CATASTROPHIC BEHAVIOUR OF BACKDRAFT IN COMPARTMENT FIRES

W.G. Weng and W.C. Fan

State Key Laboratory of Fire Science

CHINA

ABSTRACT

Backdraft is a fire phenomenon in which the hot gasses formed from combustion at low oxygen levels rapidly combust due to abruptly supplying abundant fresh air. This transient phenomenon is an example of typical catastrophe behaviour. In this paper, a simplified mathematical model of backdraft in compartment fires is established based on an energy balance equation. Its catastrophe manifold function equation based on catastrophe theory was created, and the corresponding relationship between system control variables and fire operating conditions was studied. It is indicated that the catastrophe form of backdraft is a swallowtail catastrophe, and whether backdraft takes place is determined based on its bifurcation set.

Keywords: Backdraft, catastrophe, swallowtail catastrophe.

INTRODUCTION

Fire is a very complex phenomenon involving combustion, heat transfer and fluid mechanics. Some fire phenomena such as backdraft in a compartment are even more complicated as the heat and mass produced are confined in the enclosure[1]. Backdraft can develop from fires of either ordinary combustibles or ignitable liquids that become oxygen starved yet continue to generate a fuel-rich environment. If the fresh air is allowed to flow into the vitiated space, such as by opening a door or breaking window glass, a gravity current of colder air will flow into the space while the hot fuel-rich gasses flow out through the top of the opening. The air and fuel-rich gasses will mix along the interface of the two flow streams. Once a localised flammable mixture is formed and is in contact with an ignition source, the fuel-rich gasses will combust acutely, the temperature will rise rapidly and the initial fire stage will develop into flashover or deflagration. The deflagration will cause the gasses to heat and expand within the fire space, thus forcing unburned gasses out of the open vent ahead of the flame front. These gasses will mix with additional air outside of the fire space. As the flame traverses the room and penetrates the doorway, it ignites the gasses outside the space resulting in a fireball and a blast wave. This phenomenon is termed backdraft[2-4]. Such transition phenomena are suggestive of a catastrophe process. Therefore, it is possible to analyse this catastrophe behaviour of backdraft in compartment fires and create a catastrophe manifold function equation based on catastrophe theory.

Catastrophe theory is concerned with discontinuities. It is a special form of bifurcation theory in which smooth incremental changes in system control variables can result in "jump" between stable and unstable behaviours[5-6]. Thom[7] suggested a number of standard or basic forms of catastrophes and each of these had a manifold or hypersurface that defined the equilibrium points for the system. Hence it is possible to manipulate the system equations into the standard form of one of these catastrophes. Catastrophe theory has been applied to the stability analysis of ships, airplanes and structural systems[8]. In this paper, catastrophe theory is applied to analyse backdraft in compartment fires.

Even though backdraft is known to be related to hot smoke components, temperature, and environment conditions, there are currently no sound mathematical models. Therefore, the study on backdraft is deemed as one of the difficulties of fire science[9]. The present study methods of backdraft are restricted to experiments[3]. However, due to its suddenness, it is difficult to understand the physical mechanism using present experimental techniques. Armed with a catastrophe theory, a mathematical model for backdraft can be investigated with a view to verifying the predictive power. Large fire simulation models can then built on this sound framework of knowledge of fundamental physical events, allowing the model to evolve and to incorporate finer details of structural form. Additionally, the model can be used to determine the methods of escaping the occurrence of backdraft without expensive experimental studies.

The simplified mathematical model of backdraft in compartment fires based on an energy balance equation and its catastrophe manifold function equation (based on catastrophe theory) will be presented in the following section. The corresponding relationship between system control variables and operating conditions will be discussed. Conclusions and future work are then presented.

Simplified mathematical model of backdraft

Vitiated compartment fires may develop in two situations as shown in Figure 1. The development of the first situation is 1-2-3-4-5-6, i.e. ignition, free combustion, combustion with lack of oxygen or smouldering, backdraft, developed combustion and decay. The development of another situation is 1-2-3-7, i.e. ignition, free combustion, combustion with lack of oxygen or smouldering and decay. Backdraft in compartment fires can take place only in the first situation. There are three key time-variables to differentiate these stages. t_c is the transform time from combustion to combustion with lack of oxygen or smouldering, t_o is the time of supplying oxygen, and t_d is the transform time from developed combustion to decay.

In order to establish the mathematical model of backdraft in compartment fires, the following assumptions are introduced:

- The compartment can be divided into two zones, which may be represented by average temperatures.
- The wall surfaces surrounding the lower zone and the corresponding wall surfaces below the thermal discontinuity are assumed to be at an initial temperature.

- The flame in compartment at t_o is entirely extinct.
- The hot gas and the colder air in the compartment at t_o is assumed to form steady gravity currents below the thermal discontinuity, i.e. the reaction zone of the hot gas and the colder air is only below the thermal discontinuity.

The equation of energy conversation takes the form of a heat balance:

$$mc_p \frac{dT}{dt} = G - L \tag{1}$$

where m is the total mass in the hot layer in the compartment, c_p is its specific heat capacity (at constant pressure), T is the temperature of the hot zone, t is time, G is the rate of heat gain of the hot zone in the compartment, and L is the rate of heat loss from the hot zone in the compartment.

$$G = C_0^n QDA_V lk_0 \exp(-\frac{E}{RT}) \tag{2}$$

where C_0 is the concentration of combustible mixture, n is the reaction order, Q is the heat of combustion, D is the fractional height of the thermal discontinuity plane, A_v is the area of the vent, l is the length of reaction zone, k_0 is pre-exponential factor, E is the activation energy, and R is the gas constant.

$$L = \dot{m}_{out} c_p (T - T_0)(1-D) + [A_U - (1-D)A_V]h_c(T - T_W) + (1-D)A_V h_V (T - T_0)$$

$$+ \alpha_g \sigma [A_U - (1-D)A_V](T^4 - T_W^4) + \alpha_g \sigma [A_L + (1-D)A_V](T^4 - T_0^4) \tag{3a}$$

$$\dot{m}_{out} = \frac{2}{3} c_d \rho_0 A_V \sqrt{2g(1-D)H_V \frac{T_0}{T}(1 - \frac{T_0}{T})} \tag{3b}$$

where $\dot{m}_{out}$ is the total mass flow of gas from the opening, c_d is the discharge coefficient, ρ_0 is the initial density, g is the acceleration due to gravity, H_v is the height of the vent. A_u is the surface area of wall surrounding the gas layer, h_c is the convective heat transfer coefficient for the hot wall surface, T_w is the surface temperature of the walls surrounding the hot zone, h_v is the convective heat transfer constant for the vent, α_g is the emissivity of the gas layer, and A_L is the surface area of walls surrounding the lower zone.

Introducing the dimensionless transform and assuming[10] $\frac{(T - T_0)^{3/2}}{TT_0^{1/2}} \approx \frac{T - T_0}{2T_0}$ yields:

$$\theta = \frac{T - T_0}{T_0 \varepsilon}, \ \varepsilon = \frac{RT_0}{E}, \ \tau = \frac{t}{t_*}, \ t_* = \frac{mc_p \varepsilon}{C_0^n QDA_V lk_0 \exp(-\frac{E}{RT_0})}, \ \beta = \frac{\theta_W}{\theta}$$

$$\varphi_{out} = \frac{t_{out}}{t_*},\ t_{out} = \frac{mc_p}{\dot{m}'_{out} A_V c_p (1-D)},\ \varphi_{c,H} = \frac{t_{c,H}}{t_*},\ t_{c,H} = \frac{mc_p}{[A_U - (1-D)A_V]h_c},$$

$$\varphi_{c,L} = \frac{t_{c,L}}{t_*},\quad t_{c,L} = \frac{mc_p}{(1-D)A_V h_V},\quad \varphi_{R,L} = \frac{t_{R,L}}{t_*},\quad t_{R,L} = \frac{mc_p}{\alpha_g \sigma [A_U - (1-D)A_V] T_0^3},$$

$$\varphi_{R,W} = \frac{t_{R,W}}{t_*},\ t_{R,W} = \frac{mc_p}{\alpha_g \sigma [A_L + (1-D)A_V] T_0^3},\ \dot{m}'_{out} = \frac{1}{3} c_d \rho_0 \sqrt{2g(1-D)H_V}$$

where θ is the dimensionless temperature, ε is a small quantity, τ is the dimensionless time, t_* is the characteristic time of heating of the upper layer by heat from reaction zone, θ_W is the dimensionless temperature of walls, β is the temperature factor of walls, t_{out} is the characteristic time for enthalpy flow from the vent, φ_{out} is its dimensionless scale, $t_{R,j}(j=W,L)$ is the characteristic time for radiative heat transfer from the hot zone to the hot walls and to the lower zone, $\varphi_{R,j}(j=W,L)$ is its dimensionless scale, $t_{c,K}(K=H,L)$ is the characteristic time for heat convection from the hot layer to the wall surfaces and the vent surface, and $\varphi_{c,K}(K=H,L)$ is its dimensionless scale.

Equation 1 becomes:

$$\frac{d\theta}{d\tau} = \exp\left(\frac{\theta}{1+\varepsilon\theta}\right) - \frac{1}{\varphi_{out}}\theta - \frac{1}{\varphi_{c,H}}(\theta - \theta_W) - \frac{1}{\varphi_{c,L}}\theta$$

$$- \frac{1}{\varphi_{R,W}}\frac{1}{\varepsilon}[(1+\varepsilon\theta)^4 - (1+\varepsilon\theta_W)^4] - \frac{1}{\varphi_{R,L}}\frac{1}{\varepsilon}[(1+\varepsilon\theta)^4 - 1] \tag{4}$$

where the approximation of the quadratic multinomial is used[11]:

$\exp\left(\frac{\theta}{1+\varepsilon\theta}\right) = \theta^2 + (e-2)\theta + 1$, and the following parameters are introduced:

$$a_1 = (e-2) - \frac{1-\beta}{\varphi_{c,H}} - \frac{1}{\varphi_{out}} - \frac{1}{\varphi_{c,L}} - \frac{4(1-\beta)}{\varphi_{R,W}} - \frac{4}{\varphi_{R,L}},$$

$$a_2 = 1 - \frac{6\varepsilon(1-\beta^2)}{\varphi_{R,W}} - \frac{6\varepsilon}{\varphi_{R,L}},\ a_3 = -4\varepsilon^2\left(\frac{1-\beta^3}{\varphi_{R,W}} + \frac{1}{\varphi_{R,L}}\right),\ a_4 = -\varepsilon^3\left(\frac{1-\beta^4}{\varphi_{R,W}} + \frac{1}{\varphi_{R,L}}\right)$$

Equation 4 is transformed as:

$$\frac{d\theta}{d\tau} = a_4\theta^4 + a_3\theta^3 + a_2\theta^2 + a_1\theta + 1 \equiv \frac{\partial U}{\partial \theta} \tag{5}$$

$$U \sim \frac{a_4}{5}[\theta^5 + \frac{5a_3}{4a_4}\theta^4 + \frac{5a_2}{3a_4}\theta^3 + \frac{5a_1}{2a_4}\theta^2 + \frac{5}{a_4}\theta] \tag{6}$$

The diffeomorphic mapping is defined as:

$$\Theta : \begin{cases} \theta \to x - \dfrac{a_3}{4a_4} \\ u \to \dfrac{5}{a_4}(\dfrac{a_2}{3} - \dfrac{a_3^2}{8a_4}) \\ v \to \dfrac{5}{16a_4}(\dfrac{a_3^3}{a_4^2} - \dfrac{4a_2a_3}{a_4} + 8a_1) \\ w \to \dfrac{5}{16a_4}[\dfrac{a_2a_3^2}{a_4^2} - \dfrac{4a_1a_3}{a_4} - \dfrac{3a_3^4}{16a_4^3} + 16] \\ k \to \dfrac{1}{256a_4^2}[\dfrac{a_3^5}{a_4^3} - \dfrac{20a_2a_3^3}{3a_4^2} + \dfrac{40a_1a_3^2}{a_4} - 320a_3] \end{cases}$$

From this analysis the following equation is obtained:

$$U \sim \frac{a_4}{5}(x^5 + ux^3 + vx^2 + wx + k) \tag{7}$$

It is obvious that Equation 7 is the potential function of a swallowtail catastrophe.

The equilibrium surface M is:

$$U' = 5x^4 + 3ux^2 + 2vx + w = 0 \tag{8}$$

The singularity set S is:

$$U'' = 20x^3 + 6ux + 2v = 0 \tag{9}$$

By removing x from Equations 8 and 9, the equation of bifurcation set B is obtained. It is a curved surface of 3-dimensional control space C. The schematic outline is shown in Figure 2.

Operating conditions

In this paper, the control variables v and w are discussed while conserving the invariance of the control variable $u(u \geq 0, < 0)$. This is because of the symmetry characteristic of the bifurcation set of the swallowtail catastrophe. This is done in order to discuss directly the corresponding relationships of control variables and operating conditions. The conclusions will now be given.

$u \geq 0$

Figure 3 shows the bifurcation set of the swallowtail catastrophe at the control variable $u \geq 0$ ($u = 10$).

The following results are shown with reference to Figure 3:

(1) The space *v*-*w* can be divided into six regions in which the borderline between Region IV and Region II, III is defined according to Equation 10:

$$\begin{cases} v = -10x^3 - 3ux \\ w = 15x^4 + 3ux^2 \end{cases} \tag{10}$$

(2) From, $\theta = x - a_3/4a_4$, x has physical meaning only at $x > a_3/4a_4\,(\geq 0)$ Based on Equation 10: v is the odd function of x, and w is the even function of x. Regions III, IV, V and VI are the ones in which the operating points may not be reached.

(3) The potential function $U(a_4 < 0)$ of Region I based on Equation 7 shows that U has two critical points, one maximal point and one minimal point, and both corresponding x are positive. The maximal point is an unstable point, and the minimal point is a stable point. However, x_{min} corresponding to the minimal point is less than x_{max} corresponding to the maximal point. So x will increase with U decreasing at $x > x_{max}$ until backdraft takes place, so Region I is a backdraft region. Otherwise, x will fall to x_{min} with U decreasing at $0 < x < x_{max}$, and Region I will be an extinction region.

(4) The potential function U of Region II has no critical points, i.e. x has no real roots. But x will increase with U decreasing until backdraft takes place. So Region II is a backdraft region.

From this analysis, Regions III, IV, V and VI are the ones in which the operating points may not be reached. Region II is a backdraft region ($x > a_3/4a_4$). Region I is a backdraft region at $x > x_{max}$, and an extinction region at $0 < x < x_{max}$ ($x > a_3/4a_4$).

$u < 0$

Figure 4 shows the bifurcation set of a swallowtail catastrophe at the control variable $u < 0$ ($u = -50$).

The results derived from Figure 4 include:

(1) The space $v - w$ can be divided into ten regions, in which the borderline between Regions I, III, IV, VI, VIII, IX and Regions II, V, VII, X is defined according to Equation 10.

(2) Firstly, the operating conditions of Region I, III, IV, VI, VIII and IX are studied. Based on their symmetry characteristics, only the conditions at $v = 0$ are considered. Therefore, the solution of Equation 8 is:

$$x^2 = \frac{1}{10}[-3u \pm \sqrt{9u^2 - 20w}] \tag{11}$$

(a) At Regions III and IV, i.e. $w = 9u^2/20$, Equation 11 has no real roots, and the potential function U has no critical points. But x will increase with U decreasing until backdraft takes place so Regions III and IV are backdraft regions.

(b) At Regions I and VI, i.e. $0 < w < 9u^2/20$, Equation 11 has four real roots, two positive and two negative, and the potential function U has four critical points, two maximal points and two minimal points. The maximal point corresponding to the positive root is an unstable point, and the minimal point corresponding to the positive root is a stable point. But x_{min} corresponding to the minimal point is less than x_{max} corresponding to the maximal point, so x will increase with U decreasing at $x > x_{max}$ until backdraft takes place. Therefore, Regions I and VI are backdraft regions. Otherwise, x will fall to x_{min} with U decreasing at $0 < x < x_{max}$, and Regions I and VI are extinction regions.

(c) At Regions VIII and IX, i.e. $w < 0$, Equation 11 has two real roots, and the potential function U has two critical points, one maximal point and one minimal point. But x_{max} corresponding to the maximal point is more than zero, and x_{min} corresponding to the minimal point is less than zero. So x will increase with U decreasing at $x > x_{max}$ until backdraft takes place, so Regions VIII and IX are backdraft regions.

(3) Based on the potential function U of Region II, U has two critical points, one maximal point and one minimal point, and both corresponding x are positive. The maximal point is an unstable point, and the minimal point is a stable point. But x_{min} corresponding to the minimal point is less than x_{max} corresponding to the maximal point, x will increase with U decreasing at $x > x_{max}$ until backdraft takes place, so Region II is a backdraft region. Otherwise, x will fall to x_{min} with U decreasing at $0 < x < x_{max}$, and Region II is an extinction region.

(4) The potential function U of Region V has two critical points, one maximal point and one minimal point, but both corresponding x are negative. However, $x(>0)$ will increase with U decreasing until backdraft takes place. Therefore, Region V is a backdraft region.

(5) The potential functions U of Region VII and X each have two critical points, one maximal point and one minimal point. But x_{max} corresponding to the maximal point is positive, and x_{min} corresponding to the minimal point is negative, so x will increase with U decreasing at $x > x_{max}$ until backdraft takes place and Regions VII and X are backdraft regions.

This analysis has shown that Regions III, IV and V are backdraft regions ($x > a_3/4a_4$), Regions I, II, VI, VII, VIII, IX and X are backdraft regions at $x > x_{max}$, and Regions I, II and VI are extinction regions at $0 < x < x_{max}$ ($x > a_3/4a_4$).

CONCLUSIONS AND FUTURE WORK

A simplified mathematical model of backdraft in compartment fires has been established based on an energy balance equation and its catastrophe manifold function equation based on catastrophe theory. In addition, the corresponding relationship between system control variables and operating conditions is discussed at the control variable $u \geq 0$ and < 0. The catastrophe form of backdraft was found to be a swallowtail catastrophe. In the bifurcation set of the swallowtail catastrophe, few regions at limited conditions are backdraft regions. Therefore, in order to determine whether backdraft takes place in the development of a fire, the operating point must be in a backdraft region. The critical temperature of backdraft is also evaluated from the bifurcation set of the swallowtail catastrophe.

Future work should focus on practical applications for backdraft based on catastrophe theory. For example, performance design for building fires to determine the materials of buildings to avoid the occurrence of backdraft. Other future work should extend catastrophe theory to typical catastrophe behaviour of fire phenomena such as flashover in compartment fires.

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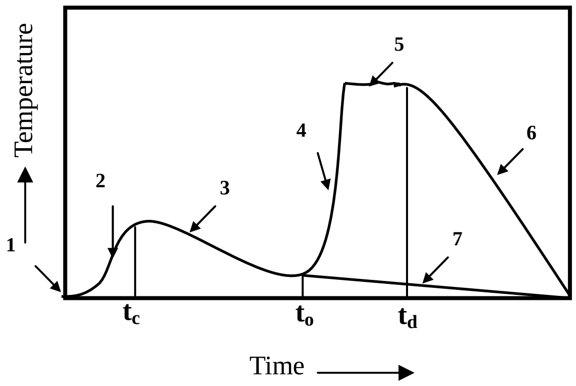

1-Ignition 2-Free combustion 3-Smouldering 4-Backdraft
5-Developed combustion 6-Decay 7-Decay

Figure1: Schematic outline of the development of vitiated compartment fires.

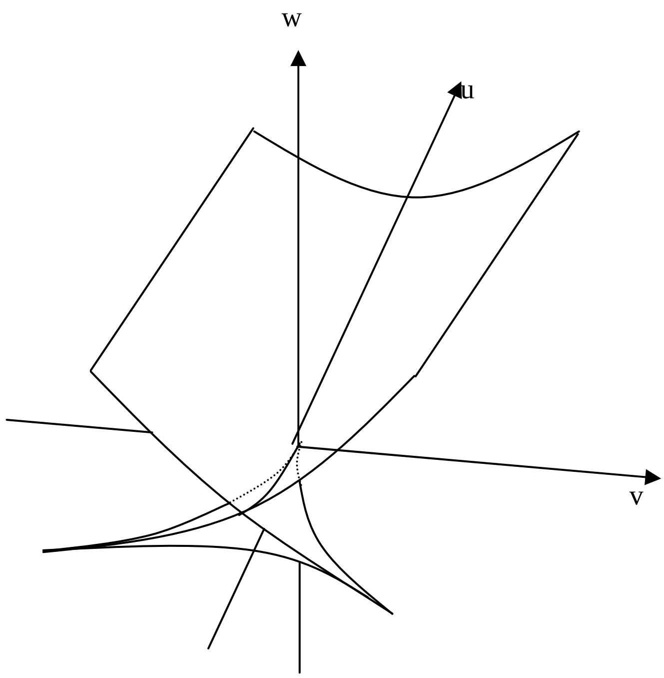

Figure 2: Schematic outline of bifurcation set for swallowtail catastrophe [5].

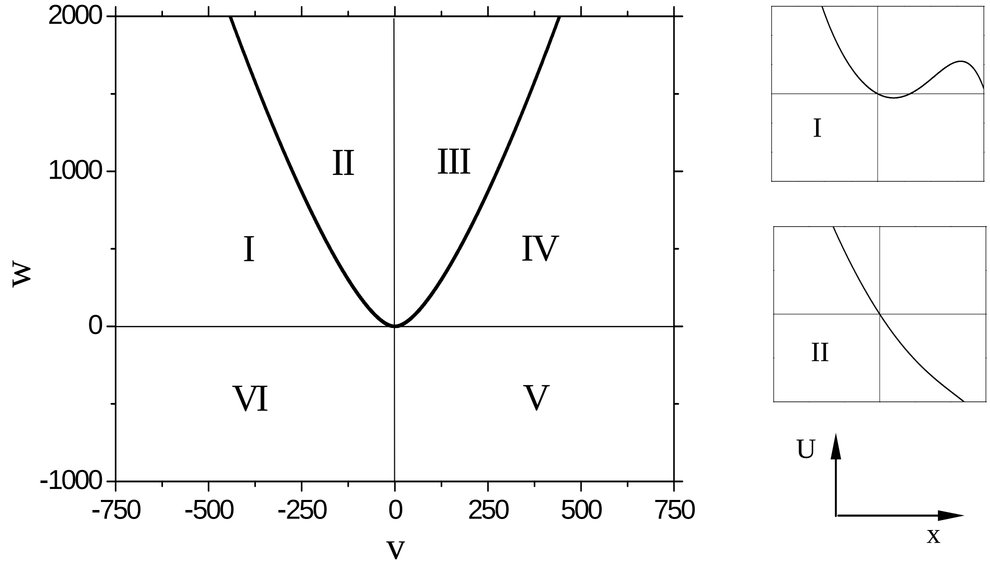

Figure 3: Bifurcation set of swallowtail catastrophe at $u \geq 0$ (small figures are potential functions).

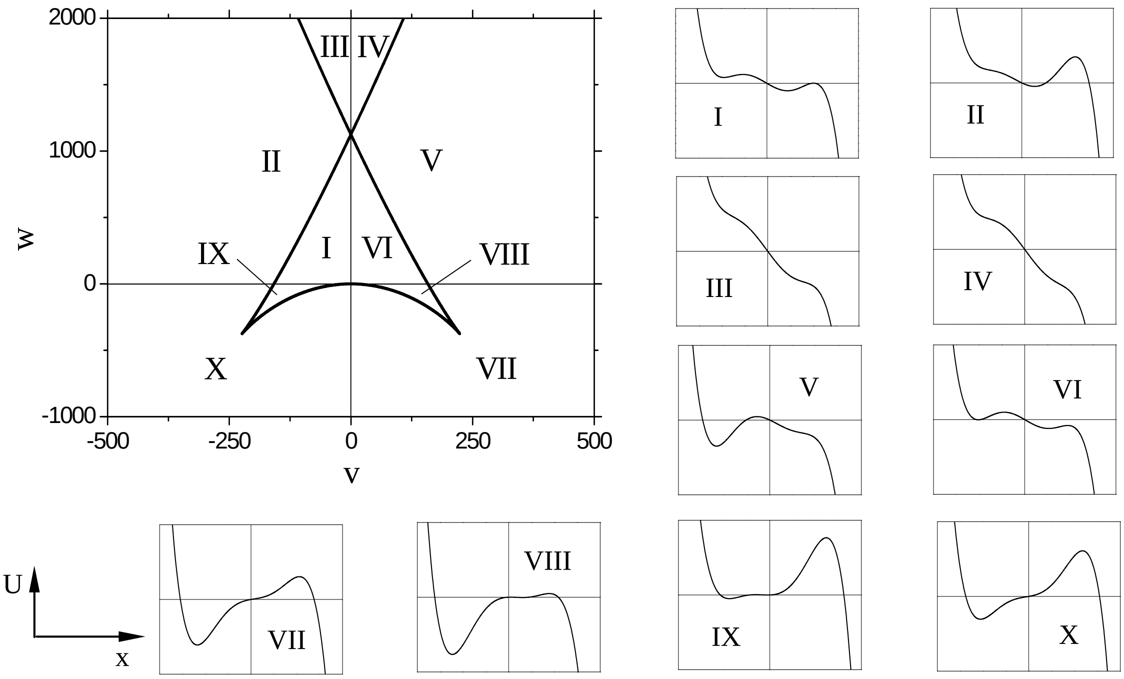

Figure 4: Bifurcation set of swallowtail catastrophe at $u < 0$ (small figures are potential functions).