used to determine the stability or otherwise of the state:

$$\begin{array}{ll} \lambda > 0 & \text{Unstable} \\ \lambda < 0 & \text{Stable} \end{array} \tag{6}$$

The two differential equations are integrated using a fourth/fifth order adaptive step size Runge-Kutta. The approach of "Path followed" [26] was employed to locate attractors and basins of attraction. In the simplest two-dimensional case analyzed here the fire trajectories evolve on a control surface with slow variables/parameters (wall temperature parameter and fire radius) effectively controlling the evolution of the layer temperature. Bishop *et al.* found that the surface has the form of a cusp catastrophe (i.e., for some values of fire radius and wall temperature parameter the layer temperature has more than one possible value). The phenomenon of flashover can be identified with a trajectory encountering the cusp, undergoing a catastrophe, and jumping from the lower to the upper surface. The effect of a change in ventilation on the form of the control surface has also been investigated in this paper. The model described above was tested by the experiments of Holborn *et al.* [27] finished in a small-scale compartment. A comparison between experimental data and theoretical predictions obtained from model simulations were shown to be in reasonable agreement.

Graham, Makhviladze, and Roberts [28] also adopted similar stability analysis to investigate the flashover development. The following equations were used to obtain the critical temperature T for the occurrence of flashover:

$$G(T) = L(T) \tag{7a}$$

$$G'(T) = L'(T) \tag{7b}$$

Actually this criterion is equivalent to that of Bishop *et al.*, which can be shown by comparison of Eq. (7) and Eq. (6). The critical conditions have been given in a general form in terms of two similarity parameters for walls of large and low thermal inertia. The flashover induction time was also obtained analytically. Calculation of the flashover induction time allows for an evaluation of the total time for fire growth from ignition to flashover.

Flashover is a typically nonlinear phenomenon that has yet been far from being resolved. However, as suggested by previous work, nonlinear methods can effectively help construct models of flashover that relate fire behavior to the physical characteristics of the system. This improved understanding should also facilitate the formulation of effective regulations to govern fire safety.

Stability of premixed flame

Considerable progress has already been achieved in the understanding of the nature and character of spontaneous instabilities in premixed flames in the past two decades. It is well known that two decades ago the mathematical theory of steady plane flames had already been in principle complete, and there are several monographs on the subject (e.g. Williams [29], Kanury [30], Glassman [31]). At that time, however, the situation was far less satisfactory in regard to nonsteady phenomena in curved flames. Some experiments had shown that there is a class of flames that prefer a characteristic two- or three-dimensional structure rather than a plane flame [32,33]. It was noticed that cellular structure tends to appear when the combustible mixture is deficient in the light reactant (e.g. rich hydrocarbon-air or lean hydrogen-air mixtures). Spinning propagation of luminous flames were also observed [34]. These observed instabilities, however, had not yet been well explained theoretically at that time. While during the following five years or so, say, in the late 1970's and early 1980's, marked progress was achieved in this field. Plenty of theoretical and experimental results were obtained which helped understand the nature of the curved flame and the transition from steady plane flame to it. These achievements were largely due to the linear and nonlinear stability analyses methods that have penetrated combustion theory from nonlinear science. It is now well established that there exist two basic types of instabilities which can affect the mode of propagation of an adiabatic flame in the absence of gravity. One is a hydrodynamic instability, first described by Landau [35], which arises due to the effect of the thermal expansion of the gas. The other type of instability, first proposed by Zeldovich [36], is diffusional-thermal in nature and is directly related to the relative rates of diffusion for temperature and species concentration. Thus, if the Lewis number of the mixture is sufficiently small, corresponding to a deficiency in the lighter component of the mixture of fuel and oxidizer, the flame will experience a cellular instability [37]. On the other hand, if the Lewis number is sufficiently large, corresponding to a deficiency in the heavier component of the mixture, the flame will experience a pulsating instability [38].

Linear stability analysis has been already used widely in different fields, and it can be used to determine whether or not the basic solution is stable to infinitesimal disturbances. It was also used in the investigation of the thermodiffusive flame instability. As early as the mid-1940s, Zeldovich [36] proposed a qualitative explanation for why cellular flames tend to form in mixtures that are deficient in the light reactant. A mathematical model was constructed and linear analysis of the stability of a plane flame front to long-wave disturbances yields the following dispersion relation [39]:

$$\sigma = D_{th}[\frac{1}{2}\beta(1 - Le) - 1]k^2 \tag{8}$$

where $\beta = E(T_b - T_u) / RT_b^2$ with T_u the temperature of the unburned cold mixture and T_b the temperature of the burned gas; E is the activation energy, and R is the universal gas constant; Lewis number Le is the ratio of the thermal diffusivity D_{th} of the mixture and the molecular diffusivity D_{mol} of the limiting reactant (the reactant that is entirely consumed in the reaction, which is assumed to be strongly deficient); σ is the rate-of-instability parameter; and k is the wave vector of the disturbance of the flame front F with $F \sim \exp(\sigma t + i\mathbf{k}\cdot\mathbf{x})$. Thus the flame is stable only if the mobility of the limiting reactant is sufficiently low ($Le > Le_c = 1 - 2/\beta$). At $Le < Le_c$ the flame is unstable.

However, as was pointed out later [37], a flame, though possibly unstable to long-wave disturbances, is nevertheless always stable to short-wave disturbances. At $Le \cong Le_c$ using linear instability analysis the dispersion relation incorporating the relaxation effect of short-wave disturbances is

$$\sigma = D_{\text{th}}[\frac{1}{2}\beta(1 - Le) - 1]k^2 - 4D_{\text{th}}l_{\text{th}}^2 k^4 \qquad (9)$$

where l_{th} is the thermal thickness of the flame. Thus, when the flame is unstable ($Le < Le_c$), there is a wavelength $\lambda_c = 2\pi/k_c$ corresponding to the highest amplification rate of small disturbances (maximum σ).

What happens to the flame front after the development of progressive disturbances? As remarked previously, in contrast to a linear stability analysis, the nonlinear analysis can be used to determine the new solution to which disturbances of the basic solution evolve. Consider a curved flame front in the constant-density model. The following rigorous asymptotic nonlinear relation was derived provided $Le - Le_c$ is a small parameter [40]:

$$F_t + \tfrac{1}{2}U_b(\nabla F)^2 + D_{\text{th}}[\frac{1}{2}\beta(1 - Le) - 1]\ \nabla^2 F + 4D_{\text{th}}l_{\text{th}}^2\nabla^4 F \ = 0 \qquad (10)$$

where F is the perturbation of a plane flame front; U_b is the velocity of the undisturbed plane flame front relative to the burned gas. Qualitative arguments in favor of such a mechanism of nonlinear stabilization have been put forward by Manton, von Elbe, and Lewis [41], Markstein[42], Petersen and Emmons[43]. Numerical experiments on this equation have shown that when a plane flame is disturbed it ultimately evolves into a cellular flame, with characteristic cell size somewhat greater than λ_c. This structure is essentially nonsteady, the cells being in a state of continual chaotic self-motion [44]. This phenomenon was later reconfirmed in experiments performed by Sabathier, Boyer, and Clavin [45]. Thus, by means of stability analysis methods, despite its simplicity the one-reactant constant-density model proved sufficiently rich not only to provide an adequate description of the sensitivity of flame stability to the composition of the mixture and to predict the characteristic size of the cells, but also to describe the chaotic self-motion of the cells.

Stability analysis methods were also used in the investigations of effects of acceleration [46], heat loss [47] and stretching [48] on the stability of flame and satisfactory results were obtained.

Compared with diffusional-thermal instability, hydrodynamic instability is the more difficult to analyze because it involves a full coupling of the equations of fluid dynamics with the diffusion equations for species concentrations and temperature. However, nonlinear stability analysis also yielded a nonlinear evolution equation for the perturbation of the flame front and the numerical results which coincided with the experiments [44].

Stability analysis was also used in the investigation of the flame propagation in vertical channel and bifurcation to bimodal cellular flames was described [49].

For review of nonlinear stability and bifurcation in the transition from laminar to turbulent flame propagation, see, Margolis [6]. From it we can see that instability analysis, especially nonlinear instability analysis plays a very important role in the development of the theory of the flame transition.

Besides the stability analysis, derect mathematical modeling can also be used to investigate the bifurcation of fire dynamical systems. Bayliss and Matkowsky [7] numerically solved the equations governing two models of gasless combustion which exhibit pulsating solutions by the so-called Chebyshev pseudospectral method. In this work, instability analysis was not adopted. The governing equations were directly computed numerically, and the bifurcations to chaos were investigated in detail. Xie and Fan [68] established the nonlinear mathematical model of the mine ventilation system. By means of Runge-Kutta method some bifurcation phenomena of mine ventilation system were firstly numerically found.

To date stability and bifurcation analyses methods have played a very important role in the development of the theory of fire dynamics. It can be expected that these methods will continue to help understand more complicated phenomena in fire processes.

CHARACTERIZATION OF CHAOTIC BEHAVIOR IN FIRE PROCESSES

Power spectrum analysis

The power spectrum indicates whether the system is periodic or quasiperiodic. The power spectrum of a periodic system with frequency ω has Dirac δ' s at ω and its harmonics 2ω, 3ω,.... A quasiperiodic system with basic frequencies $\omega_1,\ldots,\omega_k$ has δ' s at these positions and also at all linear combinations with integer coefficients. In experimental power spectra, the Dirac δ' s are not infinitely sharp; they have at least an "instrumental width" $2\pi/T$, where T is the length of the time series used. The linear combinations of the basic frequencies $\omega_1,\ldots,\omega_k$ are dense in the reals if $k>1$, but the amplitudes corresponding to complicated linear combinations are experimentally found to be small. In fact, $k=2$ is common, and higher k's are increasingly rare, because the nonlinear couplings between the modes corresponding to the different frequencies tend to destroy quasiperiodicity and replace it by chaos (see Ruelle and Takens [50]). Nonquasiperiodic systems are usually chaotic. Although their power spectra still may contain peaks, those are more or less broadened (they are no longer instrumentally sharp). Furthermore, a noisy background of broadband spectrum is present. In general, power spectra are very good for the visualization of periodic and quasiperiodic phenomena and their separation from chaotic time evolutions.

Here we address the application of power spectrum analysis in the identification, description, and characterization of nonperiodic flame dynamics. In mid-1980's four nonsteady modes of propagation were observed on burner-stabilized laminar premixed flames [51,52,53], which were the axial mode, the radial mode, the spiral mode, and the "drumhead" mode. The stability boundaries for the occurrence of these modes were given experimentally by et-Hamdi *et al.* [53]. Later, et-Hamdi, Gorman,and Robbin [21] constructed a classification scheme based on an analysis of power spectra to identify and classify chaotic dynamics of the premixed flame. Here we outlined some interesting results of their experiments.

In pulsating flames chaotic dynamics can occur when a periodic mode is forced toward the extinction boundary. The power spectrum of a periodic radial mode as compared with that of a radial-extinction mode. Chaotic dynamics is indicated by the broad fall-off in the power spectrum. The sharp peaks corresponding to the dominant radial oscillation are also present.

Chaotic dynamics can also occur in the transition region between two periodic modes. In the axial mode the flame front pulsates along the burner axis. In the spiral mode the most intense region of the flame front moves in a complicated spiral path. As the experimental parameters are varied, these modes can interact. In the power spectra taken at representative points in the axial-radial transition only sharp peaks were observed corresponding to the axial mode, the radial mode, their combinations and harmonics. While in the power spectra taken at representative points in the axial-spiral transition, the sharp peaks associated with each of the modes sit on top of a broad fall-off.

Chaotic dynamics is also found in cellular flames. The power spectrum of the unsteady cellular flame has a single broad component, extending from zero and decreasing monotonically to around 50 Hz.

A principal difficulty in experimental nonlinear science is the distinction between nonperiodic dynamics caused by deterministic processes (chaos) and that caused by stochastic processes (noise). Upon the above characteristics of power spectra in different unsteady modes this distinction is usually framed by the question, 'Is it chaos or is it noise?' and many tests have been put forward to make this distinction, usually by computing quantities of interest in nonlinear science such as the Lyapunov exponent or the fractal dimension of the chaotic attractor. However, Osborne and Provenzale [54] have pointed out the inability of the fractal dimension to conclusively distinguish between stochastic and deterministic dynamics. In addition, the computation of these quantities often require large accurate data sets which are difficult to obtain in combustion experiments because of variations in the experimental parameters. Sigeti and Horsthemke[55] have argued that power spectra can be used to make the distinction between deterministic and stochastic systems. They argued that the high frequency fall-off of the power spectrum would be exponential for deterministic processes and a power-law for stochastic processes. According to this argument, el-Hamdi *et al.* [21] proved that power spectra can be used to classify the nonperiodic dynamics of the flame front by a comparison of log-log and semi-log plots of the high frequency fall-off of the power spectrum. They also argued that the analysis of Sigeti and Horsthemke is incomplete and proposed that there are at least three classifications: systems with an exponential fall-off, systems with a power-law fall-off, and systems in which both exponential and power-law fall-offs give equally good fits.

From the above review of the power spectrum analysis in the identification of the chaotic dynamics in premixed flames, it can be concluded that for systems whose dynamics is difficult to investigate by theoretical model, power spectrum analysis can play an important role. However, it should be pointed out that the analysis of the chaotic motions themselves does not benefit much from the power spectra, because (being squares of absolute values) they lose phase information, which is essential for the understanding of what happens on a strange attractor.

Fractal in Fire Dynamics

A contemporary development with the emergence of chaotic dynamics has been the growing realization of the fractal nature of the world around us. Intuitively, the dimension of a space is the minimal number of coordinates needed to specify in it the location of a point. Geometrical objects with integer dimensions, such as zero-dimensional points, one-dimensional lines, two-dimensional surfaces, three-dimensional bodies and four-dimensional space-time are familiar notions. These topological dimensions do not change under continuous deformation of the objects. The characterization of chaotic attractors calls for fractal, i.e., not necessarily integer, dimensions, introduced by mathematicians at the beginning of this century, but made a regular tool of science only during the past two decades. Credit must be given to B. B. Mandelbrot who coined the term fractal and did a good job of popularizing the idea of fractal geometry.

Theory of fractal has been used widely in the understanding of fire dynamics. By means of the concept of fractal, marked progress has been achieved in modeling the premixed turbulent flame, and here we review the corresponding results.

Experimental evidence showed that reaction in premixed turbulent flames at moderate turbulence levels is confined to thin sheets usually referred to as flamelets [59]. Depending on turbulence conditions the flamelets may form a continuous sheet, several sheets, or may be fragmented into many separate sheets. Analyses for premixed laminar flames have shown that at low to moderate levels of turbulence the combustion rate per unit flamelet surface area is approximately that of an unstrained laminar flame, i.e., $\rho_0 u_0$, the product of the reactant gas density and the unstrained laminar flame speed. In this circumstance, only the knowledge of flamelet surface area is needed to determine combustion rates.

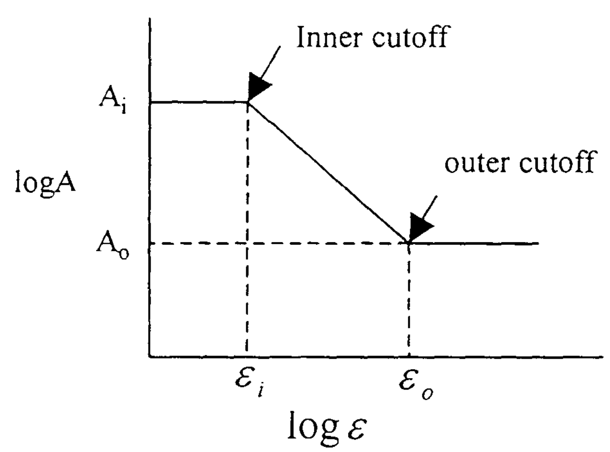

Figure 2. Postulated variation of measured surface area in the presence of inner and outer cutoffs

According to the definition of fractal dimension, the measured surface area $A(\varepsilon)$ is given by

$$A(\varepsilon) \propto \varepsilon^{2-D} \tag{11}$$

The range of scales over which the power laws of the type Eq. (11) holds is bounded by cutoffs on both ends imposed by physical limits (Fig.2). An appropriate length for this minimum scale or inner cutoff would appear to be the Kolmogorov scale, η, if the Prandtl or Schmidt number is one. Similarly, if there is a maximum length scale of surface wrinkling, say the turbulence integral scale l, which is less than the size of the underlying space, one would expect A to be independent of ε for ε greater than this maximum scale.

The generally accepted theory of the effect of turbulence on premixed flame structure and flame speed was introduced by Damkohler. He theorized that eddies larger than the flame thickness affect flame structure by convective distortion of the flame front, while eddies smaller than the flame thick-ness increase the heat and mass transport rates within the flame front and thereby alter the local laminar flame speed. Many practical combustion systems, however, operate in a combustion regime where the small-scale effects can be neglected. This regime of combustion is referred to as the reaction sheet regime, and is characterized by ratios of the Kolmogorov length scale to the flame thickness $(\eta/\delta_L)>1$ and ratios of the characteristic flow time scale to chemical time scale, i.e. the Damkohler number $\mathrm{Da}_L \gg 1$. Under these conditions and in the absence of significant flame stretch, the ratio of the turbulent to laminar flame velocity should be proportional to the ratio of the instantaneous lame surface area of the turbulent flame to the flow cross section area, i.e.,

$$\frac{u_t}{u_l} \approx \frac{A_t}{A_0} \tag{12}$$

where u_t and u_l are the turbulent and laminar flame velocities, respectively; A_t is the area of the turbulent flame, and A_0 is the flow cross section. Then the fractal geometry yields the following relationship [23]

$$\frac{A_t}{A_0} = \frac{A_i}{A_o} = \left(\frac{\varepsilon_o}{\varepsilon_i}\right)^{D-2} \tag{13}$$

For nonreacting turbulent flows, Mandelbrot [60] proposed a fractal dimension of 2(2/3) for Gauss-Kolmogorov turbulence and 2(1/2) for Gauss-Bergers turbulence.

Started with (13) and in terms of the following relation [61] for homogenous, isotropic turbulence

$$l/\eta = A_t^{1/4} R_l^{3/4} \tag{14}$$

where $R_l = u'l/\nu$, Gouldin [23] obtained a prediction for u_t under the assumption that flamelets propagate at the unstrained laminar flame speed, u_0:

$$u_t/u_0 = \{[1-(1-A_t^{-1/4}R_l^{-3/4})\exp(-(A_t/R_l)^{1/4}\,u'/u_0)]A_t^{1/4}R_l^{3/4}\}^{D-2} \tag{15}$$

where D is the fractal dimension and has a value of 2.32 to 2.4 as inferred from experiment [62] and by analysis [63,64]. A_t=0.37, based on turbulent pipe flow data.

In the analysis of Gouldin, the flame surface was assumed to behave largely as a passive scalar surface at high Reynolds numbers. However, Kerstein [65] rejected the passive scalar assumption. Without this assumption and with the help of dimension analysis he derived directly from existed relations that the fractal dimension of turbulent premixed flames is 7/3.

A freely propagating premixed turbulent flame was examined over a range of turbulent Reynolds numbers from 52 to 1431 and Damkohler numbers from 10 to 889 [66]. This investigation emphasizes the effects of changing turbulence intensity and laminar flame speed on the flame front fractal dimension. The fractal dimension was found to increase with u'/u_L, from 2.13 at u'/u_L of 0.25 to 2.32 at of 11.9. These results were found to agree well with results obtained in other studies using different flame configurations. The inner cutoff was not observed due to experimental limitations, however, the Gibson scale and an expression for the inner cutoff proposed by Gouldin were both found to over predict the inner cutoff at low values of u'/u_L.

A heuristic relationship was formulated which predicts the observed change in flame surface fractal dimension as a function of the turbulence intensity to laminar flame speed ratio. The model assumes that at high turbulence intensity, the effect of turbulent diffusion dominates. Whereas at low turbulence intensities, the process of flame propagation dominates, smoothing the flame surface. At intermediate turbulence intensities, the flame structure is determined by the competition between these two processes which can be represented by the expression,

$$D_F = \frac{D_L}{\frac{u'}{u_L}+1} + \frac{D_T}{\frac{u_L}{u'}+1} \tag{16}$$

Using the heuristic fractal dimension relationship and the Kolmogorov and integral scales as inner and outer cutoffs, respectively, a fractal turbulent flame speed model is obtained which depends on u', u_L, and l.

Another analysis was finished by Gulder [67] who adopted a variable inner cutoff scale as a function of turbulent and molecular diffusivities. With a fractal dimension of 7/3 and an outer cutoff approximating the integral length scale of turbulence, the analysis yielded

$$u_t/u_L \propto (u'/u_L)^{1/2}\, Re_L^{1/4} \tag{17}$$

This model suggested a wrinkled flame structure influenced by a turbulent length scale as well as the turbulence intensity.

For the turbulent diffusional flame surface, a one-dimensional modeling was presented by Zou [69] by means of fractal method, and the relationship between the surface fractal dimension and the flame propagation speed was obtained.

From the above review, it can be seen that fractal geometry description of turbulent flames is a promising means of treating complex problems in combustion dynamics.

OUTLOOK

Our review has led from the introduction of basic nonlinear dynamical theory to applications of the theory in fire dynamics. In this review we have tried to introduce nonlinear methods by presenting those topics and examples that seem to have risen to the very important fare in fire dynamics. However, for reasons of length choices must be made and the topics selected certainly reflect our own preferences. However, we have tried to represent what most fire researchers seem to feel are the important topics. The purpose of this review is to make nonlinear science accessible to a large number of fire scientists. The results presented here are the combined achievement of many investigators, only incompletely cited. For several other recent development of nonlinear fire dynamics, see [70~76]. We believe that the advanced nonlinear theory such as methods of chaos control [77,78] should lead to a better understanding of the complicated fire dynamics. We hope that the present review serves as an encouragement for the understanding of the nonlinear fire behavior in a better "nonlinear" way.

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REFERENCES

1. Lakshmikantham, V., Matrosov, V. M., and Sivasundaram, S., Vector Lyapunov Functions and Stability Analysis of Nonlinear Systems, Kluwer Academic Publishers, Dordrecht, 1991.
2. Bishop, S. R. et al., Dynamic Modeling of Building Fires, Appl. Math. Modeling, 17, 170, 1993.
3. Sivashinsky, G. I., Instabilities, Pattern Formation, and Turbulence in Flames, Ann. Rev. Fluid Mech., 15, 179, 1983.
4. Koylu, U. O. et al., Fractal and Projected Structure Properties of Soot Aggregates, Combust. Flame, 100, 621, 1995.
5. Sanchez, A. L. et al., Relationships Between Bifurcation and Numerical Analyses for Ignition of Hydrogen-Air Diffusion Flames, Combus. Flame, 105, 569, 1996.
6. Margolis, S. B. and Matkowsky B. J., Nonlinear Stability and Bifurcation in the Transition from Laminar to Turbulent Flame Propagation, Combust. Sci. and Tech., 34, 45, 1983.
7. Bayliss, A. and Matkowsky B. J., Two Routes to Chaos in Condensed Phase Combustion, SIAM J. Appl. Math., 50, 437, 1990.
8. Chow, S. N. and Hale, J. K., Methods of Bifurcation Theory, Springer-Verlag, 1982.
9. Keller, J. B. and Antman, S. (Eds.), Bifurcation Theory and Nonlinear Eigenvalue Problems, W. A. Benjamin, 1969.
10. Arnold, V. I., Catastrophe Theory, Springer-Verlag, 1984.
11. Thom, R., Structural Stability, catastrophe Theory and Applied Mathematcs, SIAM Rev., 1977.
12. Gilmore, R., Catastrophe Theory for scientists and Engineers, John Wiley & Sons, 1981.
13. Hao, B. L., H., Elementary Symbolic Dynamics and Chaos in Dissipative Systems, World Scientific, Singapore, 1989.
14. Ruelle, D. and Takens, F., On the Nature of Turbulence, Commun. Math. Phys., 20, 167, 1971.
15. Parker, T. S. and Chua, L. O., Practical Numerical Algorithms for Chaotic Systems, Springer-Verlag, New York, 1989.
16. Rasband, S. N., Chaotic Dynamics of Nonlinear Systems, John Wiley & Sons, New York, 1990.
17. Jackson, E. A., Perspectives of Nonlinear Dynamics, Cambridge University Press, New York, 1990.
18. Eckmann, J. P. et al., Liapunov Exponents from Time Series, Phys. Rev. A, 34, 4971, 1986.
19. Sano, M. and Sawada, Y., Measurement of the Lyapunov Spectrum from a Chaotic Time Series, Phys. Rev. Lett., 55, 1082, 1985.
20. Bryant, P., Brown, R. and Abarbanel, H. D. I., Lyapunov Exponents from Observed Time Series, Phys. Rev. Lett., 65, 1523, 1990.
21. el-Hamdi, M., Gorman, M. and Robbins, K. A., Deterministic Chaos in Laminar Premixed Flames: Experimental Classification of Chaotic Dynamics, Combust. Sci. and Tech., 94, 87, 1993.
22. Mandelbrot, B. B., The Fractal Geometry of Nature, Freeman, New York, 1983.

23. Gouldin, F. C., An Application of Fractals to Modeling Premixed Turbulent Flames. Combust. Flame, 68, 249,1987.
24. Thomas, P. H. et al., Flashover and Instabilities in Fire Behavior, Combust. Flame. 38, 159, 1980.
25. Bishop, S. R. et al., Nonlinear Dynamics of Flashover in Compartment Fires, Fire Safety Journal, 21, 11,1993.
26. Foale, S. and Thompson, J. M. T., Geometrical Concepts and Computational Techniques of Nonlinear Dynamics, Comput. Methods Appl. Mech. Eng., 89, 381,1991.
27. Holborn, P. G. et al., Experimental and Theoretical Models of Flashover, Fire Safety Journal, 21, 257, 1993.
28. Graham, T. L., Makhviladze, G. M. and Roberts, J. P., On the Theory of Flashover Development, Fire Safety Journal, 25, 229,1995.
29. Williams, F. A., Combustion Theory, Addison-Wesley, 1965.
30. Kanury, A. M., Introduction of Combustion Phenomena, Gordon & Breach, New York, 1975.
31. Glassman, I., Combustion, Academic, New York, 1977.
32. Smithells, S. and Ingle, K., The Structure and Chemistry of Flames, J. Chem. Soc., 61, 204, 1882.
33. Markstein, G. H., Cell Structure of Propane Flames Burning in Tubes, J. Chem. Phys., 17, 428, 1949.
34. Gololobov, I. M., Granovsky, E. A. and Gostintsev, Y. A., On Two Combustion Regimes at the Limit of Luminous Flame Propagation, Fiz. Goreniya Vzryva, 17, 28. 1981.
35. Landau, L. D., On the Theory of Slow Combustion, Acta Physicochim, URSS, 19, 77,1944.
36. Zeldovich, Y. B., Theory of Combustion and Detonation of Gases, Acad. Sci. USRR, 1944, (In Russian).
37. Sivashinsky, G. I., Diffusional-thermal Theory of Cellular Flames, Combust. Sci. and Tech., 15, 137, 1977.
38. Matkowsky, B. J. and Olagunju, D. O., Propagation of a Pulsating Flame Front in a Gaseous Combusitible Mixture, SIAM J. Appl. Math., 44, 344, 1984.
39. Barenblatt, G. I., Zeldovich, Y. B. and Istratov, A. G., On Diffusional-thermal Stability of a Laminar Flame, Zh. Prikl. Mekh. Tekh. Fiz., 4, 21, 1962.
40. Sivashinsky, G. I., Nonlinear Analysis of Hydrodynamic Instability in Laminar Flames, Part I. Derivation of Basic Equations, Acta Astronaut., 4, 1177,1977.
41. Manton, J., von Elbe, G. and Lewis, B., Nonisotropic Propagation of Combustion Waves in Explosive Gas Mixtures and the Development of Cellular Flames, J. Chem. Phys., 20, 153, 1952.
42. Markstein, G. H., Nonisotropic Propagation of Combustion Waves, J. Chem. Phys., 20, 1051, 1952.
43. Petersen, R. E. and Emmons, K. W., The Stability of Laminar Flames, Phys. Fluids, 4, 456, 1961.
44. Michelson, D. M. and Sivashinsky, G. I., Nonlinear Analysis of Hydrodynamic Insability in Laminar Flames, Part II, Numerical Experiments, Acta Astronaut., 4, 1207,1977.
45. Sabathier, F., Boyer, L. and Clavin, P., Experimental Stydy of a Weak Turbulent Premixed Flame, Prog. Astronaut. Aeronaut., 76, 246,1981.

46. Michelson, D. M. and Sivashinsky, G. I., Acceleration Effects on the Stability of Flame Propagation, SIAM J. Appl. Math., 37, 669, 1979.
47. Sivashinsky, G. I. and Matkowsky, B.J., On Stability of Nonadiabatic Flames, SIAM J. Appl. Math., 40, 255, 1981.
48. Law, C. K., Ishizuka, S. and Mizomoto, M., Lean-limit Extinction of Propane/Air Mixtures in the Stagnation-point Flow, 18th Symp. (Int.) Combust., p1791, The Combustion Institute, 1981.
49. Margolis, S. B. and Sivashinsky, G. I., Flame Propagation in Vertical Channels: Bifurcation to Bimodal Cellular Flames, SIAM J. Appl. Math., 44, 344, 1984.
50. Eckmann, J. P. and Ruelle, D., Ergodic Theory of Chaos and Strange Attractors, Rev. Mod.Phys., 57, 617, 1985.
51. Blackshear, J. I., Mapp, J. W. and Gorman, M., An Experimental Study of Pulsating Low-pressure Flames, Combust. Sci. Technol., 35, 311, 1984.
52. Mapp, J. W., Blackshear, J. I. And Gorman, M., Nonsteady, Nonplanar Modes of Propagation in Premixed Burner-Stabilized, Combust. Sci. Technol., 43, 217, 1985.
53. el-Hamdi, M. et al., Stability Boundaries of Periodic Modes of Propagation in Burner-Stabilized Methane-Air Flames, Combust. Sci. Technol., 55, 33, 1987.
54. Osborne, A. R. and Matkowsky, B. J., Finite Correlation Dimension for Stochastic Systems, Physica D, 35, 357, 1989.
55. Sigeti, D. and Horsthemke, W., High Frequency Power Spectra of Chaotic Systems, Phys. Rev., 35A, 2276, 1987.
56. Grassberger, P., Generalized Dimensions of Strange Attractors, Phys. Lett., A97,227,1983.
57. Hentschel, H. G. E. and Procaccia, I., The Infinite Number of Generalized Dimensions of Fractals and Strange Attractors, Physica D, 8, 435, 1983.
58. Ott, E., Chaos in Dynamical Systems, Cambridge University Press,1993.
59. Peters, N., Laminar Flamelet Concepts in Turbulent Combustion, 21st Symp. (Int.) on Combust., p1231, The Combustion Institute, 1986.
60. Mandelbrot, B. B., On the Geometry of Homogeneous Turbulence, With Stress on the Fractal Dimension of the Iso-Surfaces of Scalars, J. Fluid Mech., 72, 401, 1975.
61. Tennekes, H. and Lumley, J. L., A First Course in Turbulence, MIT Press, 1972.
62. Lovejoy, S., Area-Perimeter Relation for Rain and Cloud Areas, Science, 216, 185, 1982.
63. Hentschel, H. G. E. and Procaccia, I., Relative Diffusion in Turbulent Media: The Fracta Dimension of Clouds, Phys. Rev., A29, 1461, 1984.
64. Screenivasan, K. R. and Meneveau, C., The Fractal Facets of Turbulence, J. Fluid Mech., 173, 357, 1986.
65. Kerstein, A. R., Fractal Dimension of Turbulent Premixed Flames, Combust. Sci. tech. and Tech., 60, 441, 1988.
66. North, G. L. and Santavicca, D. A., The Fractal Nature of Premixed Turbulent Flames, Combust. Sci. and Tech., 72, 215, 1990.
67. Gulder, O. L., Turbulent Premixed Combustion Modelling Using Fractal Geometry, 23 Symp. (Int.) on Combust., p835, The Combustion Institute, 1990.
68. Xie, Z. and Fan, W., Nonlinear Dynamics Study on Mine Ventilation System, Proceedings of the Second Japan-China Bilateral Symposium on Safety and Environmental Engineering, Eds. Tamai, N. and Cheng, X., Japan, 1997.
69. Zou, Y., The Fractal Characterization of the Turbulent Diffuse Flame Surface, M. D. Thesis, China University of Science and Technology, Hefei, China, 1997.

70. Frankel, M. L., On the Nonlinear Dynamics of Flame Fronts in Condensed Combustible Matter, Phys. Lett., A140, 405, 1989.
71. Liberman, M. A. et al., Stability of a Planar Flame Front in the Slow-Combustion Regime, Phys. Rev. E, 49, 445, 1994.
72. Weeratunga, S., Buckmaster, J. and Johnson, R. E., A Flame-Bubble Analogue and Its Stability, Combust. Flame, 79, 100, 1990.
73. Bychkov, V. V. and Liberman, M. A., Stability and the Fractal Structure of a Spherical Flame in a Self-Similar Regime, Phys. Rev. Lett., 76, 2814, 1996.
74. Nicolleau, F. and Mathieu, J., Eddy Break-Up Model and Fractal Theory: Comparisons With Experiments, Int. J. Heat Mass Trans., 37, 2925, 1994.
75. Davis, M. R. and Li, H. L., Evaluation of Fractal Dimension for Mixing and Combustion By the Schlieren Method, Experiments in Fluids, 21, 248, 1996.
76. Pocheau, A. and Queiros-Conde, D., Transition From Euclidean to Fractal Forms Within a Scale-Covariant Process, Europhys. Lett., 35, 439, 1996.
77. William, L. D., Spano, M. L. and Lindner, J. F., Techniques for the Control of Chaos, Physica D, 86, 198, 1995.
78. Abarbanel, H. D. I. Et al., The Analysis of Observed Chaotic Data in Physical Systems, Rev. Mod. Phys., 65, 1331, 1993.