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Numerical Modeling of the Downward Flame Spread: The Effect of Opposed Forced Flow

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ABSTRACT

The process of downward flame spread over the thin bed of combustible material is studied numerically by solving two-dimensional steady-state conservation equations. The algorithm for the prediction of flame spread rate is based on the principle of minimal entropy production. The primary purpose of investigation is focused on the influence of opposed ambient flow (forced and buoyancy) on the flame spread behavior. Achieved results showed the appropriate evaluation, in comparison with experimental data, of the overall effect of the opposed flow and of the limit of stable flame propagation.

KEYWORDS: Opposed flow, downward flame spread, numerical modeling

NOMENCLATURE

C - Specific heat;
D - Diffusion coefficient;
E - Activation energy;
g - Gravity acceleration;
J - Thermodynamic flux;
k - Preexponential factor;
L_0 - Initial thickness of fuel bed;

n - Coordinate normal to the solid fuel's surface;
P - Entropy production;
p - Pressure;
Q - Effective heat of reaction;
R - Specific gas constant;
R_0 - Universal gas constant;
T - Temperature;
u, v - Velocity components;
u_f - Steady flame spread rate;
v_s - Linear pyrolysis rate;
W - Chemical reaction's rate;
X - Generalized force;
x_b - Burnout position;
Y - Mass fraction.

Greek
$\delta(x)$ - Thickness of solid fuel bed;
δ^* - Boundary layer thickness;
λ - Thermal conductivity;
μ - Viscosity;
ν - Stoichoimetric coefficient;
ξ - Reaction's coordinate;
ρ - Density.

Subscripts
a - Ambient;
b - Buoyancy;
F - Fuel;
O - Oxidizer;
s - Solid phase;
W - Chemical reaction;
X, Y - Thermal conduction in x and y - directions.

INTRODUCTION

By the effect of ambient free-stream flow on the flame spread process over the surface of combustible material, two modes are usually distinguished: opposed-flow and flow-assisted flame spread. First of them results the steady flame propagation within the reasonable interval between ignition and extinction. For the other mode, the existence of steady flame spread regime is not necessarily achieved, especially in the case of small-scale flames (accelerating upward flame propagation is a typical example). Since the physical background of the mathematical model presented below is the description of fundamentally stationary process, it has to be outlined clearly that unsteady flame spread is not considered here.

The development of theory of opposed-flow flame spread [1] (steady flame spread, by our approach) formulated in terms of self-contained partial differential equations of conservation laws which are able to describe the essence of flame propagation has started from the classical study of de Ris [2]. His model assumes a simplified description of flow dynamics in the gas phase (Oseen approximation) and, correspondingly, resulting formulas for flame spread rate show the simplified dependence upon the opposed flow velocity: independence of u_f upon u_a for the thermally thin fuel beds and linear increase of u_f as u_a increases for semi-infinite thickness of fuel bed. These results, while reasonable in some cases, can not predict the overall effect of opposed flow on the flame spread rate observed experimentally [3], especially concerning the extinction limit.

In the comprehensive numerical models of Di Blasi *et al* [4] and Bhattacharjee *et al* [5] two-dimensional Navier-Stokes equations are used in the gas phase which allow to analyze the effect of flow dynamics on flame spread process in details. In our previous study [6] the same gas-phase equations are employed but the approach to the determination of the flame spread rate is of essentially different physical nature. The steady flame spread is considered as a stationary state of non-equilibrium thermodynamic system, which is characterized by the minimal entropy production. The evaluation of this approach relatively to one-dimensional flame propagation over premixed gases [7] and two-dimensional diffusion flame spread over solid fuels [6,8] has shown the appropriate agreement with well-known basic features of flame propagation. However, the investigation of the effect of opposed forced flow performed in [8] was limited by the simplified definition of gas-phase flow dynamics. The present study is devoted to the investigation of the effect of ambient free-stream flow on the downward flame spread on the base of two-dimensional gas-phase momentum equations describing the viscous motion of reacting gas.

FORMULATION

The model of two-dimensional downward flame spread over the solid combustible is shown in Figure 1. The problem is formulated in the steady-state coordinate system fixed on the moving flame front. The governing equations for conservation of momentum, energy and mass in the gas phase are

$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=\frac{\partial}{\partial x}\mu\frac{\partial u}{\partial x}+\frac{\partial}{\partial y}\mu\frac{\partial u}{\partial y}-\frac{\partial p}{\partial x}+(\rho_a-\rho)g \quad (1)$$

$$\rho u\frac{\partial v}{\partial x}+\rho v\frac{\partial v}{\partial y}=\frac{\partial}{\partial x}\mu\frac{\partial v}{\partial x}+\frac{\partial}{\partial y}\mu\frac{\partial v}{\partial y}-\frac{\partial p}{\partial y} \quad (2)$$

$$\frac{\partial \rho u}{\partial x}+\frac{\partial \rho v}{\partial y}=0 \quad (3)$$

$$C\rho u\frac{\partial T}{\partial x}+\rho v\frac{\partial T}{\partial y}=\frac{\partial}{\partial x}\lambda\frac{\partial T}{\partial x}+\frac{\partial}{\partial y}\lambda\frac{\partial T}{\partial y}+Q\rho W \quad (4)$$

$$\rho u\frac{\partial Y_O}{\partial x}+\rho v\frac{\partial Y_O}{\partial y}=\frac{\partial}{\partial x}\rho D\frac{\partial Y_O}{\partial x}+\frac{\partial}{\partial y}\rho D\frac{\partial Y_O}{\partial y}-\nu_O\rho W \quad (5)$$

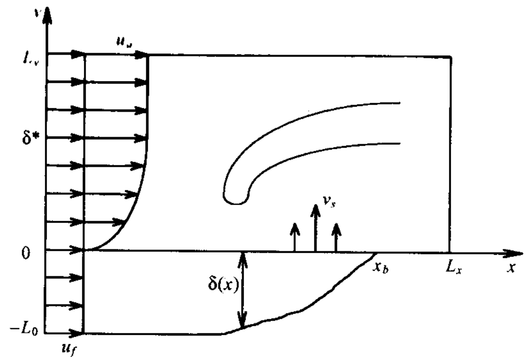

FIGURE 1. Flame spread model.

$$\rho u\frac{\partial Y_F}{\partial x}+\rho v\frac{\partial Y_F}{\partial y}=\frac{\partial}{\partial x}\rho D\frac{\partial Y_F}{\partial x}+\frac{\partial}{\partial y}\rho D\frac{\partial Y_F}{\partial y}-\nu_F\rho W \tag{6}$$

$$\rho=p/RT\,. \tag{7}$$

The balance of energy in the solid fuel is expressed as

$$C_s\rho_s u_f\frac{\partial T_s}{\partial x}=\frac{\partial}{\partial x}\lambda_s\frac{\partial T_s}{\partial x}+\frac{\partial}{\partial y}\lambda_s\frac{\partial T_s}{\partial y}+Q_s\rho_s W_s \tag{8}$$

The gas-phase combustion reaction and solid fuel pyrolysis both are described by Arrhenius-type formulas:

$$W=kY_OY_F\exp(-E/R_0T) \tag{9}$$

$$W_s=k_s\exp(-E_s/R_0T_s) \tag{10}$$

Boundary conditions, correspondingly to the configuration shown in Figure 1, are

$$y=0\,(0<x<x_b):\ u=u_f \tag{11}$$

$$\rho v=\rho_s v_s \tag{12}$$

$$-\lambda\frac{\partial T}{\partial y}+\rho vCT=-\lambda_s\frac{\partial T_s}{\partial y}+\rho_s v_s C_s T_s \tag{13}$$

$$T=T_s \tag{14}$$

$$-\rho D\frac{\partial Y_O}{\partial y}+\rho vY_O=0 \tag{15}$$

$$-\rho D\frac{\partial Y_F}{\partial y}+\rho vY_F=\rho_s v_s \tag{16}$$

$$y=0\,(x>x_b):\quad \rho v=0,\ \frac{\partial T}{\partial y}=0,\ \frac{\partial u}{\partial y}=0 \tag{17}$$

$$x=0:\quad u=\begin{cases}u_f+u_a\left[\frac{3}{2}\left(\frac{y}{\delta^*}\right)-\frac{1}{2}\left(\frac{y}{\delta^*}\right)^3\right], & \text{for } y<\delta^*\\ u_f+u_a, & \text{for } y>\delta^*\end{cases} \tag{18}$$

$$v=0,\ T=T_a,\ Y_F=0,\ Y_O=Y_{O.a} \tag{19}$$

$$y=L_y:\quad u=u_f+u_a,\ \frac{\partial v}{\partial y}=0,\ T=T_a,\ Y_F=0,\ Y_O=Y_{O.a} \tag{20}$$

$$x=L_x:\quad \frac{\partial u}{\partial x}=0,\ \frac{\partial v}{\partial x}=0,\ \frac{\partial T}{\partial x}=0,\ \frac{\partial Y_F}{\partial x}=0,\ \frac{\partial Y_O}{\partial x}=0 \tag{21}$$

$$y=-\delta(x):\quad \frac{\partial T_s}{\partial n}=0 \tag{22}$$

Here boundary condition (18) is introduced to define the structure of input forced flow [9] since the boundary $x=0$ does not coincide with the edge of fuel bed sample.

The mathematical statement defined by (1)-(8) provides eight equations for eight variables which are $u, v, p, T, Y_O, Y_F, \rho, T_s$. Besides them, conservation equations and boundary conditions contain three additional unknown parameters: linear pyrolysis rate v_s, burning surface profile $\delta(x)$ and flame spread rate u_f which are the eigenvalues of the problem. Therefore, they should be determined through the other variables as a result of the problem's solution. The expressions for the first two of them have been defined through the integration of the mass balance equation in the solid fuel, which results the following [6]

$$v_s=\int_{-\delta(x)}^{0}W_s\,dy \tag{23}$$

$$\delta(x)=L_0-\frac{1}{u_f}\int_0^x v_s\,dx \tag{24}$$

Determination of the flame spread rate u_f is the corner-stone of every flame spread theory which uses the steady-state conservation equations. A number of studies [1] have been reported which were analyzed [8] relatively to the approach involved to outline the algorithm for the determination of flame spread rate. In our previous studies [6, 8] the principle of minimal entropy production has been employed to specify the eigenvalue problem of mathematical formulation described above.

The summary entropy production within the solid fuel considered as non-equilibrium thermodynamic system is expressed in the following form [6]

$$P = \int_{0}^{\infty}\int_{-\delta}^{0} (J_X X_X + J_Y X_Y + J_W X_W) dy dx \tag{25}$$

where thermodynamic fluxes and generalized forces due to the thermal conduction and chemical reaction are

$$J_X = -\lambda_s \frac{\partial T_s}{\partial x}, \quad X_X = -\frac{1}{T_s^2}\frac{\partial T_s}{\partial x} \tag{26}$$

$$J_Y = -\lambda_s \frac{\partial T_s}{\partial y}, \quad X_Y = -\frac{1}{T_s^2}\frac{\partial T_s}{\partial y}. \tag{27}$$

$$J_W = \rho_s W_s, \quad X_W = Q_s\left(\frac{1}{T_s} - \frac{1}{T_s^*}\right) - R\ln\xi \tag{28}$$

Here T_s^* is the maximal temperature in the solid fuel and reaction's coordinate ξ of solid fuel's pyrolysis is described correspondingly to fuel bed thickness as

$$\xi(x) = 1 - \delta(x)/L_0 \tag{29}$$

Finally, the algorithm for the prediction of flame spread rate is based on the principle of minimal entropy production. Among the possible solutions corresponding to every assigned value of flame spread rate u_f, the stationary one is that which provides the minimum of integral (25) and corresponding value of u_f is presumed to be the searched steady flame spread rate.

The numerical solution of the problem statement defined above is obtained by combined finite difference (for gas phase) – finite element (for solid fuel) procedure described in [6].

RESULTS AND DISCUSSION

The prediction of steady rate of downward flame spread over the thin sheet of paper has been carried out here. The gas phase and solid fuel properties used in the calculations are listed in Table 1. These values have been proposed by Frey and T'ien [10] except of the preexponential factor of chemical reaction of combustion in the gas phase which has been selected [6] for better correlation with the experimental data. Diffusion coefficient is expressed as $\rho D = \mathrm{Le}\lambda / C$ and viscosity as $\mu = \mathrm{Pr}\lambda / C$, where Lewis number $\mathrm{Le} = 1$ and Prandtl number $\mathrm{Pr} = 1$.

Figure 2 presents the results of calculation of entropy production under the various values of opposed flow velocities for $Y_{O,a} = 0.21$. The point of minimum (triangle symbol) indicating the searched flame spread rate moves slightly to the smaller values as opposed flow velocity increases. As the u_a increases the minimum point itself becomes marked more weakly.

TABLE 1. Gas phase and solid fuel properties.

Symbol	Gas	Solid	Unit
C	1005.6	1257	J/(kg K)
λ	0.0254	0,1257	W/(m K)
ρ	1.0	650	kg/m^3
p	10^5	–	Pa
Q	1.68×10^7	-7.54×10^5	J/kg
k	5×10^8	10^{10}	1/s
E	62850	125700	J/mol
ν_F	1.0	–	
ν_O	1.185	–	
T_a	300	300	K
L_0	–	0.095	mm

Under some value of ambient velocity ($u_a > 0.7$ m/s for $Y_{O,a} = 0.21$ as curves 4 and 5 of Fig.2 show) the minimum point vanishes and monotonous character of the distribution of entropy production appears. The same dependence is observed for higher ambient oxidizer mass fraction ($Y_{O,a} = 0.30$) which is shown in Fig.3. In this case the minimum point on the distribution of entropy production corresponding to extinction limit (curve 5 of Fig.3) becomes even more sharp. For the higher values of u_a the minimum point also vanishes.

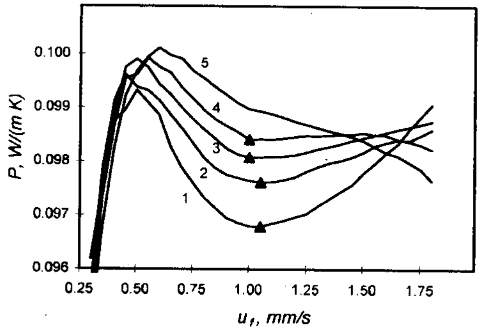

FIGURE 2. Dependence of entropy production distribution versus flame spread rate upon ambient opposed flow velocity. $Y_{O,a} = 0.21$.

Curves: 1- $u_a = 0$, 2- $u_a = 0.15$ m/s, 3- $u_a = 0.3$ m/s, 4- $u_a = 0.7$ m/s, 5- $u_a = 1.0$ m/s.

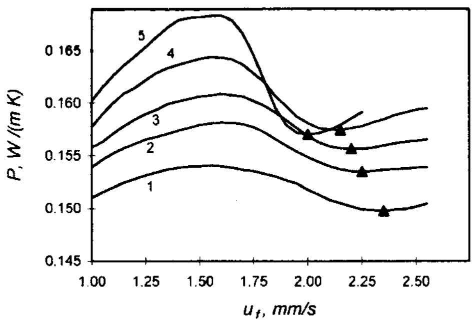

FIGURE 3. Dependence of entropy production distribution versus flame spread rate upon ambient opposed flow velocity. $Y_{O,a} = 0.30$.

Curves: 1- $u_a = 0$, 2- $u_a = 0.2$ m/s, 3- $u_a = 0.4$ m/s, 4- $u_a = 0.8$ m/s, 5- $u_a = 2.2$ m/s.

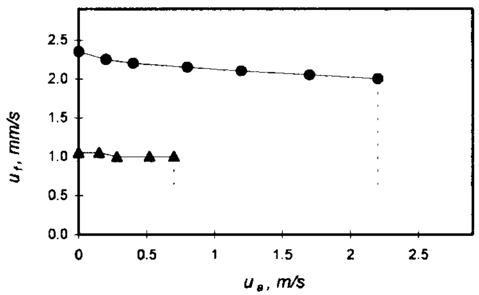

FIGURE 4. Dependence of flame spread rate upon the ambient flow velocity.
Curves: Δ - $Y_{O,a} = 0.21$, O - $Y_{O,a} = 0.30$.

The overall dependence of flame spread rate upon the ambient opposed flow is shown in Fig.4. Comparing these data with the well-known results of measurements [3,11] a satisfactory agreement may be concluded.

However, last conclusion may be addressed properly to the range of opposed flow velocity before the extinction limit shown in Fig.4. Beyond this limit the proposed approach fails to calculate the steady flame spread rate since there is no minimum point on the distribution of entropy production as shown in Fig.2. This feature relates to fundamental physical difference

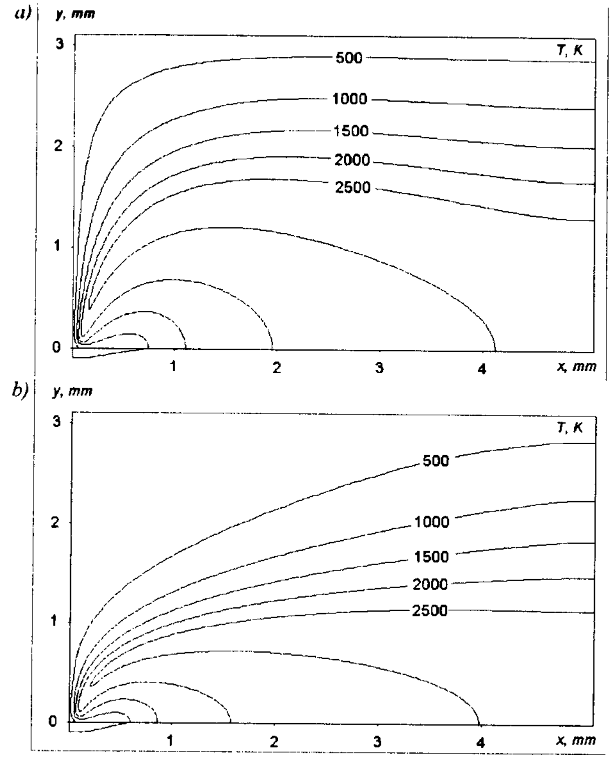

FIGURE 5. Distribution of temperature field in the flame zone.
(a) - $u_a = 0$, (b) - $u_a = 0.1$ m/s.

between 'stationary' and 'steady' flame propagation. The experimental investigation of Hirano *et al* [11] showed the existence of two regimes of downward flame spread in opposed forced flow. Unless u_a reaches some value the stable regime of a self-similar process of flame propagation is observed. Above this stable limit of u_a the flame spread process becomes *locally* unstable. Actually the flame is able to propagate along the surface but spread rate itself is determined as a time-averaged value. Therefore, such process is essentially unsteady and can not be considered as a stationary thermodynamic state of the system and flame spread rate can not be predicted by our approach beyond the stable regime of flame spread.

Another aspect of the influence of ambient flow on the flame spread rate relates to the effect of natural convection occurring around the flame zone. The distribution of temperature field in the flame zone in the absence of ambient flow shown in Fig.5a would be considered as somewhat unrealistic comparing with the experimental data (*e.g.* [12]). Actually, in the case of downward flame spread the buoyancy flow is formed ahead of flame leading edge due to the temperature difference between flame zone and surrounding. The order of magnitude of such flow's velocity can be estimated [9] as $u_b \propto [gl(T_f - T_a)/T_f]^{1/2}$, where characteristic length $l \propto \lambda / C\rho u_b$, which results the following [13]

$$u_b \propto B\left(\frac{g\lambda}{\rho_a C}\frac{T_f - T_a}{T_a}\right)^{1/3} \qquad (30)$$

Here T_f is the temperature in flame zone and B is a constant. Figure 5b presents the distribution of temperature field under the input opposed flow of some characteristic buoyancy velocity. In this case temperature field becomes much more realistic.

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