

DERIVATION OF GENERALISED STAGNATION FLOW EQUATIONS FOR A MULTICOMPONENT, REACTING GAS MIXTURE

M. T. Matthews, B. Z. Dlugogorski and E. M. Kennedy

Department of Chemical Engineering, The University of Newcastle

AUSTRALIA

ABSTRACT

This paper presents an overview of bifurcation theory and the continuation method used to predict extinction limits of flames. Complete derivation of the continuation method is presented, together with the physical and mathematical assumptions applied to simplify the governing equations.

INTRODUCTION

Prediction of the ignition and extinction limits of exothermic reactions has many important applications. Since methane is the major constituent of natural gas, an understanding of the ignition and extinction properties of methane is crucial if combustion processes are to be employed safely and efficiently in industry.

In general, there are two classes of gaseous fuel flames – premixed and diffusion flames. In premixed flames, the reactants (fuel and oxidiser) are premixed before entering the reaction zone. If the composition and physical conditions of the combustible gases are specified, the compositions, pressure and temperature of the burnt gas in the final combustion state and the characteristics of the flame are determined uniquely¹. In diffusion flames the fuel and oxidiser are initially separated and mix in the same region that the reaction occurs. Hence combustion occurs at an interface between the fuel gas and the oxidant gas, and the burning process depends more on the rate of mixing of the fuel and oxidiser than on the rates of the chemical processes involved¹. Steady-state premixed and diffusion flames have been the subjects of extensive theoretical and computational research. Flame characteristics have been analysed primarily for two flow configurations – the flat plate boundary layer²⁻⁴ and stagnation point flow⁵⁻¹¹. Work on these configurations has comprised of experimental, analytical and numerical investigations.

These two configurations are convenient because the flow field is well understood and the two spatial

dimensions of the system can be reduced to one dimension by a similarity transformation⁵. Such systems are able to produce multiple steady-state solutions that characterise the ignition and extinction behaviour¹². A formal analysis on the flow past a body of revolution incorporates both configurations.

A widely applied method for analysis of the two configurations has been the perturbation or asymptotic method^{6,8-10}. However, steady-state bifurcation theory is now more widely used¹³⁻¹⁷. In many physical problems, some observable quantity depends on a parameter describing the physical state. As the parameter is increased, a critical value is reached at which the observable quantity suddenly changes its character. The critical value is called a bifurcation point¹⁸.

Ignition and extinction may be regarded as bifurcations of the system. There are several advantages of bifurcation analysis over a perturbation or asymptotic analysis, the most important being that ignition and extinction can be regarded as being the same type of bifurcation, called a simple fold bifurcation¹², and hence only one type of computational algorithm is required for both ignition and extinction limits.

This paper is organised as follows. In the next section a complete mathematical analysis for the flow past a body of revolution is presented, including an analysis of the geometry and scaling arguments that lead to the so-called boundary layer equations, as well as the transformation of these equations into a system of coupled non-linear boundary value problems – the continuation equations. Discussion of the assumptions is subsequently presented.

THEORETICAL

Consider the boundary layer formed by an incompressible Newtonian fluid flowing past a body of revolution. A plane section through the axis of symmetry of the body is shown in Figure 1. The equation of the axially symmetric surface is given by:

$$r = f(z) \quad (1)$$

An orthogonal curvilinear coordinate system can be defined such that:

- $x^1 \equiv x =$ arc length measured along the body in a plane of constant θ
- $x^2 \equiv y =$ arc length measured along straight lines normal to the body
- $x^3 \equiv \theta =$ azimuthal cylindrical coordinate measured around the axis of the body

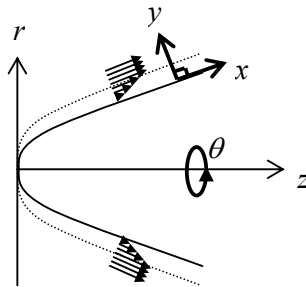


Figure 1: Flow past a body of revolution.

The shape of the body suggests planar flow, defined by:

$$v_x \equiv v_{(1)} = v_x(x, y, t), v_y \equiv v_{(2)} = v_y(x, y, t), v_\theta \equiv v_{(3)} = 0 \quad (2)$$

It can be shown via the arc length formula that:

$$\frac{dz}{dx} = [1 + f'^2]^{\frac{1}{2}} \quad (3)$$

where the prime indicates differentiation with respect to z . It can be shown that for the coordinate system defined above the metric coefficients are given by:

$$\begin{aligned} g_{11} &= \hat{\mathbf{g}}_1 \cdot \hat{\mathbf{g}}_1 = g_{xx} = \hat{\mathbf{g}}_x \cdot \hat{\mathbf{g}}_x = (1 + \kappa y)^2 \\ g_{22} &= \hat{\mathbf{g}}_2 \cdot \hat{\mathbf{g}}_2 = g_{yy} = \hat{\mathbf{g}}_y \cdot \hat{\mathbf{g}}_y = 1 \\ g_{33} &= \hat{\mathbf{g}}_3 \cdot \hat{\mathbf{g}}_3 = g_{\theta\theta} = \hat{\mathbf{g}}_\theta \cdot \hat{\mathbf{g}}_\theta = \left(f + y [1 + f'^2]^{\frac{1}{2}} \right)^2 \end{aligned} \quad (4)$$

The gradient of a scalar and the divergence, curl and vector Laplacian of a vector in terms of the unit spatial vectors $\hat{\mathbf{g}}_1, \hat{\mathbf{g}}_2, \hat{\mathbf{g}}_3 \equiv \hat{\mathbf{g}}_x, \hat{\mathbf{g}}_y, \hat{\mathbf{g}}_\theta$ and metric coefficients $g_{11}, g_{22}, g_{33} \equiv g_{xx}, g_{yy}, g_{\theta\theta}$ may be calculated using expressions found in¹⁹.

Differential mass conservation

The differential mass conservation equation is given by²⁰:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad (5)$$

With $\mathbf{v} \equiv (v_x, v_y, 0) = \mathbf{v}(x, y, t)$, $\rho = \rho(x, y)$ and Equation 4, each term of Equation 5 can be calculated with the aid of expressions found in¹⁹. Hence the differential momentum conservation equation may be written as:

$$\frac{1}{1 + \kappa y} \frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} + \frac{f'}{h} (\rho v_x) + \left(\frac{\kappa}{1 + \kappa y} + \frac{1}{h} \right) (\rho v_y) = 0 \quad (6)$$

where κ is the negative value of the curvature of the curve defined by Equation 1:

$$\kappa \equiv \frac{-f''}{[1 + f'^2]^{\frac{3}{2}}} = \kappa(x) \quad (7)$$

and the function h is given by:

$$h \equiv f [1 + f'^2]^{\frac{1}{2}} + y = h(x, y) \quad (8)$$

Differential momentum conservation

The differential momentum conservation equation is given by²⁰:

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho \mathbf{v} \cdot \nabla \mathbf{v} = (\nabla \cdot \mathbf{T}) + \rho \sum_{i=1}^N Y_i \mathbf{f}_i \quad (9)$$

where \mathbf{T} is the stress tensor and \mathbf{D} is the rate of deformation tensor given by:

$$\mathbf{T} = [-P + (\kappa - \frac{2}{3}\mu)\nabla \cdot \mathbf{v}]\mathbf{I} + 2\mu\mathbf{D} \quad , \quad \mathbf{D} = \frac{1}{2}\nabla\mathbf{v} + \frac{1}{2}(\nabla\mathbf{v})^T \quad (10)$$

Now, by assuming $\nabla \cdot \mathbf{v} = 0$, $\mu = \mu(t)$ it can be shown that:

$$\nabla \cdot \mathbf{T} = -\nabla P + \mu \nabla^2 \mathbf{v}$$

In combustion, body forces \mathbf{f}_i may be neglected^[20], and using the vector identity²¹:

$$\mathbf{v} \cdot \nabla \mathbf{v} = \frac{1}{2}\nabla(\mathbf{v} \cdot \mathbf{v}) - \mathbf{v} \times \nabla \times \mathbf{v}$$

the invariant form of the differential momentum balance becomes:

$$\frac{\partial \mathbf{v}}{\partial t} + \frac{1}{2}\nabla(\mathbf{v} \cdot \mathbf{v}) - \mathbf{v} \times \nabla \times \mathbf{v} = -\frac{1}{\rho}\nabla P + \frac{\mu}{\rho}\nabla^2 \mathbf{v} \quad (11)$$

With $\mathbf{v} \equiv (v_x, v_y, 0) = \mathbf{v}(x, y, t)$ and Equation 4, each term of Equation 11 can be calculated with the aid of expressions found in¹⁹. Hence the differential momentum conservation equation may be written as:

x-component:

$$\begin{aligned} \frac{\partial v_x}{\partial t} + \frac{v_x}{1+\kappa y} \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + \frac{\kappa}{1+\kappa y} v_x v_y = -\frac{1}{\rho(1+\kappa y)} \frac{\partial P}{\partial x} + \\ + \frac{\mu}{\rho} \left[\frac{1}{(1+\kappa y)^2} \frac{\partial^2 v_x}{\partial x^2} + \frac{v_y}{(1+\kappa y)^3} \frac{d\kappa}{dx} + \frac{2\kappa}{(1+\kappa y)^2} \frac{\partial v_y}{\partial x} - \frac{y}{(1+\kappa y)^3} \frac{d\kappa}{dx} \frac{\partial v_x}{\partial x} - \frac{\kappa^2}{(1+\kappa y)^2} v_x + \right. \\ \left. + \frac{\kappa}{1+\kappa y} \frac{\partial v_x}{\partial y} + \frac{\partial^2 v_x}{\partial y^2} - \frac{f'}{h^2} v_y + \frac{\kappa f' [1+f'^2]^{\frac{1}{2}}}{h(1+\kappa y)} v_y + \frac{f' [1+f'^2]^{\frac{1}{2}}}{h(1+\kappa y)} \frac{\partial v_x}{\partial x} + \frac{1}{h} \frac{\partial v_x}{\partial y} \right] \end{aligned} \quad (12)$$

y-component:

$$\begin{aligned} \frac{\partial v_y}{\partial t} + \frac{v_x}{1+\kappa y} \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} - \frac{\kappa}{1+\kappa y} v_x^2 = -\frac{1}{\rho} \frac{\partial P}{\partial y} + \\ + \frac{\mu}{\rho} \left[\frac{1}{(1+\kappa y)^2} \frac{\partial^2 v_y}{\partial x^2} - \frac{y}{(1+\kappa y)^3} \frac{d\kappa}{dx} \frac{\partial v_y}{\partial x} + \frac{\kappa}{1+\kappa y} \frac{\partial v_y}{\partial y} - \frac{2\kappa}{(1+\kappa y)^2} \frac{\partial v_x}{\partial x} - \frac{\kappa^2}{(1+\kappa y)^2} v_y - \right. \\ \left. - \frac{v_x}{(1+\kappa y)^3} \frac{d\kappa}{dx} + \frac{\partial^2 v_y}{\partial y^2} - \frac{1}{h^2} v_y + \frac{1}{h} \frac{\partial v_y}{\partial y} + \frac{f' [1+f'^2]^{\frac{1}{2}}}{h(1+\kappa y)} \left(\frac{\partial v_y}{\partial x} - \kappa v_x \right) \right] \end{aligned} \quad (13)$$

θ -component:

$$0 = \frac{1}{h[1+f'^2]^{\frac{1}{2}}} \frac{\partial P}{\partial \theta} \quad (14)$$

Differential energy conservation

The differential energy conservation equation is given by^{20,22}:

$$\rho \frac{\partial U}{\partial t} + \rho \mathbf{v} \cdot \nabla U = -(\nabla \cdot \mathbf{q}) + \text{tr}(\mathbf{T} \cdot \mathbf{D}) + \rho \sum_{i=1}^N Y_i \mathbf{f}_i \cdot \mathbf{V}_i \quad (15)$$

where \mathbf{q} is the heat-flux vector given by:

$$\mathbf{q} = -\lambda \nabla T + \sum_{i=1}^N \rho \bar{H}_i Y_i \mathbf{V}_i + RT \sum_{i=1}^N \sum_{j=1}^N \left(\frac{X_j D_{T,i}}{W_i D_{ij}} \right) (\mathbf{V}_i - \mathbf{V}_j) + \mathbf{q}_R \quad (16)$$

It can be shown that Equation 15 may be written as²²:

$$\begin{aligned} \rho \hat{c}_p \frac{\partial T}{\partial t} + \rho \hat{c}_p \mathbf{v} \cdot \nabla T = & -(\nabla \cdot \mathbf{q}) + \left(\frac{\partial \ln(\hat{V})}{\partial \ln(T)} \right)_{P,Y_j} \left(\frac{\partial P}{\partial t} + \mathbf{v} \cdot \nabla P \right) + \text{tr}(\mathbf{S} \cdot \mathbf{D}) + \\ & + \sum_{i=1}^N \bar{H}_i (\nabla \cdot \mathbf{j}_i - w_i) + \rho \sum_{i=1}^N Y_i \mathbf{f}_i \cdot \mathbf{V}_i \end{aligned} \quad (17)$$

where \mathbf{S} is the viscous portion of the stress tensor given by:

$$\mathbf{S} = \mathbf{T} + P\mathbf{I} \quad (18)$$

and \mathbf{j}_i is the mass flux of species i with respect to \mathbf{v} given by:

$$\mathbf{j}_i = \rho_i (\mathbf{v}_i - \mathbf{v}) \quad (19)$$

and w_i is the rate of production of species i per unit volume by homogenous chemical reactions, often expressed as the phenomenological expression of chemical kinetics:

$$w_i = W_i \sum_{k=1}^M (v_{i,k}'' - v_{i,k}') \mathcal{B}_k T^{\alpha_k} \exp \left[-\frac{E_k}{RT} \right] \prod_{j=1}^N \left(\frac{X_j P}{RT} \right)^{v_{j,k}'} \quad (20)$$

With $\nabla \cdot \mathbf{v} = 0$ Equation 18 becomes:

$$\mathbf{S} = 2\mu\mathbf{D} \quad (20)$$

For an ideal gas:

$$\left(\frac{\partial \ln(\hat{V})}{\partial \ln(T)} \right)_{P, Y_i} = 1 \quad (21)$$

From the differential mass conservation equation $\nabla \cdot (\rho \mathbf{v}) = 0 = \nabla \cdot (\rho_i \mathbf{v}_i)$, hence from Equation 19:

$$\nabla \cdot \mathbf{j}_i = 0 \quad (22)$$

In combustion, Dufour effects and body forces \mathbf{f}_i may be neglected²⁰, and it is assumed that $\mu = \mu(t)$. With these assumptions and Equations 20, 21 and 22, Equation 17 becomes:

$$\begin{aligned} \rho \hat{c}_p \frac{\partial T}{\partial t} + \rho \hat{c}_p \mathbf{v} \cdot \nabla T = \nabla \cdot (\lambda \nabla T) - \nabla \cdot \left(\sum_{i=1}^N \rho \bar{H}_i Y_i \mathbf{V}_i \right) - \nabla \cdot \mathbf{q}_R + \\ + \frac{\partial P}{\partial t} + \mathbf{v} \cdot \nabla P + 2\mu \operatorname{tr}(\mathbf{D} \cdot \mathbf{D}) - \sum_{i=1}^N \bar{H}_i w_i \end{aligned} \quad (23)$$

With $\mathbf{v} \equiv (v_x, v_y, 0) = \mathbf{v}(x, y, t)$, $\mathbf{V}_i = (0, V_{y,i}, 0)$, $\mathbf{q}_R = (0, q_{Ry}, 0)$, $T = T(x, y, t)$, $P = P(x, y, \theta)$ and Equation 4, each term of Equation 23 can be calculated with the aid of expressions found in¹⁹. The calculation of $\operatorname{tr}(\mathbf{D} \cdot \mathbf{D})$ is laborious, and is simply presented here:

$$2\mu \operatorname{tr}(\mathbf{D} \cdot \mathbf{D}) = 2\mu \left[\frac{1}{(1 + \kappa y)^2} \left(\frac{\partial v_x}{\partial x} + \kappa v_y \right)^2 + \left(\frac{\partial v_y}{\partial y} \right)^2 + \left(\frac{v_y}{h} \right)^2 \right] + \mu \left[(1 + \kappa y) \frac{\partial}{\partial y} \left(\frac{v_x}{1 + \kappa y} \right) + \frac{1}{1 + \kappa y} \frac{\partial v_y}{\partial x} \right]^2$$

Hence the differential energy conservation equation may be written as:

$$\begin{aligned} \rho \hat{c}_p \left(\frac{\partial T}{\partial t} + \frac{v_x}{1 + \kappa y} \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} \right) = \frac{1}{1 + \kappa y} \frac{\partial}{\partial x} \left[\frac{\lambda}{1 + \kappa y} \frac{\partial T}{\partial x} \right] + \frac{1}{1 + \kappa y} \frac{\partial}{\partial y} \left[\lambda (1 + \kappa y) \frac{\partial T}{\partial y} \right] + \\ + \frac{\lambda f'}{h(1 + \kappa y)} \frac{\partial T}{\partial x} + \frac{\lambda}{h} \frac{\partial T}{\partial y} - \frac{\partial}{\partial y} \left(\sum_{i=1}^N \rho \bar{H}_i Y_i V_{y,i} \right) - \\ - \left(\frac{\kappa}{1 + \kappa y} + \frac{1}{h} \right) \sum_{i=1}^N \rho \bar{H}_i Y_i V_{y,i} - \frac{\partial q_{Ry}}{\partial y} - \left(\frac{\kappa}{1 + \kappa y} + \frac{1}{h} \right) q_{Ry} + \\ + \frac{v_x}{1 + \kappa y} \frac{\partial P}{\partial x} + v_y \frac{\partial P}{\partial y} + \\ + 2\mu \left[\frac{1}{(1 + \kappa y)^2} \left(\frac{\partial v_x}{\partial x} + \kappa v_y \right)^2 + \left(\frac{\partial v_y}{\partial y} \right)^2 + \left(\frac{v_y}{h} \right)^2 \right] + \\ + \mu \left[(1 + \kappa y) \frac{\partial}{\partial y} \left(\frac{v_x}{1 + \kappa y} \right) + \frac{1}{1 + \kappa y} \frac{\partial v_y}{\partial x} \right]^2 - \sum_{i=1}^N \bar{H}_i w_i \end{aligned} \quad (24)$$

Differential chemical species conservation

The differential chemical species conservation equation is given by²⁰:

$$\rho \frac{\partial Y_i}{\partial t} + \rho \mathbf{v} \cdot \nabla Y_i = w_i - \nabla \cdot (\rho Y_i \mathbf{V}_i) \quad (25)$$

With $\mathbf{v} \equiv (v_x, v_y, 0) = \mathbf{v}(x, y, t)$, $Y_i = Y_i(x, y, t)$, $\mathbf{V}_i = (0, V_{y,i}, 0)$ and Equation 4, each term of Equation 25 may be calculated with the aid of expressions found in¹⁹. Hence the differential chemical species conservation equation may be written as:

$$\rho \frac{\partial Y_i}{\partial t} + \frac{\rho v_x}{1 + \kappa y} \frac{\partial Y_i}{\partial x} + \rho v_y \frac{\partial Y_i}{\partial y} = w_i - \frac{\partial}{\partial y} (\rho Y_i V_{y,i}) - \left(\frac{\kappa}{1 + \kappa y} + \frac{1}{h} \right) \rho Y_i V_{y,i} \quad (26)$$

Boundary-layer approximation – scaling arguments

Initially, the following dimensionless variables were defined:

$$\begin{aligned} x^* &= \frac{x}{L_0} & \mathbf{V}_i^* &= \frac{V_i}{v_0} & T^* &= \frac{T - T_\infty}{T_0 - T_\infty} & \kappa^* &= \kappa L_0 \\ y^* &= \frac{y}{L_0} & P^* &= \frac{P}{P_0} & \bar{H}_i^* &= \frac{\bar{H}_i}{\hat{c}_p (T_0 - T_\infty)} & f^* &= \frac{f}{L_0} \\ \mathbf{v}^* &= \frac{\mathbf{v}}{v_0} & t^* &= \frac{t}{t_0} & q_{Ry}^* &= \frac{q_{Ry}}{\rho v_0^3} & z^* &= \frac{z}{L_0} \\ & & & & & & w_i^* &= \frac{w_i t_0}{\rho} \end{aligned}$$

In Figure 1 it is illustrated (on an exaggerated scale) the velocity distribution in the immediate neighbourhood of the body of revolution. The thickness of this boundary layer decreases as N_{Re} increases. The thickness of the boundary layer can be magnified by introducing the following dimensionless variables:

$$\begin{aligned} y^{**} &= \sqrt{N_{Re}} y^* = \sqrt{N_{Re}} \frac{y}{L_0} & V_{y,i}^{**} &= \sqrt{N_{Re}} V_{y,i}^* = \sqrt{N_{Re}} \frac{V_{y,i}}{v_0} & \kappa^{**} &= \frac{1}{\sqrt{N_{Re}}} \kappa^* = \frac{L_0}{\sqrt{N_{Re}}} \kappa \\ v_y^{**} &= \sqrt{N_{Re}} v_y^* = \sqrt{N_{Re}} \frac{v_y}{v_0} & q_{Ry}^{**} &= \sqrt{N_{Re}} q_{Ry}^* = \sqrt{N_{Re}} \frac{q_{Ry}}{\rho v_0^3} \end{aligned}$$

Substituting the above dimensionless variables into the differential mass, momentum, energy and chemical species conservation equations and taking the limit $N_{Re} \gg 1$, as well as realising that for a fixed wall configuration $\kappa^{**} \rightarrow 0$ in the limit $N_{Re} \gg 1$, the following is obtained:

Differential mass conservation:

$$\frac{\partial(\rho v_x^*)}{\partial x^*} + \frac{\partial(\rho v_y^{**})}{\partial y^{**}} + \frac{f^{*/2}}{f^*(1 + f^{*/2})} (\rho v_x^*) = 0 \quad (27)$$

Differential momentum conservation:

x-component:

$$\frac{1}{N_{St}} \frac{\partial v_x^*}{\partial t^*} + v_x^* \frac{\partial v_x^*}{\partial x^*} + v_y^{**} \frac{\partial v_x^*}{\partial y^{**}} = -\frac{1}{N_{Ru}} \frac{\partial P^*}{\partial x^*} + \frac{\partial^2 v_x^*}{\partial y^{**2}} \quad (28)$$

y-component:

$$\frac{\partial P^*}{\partial y^{**}} = 0 \quad (29)$$

θ -component:

$$\frac{\partial P^*}{\partial \theta} = 0 \quad (30)$$

Differential energy conservation:

$$\begin{aligned} \frac{1}{N_{St}} \frac{\partial T^*}{\partial t^*} + v_x^* \frac{\partial T^*}{\partial x^*} + v_y^{**} \frac{\partial T^*}{\partial y^{**}} &= \frac{\partial}{\partial y^{**}} \left[\frac{1}{N_{Pr}} \frac{\partial T^*}{\partial y^{**}} \right] - \frac{1}{\rho} \frac{\partial}{\partial y^{**}} \left(\sum_{i=1}^N \rho \bar{H}_i^* Y_i V_{y,i}^{**} \right) - \\ &- \frac{N_{Br}}{N_{Pr}} \frac{\partial q_{Ry}^{**}}{\partial y^{**}} + \frac{N_{Br}}{N_{Ru} N_{Pr}} v_x^* \frac{\partial P^*}{\partial x^*} + \frac{N_{Br}}{N_{Pr}} \left(\frac{\partial v_x^*}{\partial y^{**}} \right)^2 - \\ &- \frac{1}{N_{St}} \sum_{i=1}^N \bar{H}_i^* w_i^* \end{aligned} \quad (31)$$

Differential chemical species conservation:

$$\frac{\rho}{N_{St}} \frac{\partial Y_i}{\partial t^*} + \rho v_x^* \frac{\partial Y_i}{\partial x^*} + \rho v_y^{**} \frac{\partial Y_i}{\partial y^{**}} = \frac{\rho}{N_{St}} w_i^* - \frac{\partial}{\partial y^{**}} (\rho Y_i V_{y,i}^{**}) \quad (32)$$

The primes in Equation 27 now indicate differentiation w.r.t. z^* . Note that in the above equations it has been assumed for the purposes of scaling arguments that all the transport properties are constant. From Equations 29 and 30 it can be seen that:

$$P^* = P^*(x^*) \Rightarrow P = P(x) \quad (33)$$

From Equation 3 it is seen that:

$$\frac{dz^*}{dx^*} = [1 + f^{*/2}]^{\frac{1}{2}}$$

hence Equation 49 may be written as:

$$\frac{\partial(\rho f^* v_x^*)}{\partial x^*} + \frac{\partial(\rho f^{**} v_y^{**})}{\partial y^{**}} = 0 \quad (34)$$

Transformation

From Equations 27 to 34 it can be seen that the differential conservation equations for flow past a fixed-wall configuration body of revolution in the limit $N_{Re} \gg 1$ are given by:

Differential mass conservation:

$$\frac{\partial(\rho f v_x)}{\partial x} + \frac{\partial(\rho f v_y)}{\partial y} = 0 \quad (35)$$

Differential momentum conservation:

$$\rho \frac{\partial v_x}{\partial t} + \rho v_x \frac{\partial v_x}{\partial x} + \rho v_y \frac{\partial v_x}{\partial y} = -\frac{dP}{dx} + \mu \frac{\partial^2 v_x}{\partial y^2} \quad (36)$$

Differential energy conservation:

$$\begin{aligned} \rho \hat{c}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} \right) &= \frac{\partial}{\partial y} \left[\lambda \frac{\partial T}{\partial y} \right] - \frac{\partial}{\partial y} \left(\sum_{i=1}^N \rho \bar{H}_i Y_i V_{y,i} \right) - \frac{\partial q_{Ry}}{\partial y} + \\ &+ v_x \frac{dP}{dx} + \mu \left(\frac{\partial v_x}{\partial y} \right)^2 - \sum_{i=1}^N \bar{H}_i w_i \end{aligned} \quad (37)$$

Differential chemical species conservation:

$$\rho \frac{\partial Y_i}{\partial t} + \rho v_x \frac{\partial Y_i}{\partial x} + \rho v_y \frac{\partial Y_i}{\partial y} = w_i - \frac{\partial}{\partial y} (\rho Y_i V_{y,i}) \quad (38)$$

The diffusion velocity \mathbf{V}_i is determined from the multicomponent diffusion equation²⁰:

$$\begin{aligned} \nabla X_i &= \sum_{j=1}^N \left(\frac{X_i X_j}{D_{ij}} \right) (\mathbf{V}_j - \mathbf{V}_i) + (Y_i - X_i) \left(\frac{\nabla P}{P} \right) + \left(\frac{\rho}{P} \right) \sum_{j=1}^N Y_i Y_j (\mathbf{f}_i - \mathbf{f}_j) + \\ &+ \sum_{j=1}^N \left(\frac{X_i X_j}{\rho D_{ij}} \right) \left(\frac{D_{T,j}}{Y_j} - \frac{D_{T,i}}{Y_i} \right) \left(\frac{\nabla T}{T} \right) \end{aligned} \quad (39)$$

Neglecting body forces, Soret effects, and pressure-gradient diffusion, Equation 39 becomes:

$$\nabla X_i = \sum_{j=1}^N \left(\frac{X_i X_j}{D_{ij}} \right) (\mathbf{V}_j - \mathbf{V}_i) \quad (40)$$

If the binary diffusion coefficients of all pairs of species are equal, $D \equiv D_{ij}$ and it can be shown that Equation 40 reduces to:

$$\mathbf{V}_i = -D\nabla[\ln(Y_i)] \quad (41)$$

Equation 41 is commonly known as Fick's Law. Recalling that $\mathbf{V}_i = (0, V_{y,i}, 0)$, from Equations 4 and expressions found in¹⁹, Equation 41 yields:

$$Y_i V_{y,i} = -D \frac{\partial Y_i}{\partial y} \quad (42)$$

The free stream (tangential) velocity at the edge of the boundary layer for stagnation flow is given by²³:

$$v_{xe} = ax = v_{xe}(x) \quad (43)$$

where a is the strain rate, which may be assumed constant for stagnation flow. Substituting Equation 43 into the differential momentum conservation equation yields:

$$\rho_e v_{xe} \frac{dv_{xe}}{dx} = -\frac{dP_e}{dx} = -\frac{dP}{dx} \quad (44)$$

where the pressure in the free stream, P_e , is equal to the pressure in the boundary layer, P , in planes of constant x since from Equation 33 $P = P(x)$. This is Bernoulli's Law for the pressure distribution in the boundary layer⁵. Substituting Equations 42 and 44 into the differential balances yields the following:

Differential mass conservation:

$$\frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} = 0 \quad (45)$$

Differential momentum conservation:

$$\rho \frac{\partial v_x}{\partial t} + \rho v_x \frac{\partial v_x}{\partial x} + \rho v_y \frac{\partial v_x}{\partial y} = \rho_e v_{xe} \frac{dv_{xe}}{dx} + \mu \frac{\partial^2 v_x}{\partial y^2} \quad (46)$$

Differential energy conservation:

$$\begin{aligned} \rho \hat{c}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} \right) &= \frac{\partial}{\partial y} \left[\lambda \frac{\partial T}{\partial y} \right] + \frac{\partial}{\partial y} \left(\sum_{i=1}^N \rho \bar{H}_i D \frac{\partial Y_i}{\partial y} \right) - \frac{\partial q_{Ry}}{\partial y} - \\ &- v_x \rho_e v_{xe} \frac{dv_{xe}}{dx} + \mu \left(\frac{\partial v_x}{\partial y} \right)^2 - \sum_{i=1}^N \bar{H}_i w_i \end{aligned} \quad (47)$$

Differential chemical species conservation:

$$\rho \frac{\partial Y_i}{\partial t} + \rho v_x \frac{\partial Y_i}{\partial x} + \rho v_y \frac{\partial Y_i}{\partial y} = w_i + \frac{\partial}{\partial y} \left(\rho D \frac{\partial Y_i}{\partial y} \right) \quad (48)$$

The boundary-layer equations for multicomponent, reacting, ideal-gas mixtures, Equations 45 to 48, are reduced to their 'locally similar' form by introducing a change of variables s , η that combines the Levy-Mangler and Howarth-Dorodnitsyn transformations^{20,24}. These change of variables are:

$$s = \int_0^x \rho_e \mu_e v_{xe} f^2 dx = s(x) \quad , \quad \eta = \frac{v_{xe} f}{\sqrt{2s}} \int_0^y \rho(x, y) dy = \eta(x, y) \quad (49)$$

The modified stream function ψ is then introduced, which satisfies the following equations:

$$\rho f v_x = \frac{\partial \psi}{\partial y} \quad , \quad \rho f v_y = -\frac{\partial \psi}{\partial x} \quad (50)$$

such that the differential mass balance is automatically satisfied. Next, a function $F(s, \eta, t)$ is introduced that is related to the modified stream function via:

$$F(s, \eta, t) = \frac{\psi(x, y, t)}{\sqrt{2s}} \quad (51)$$

The reason for the form of Equation 51 will become apparent when the velocity components in terms of the new variables defined by Equation 49 are calculated. From Equation 50:

$$v_x = v_{xe} \frac{\partial F}{\partial \eta} \quad , \quad \rho v_y = -\left(\frac{\rho_e \mu_e v_{xe} f}{\sqrt{2s}} \right) \left(F + 2s \frac{\partial F}{\partial s} + \frac{2s}{\rho_e \mu_e v_{xe} f^2} \frac{\partial F}{\partial \eta} \frac{\partial \eta}{\partial x} \right) \quad (52)$$

From Equation 43, 49 and 52 the differential momentum balance is transformed to:

$$\frac{\partial}{\partial \tau} \left[\frac{\partial F}{\partial \eta} \right] = \frac{\partial}{\partial \eta} \left[\frac{\rho \mu}{\rho_e \mu_e} \frac{\partial^2 F}{\partial \eta^2} \right] + F \frac{\partial^2 F}{\partial \eta^2} + \beta \left(\frac{\rho_e}{\rho} - \left(\frac{\partial F}{\partial \eta} \right)^2 \right) \quad (53)$$

where τ is a dimensionless time and β is a dimensionless parameter given by:

$$\tau = \frac{at}{\beta} \quad , \quad \beta = \frac{2as}{\rho_e \mu_e (v_{xe})^2 f^2} \quad (54)$$

From Equation 43, 49 and 52 the differential energy balance is transformed to:

$$\begin{aligned} \frac{\partial \Theta}{\partial \tau} = & \frac{\partial}{\partial \eta} \left[\frac{\rho \mu}{\rho_e \mu_e} \frac{1}{N_{Pr}} \frac{\partial \Theta}{\partial \eta} \right] + F \frac{\partial \Theta}{\partial \eta} + \frac{1}{\hat{c}_p (T_0 - T_\infty)} \frac{\partial}{\partial \eta} \left[\frac{\rho \mu}{\rho_e \mu_e} \frac{1}{N_{Sc}} \sum_{i=1}^N \bar{H}_i \frac{\partial Y_i}{\partial \eta} \right] - \\ & - \frac{\sqrt{2s}}{\rho_e \mu_e v_{xe} f \hat{c}_p (T_0 - T_\infty)} \frac{\partial q_{Ry}}{\partial \eta} - \beta \frac{\rho_e}{\rho} \frac{N_{Br}}{N_{Pr}} \frac{\partial F}{\partial \eta} + \frac{\rho \mu}{\rho_e \mu_e} \frac{N_{Br}}{N_{Pr}} \left(\frac{\partial^2 F}{\partial \eta^2} \right)^2 - \\ & - \frac{\beta}{\rho a \hat{c}_p (T_0 - T_\infty)} \sum_{i=1}^N \bar{H}_i w_i \end{aligned}$$

where Θ is a dimensionless temperature given by:

$$\Theta = \frac{T}{T_0 - T_\infty} \quad (55)$$

Assuming $N_{Br}/N_{Pr} \ll 1$, the differential energy conservation equation becomes:

$$\begin{aligned} \frac{\partial \Theta}{\partial \tau} = & \frac{\partial}{\partial \eta} \left[\frac{\rho \mu}{\rho_e \mu_e} \frac{1}{N_{Pr}} \frac{\partial \Theta}{\partial \eta} \right] + F \frac{\partial \Theta}{\partial \eta} + \frac{1}{\hat{c}_p (T_0 - T_\infty)} \frac{\partial}{\partial \eta} \left[\frac{\rho \mu}{\rho_e \mu_e} \frac{1}{N_{Sc}} \sum_{i=1}^N \bar{H}_i \frac{\partial Y_i}{\partial \eta} \right] - \\ & - \frac{\sqrt{2s}}{\rho_e \mu_e \nu_{xe} f \hat{c}_p (T_0 - T_\infty)} \frac{\partial q_{Ry}}{\partial \eta} - \frac{\beta}{\rho \alpha \hat{c}_p (T_0 - T_\infty)} \sum_{i=1}^N \bar{H}_i w_i \end{aligned} \quad (56)$$

From Equations 43, 49 and 52 the differential chemical species balance is transformed to:

$$\frac{\partial Y_i}{\partial \tau} = \frac{\partial}{\partial \eta} \left[\frac{\rho \mu}{\rho_e \mu_e} \frac{1}{N_{Sc}} \frac{\partial Y_i}{\partial \eta} \right] + F \frac{\partial Y_i}{\partial \eta} + \frac{\beta}{\rho \alpha} w_i \quad (57)$$

where $N_{Sc} = \mu / \rho D$.

DISCUSSION

The differential conservation equations of momentum, energy and chemical species for the flow of a multicomponent, reacting, ideal gas mixture past a fixed configuration body of revolution in the neighbourhood of a stagnation point are given by:

$$\begin{aligned} \frac{\partial}{\partial \tau} \left[\frac{\partial F}{\partial \eta} \right] = & \frac{\partial}{\partial \eta} \left[\frac{\rho \mu}{\rho_e \mu_e} \frac{\partial^2 F}{\partial \eta^2} \right] + F \frac{\partial^2 F}{\partial \eta^2} + \beta \left(\frac{\rho_e}{\rho} - \left(\frac{\partial F}{\partial \eta} \right)^2 \right) \\ \frac{\partial \Theta}{\partial \tau} = & \frac{\partial}{\partial \eta} \left[\frac{\rho \mu}{\rho_e \mu_e} \frac{1}{N_{Pr}} \frac{\partial \Theta}{\partial \eta} \right] + F \frac{\partial \Theta}{\partial \eta} + \frac{1}{\hat{c}_p (T_0 - T_\infty)} \frac{\partial}{\partial \eta} \left[\frac{\rho \mu}{\rho_e \mu_e} \frac{1}{N_{Sc}} \sum_{i=1}^N \bar{H}_i \frac{\partial Y_i}{\partial \eta} \right] - \\ & - \frac{\sqrt{2s}}{\rho_e \mu_e \nu_{xe} f \hat{c}_p (T_0 - T_\infty)} \frac{\partial q_{Ry}}{\partial \eta} - \frac{\beta}{\rho \alpha \hat{c}_p (T_0 - T_\infty)} \sum_{i=1}^N \bar{H}_i w_i \\ \frac{\partial Y_i}{\partial \tau} = & \frac{\partial}{\partial \eta} \left[\frac{\rho \mu}{\rho_e \mu_e} \frac{1}{N_{Sc}} \frac{\partial Y_i}{\partial \eta} \right] + F \frac{\partial Y_i}{\partial \eta} + \frac{\beta}{\rho \alpha} w_i \end{aligned}$$

where F is related to the modified stream function ψ via:

$$F(s, \eta, t) = \frac{\psi(x, y, t)}{\sqrt{2s}}$$

and η and s are dimensionless distances given by:

$$s = \int_0^x \rho_e \mu_e v_{xe} f^2 dx = s(x) \quad , \quad \eta = \frac{v_{xe} f}{\sqrt{2s}} \int_0^y \rho(x, y) dy = \eta(x, y)$$

and β is a dimensionless parameter, τ is a dimensionless time, and Θ is a dimensionless temperature given by:

$$\beta = \frac{2as}{\rho_e \mu_e (v_{xe})^2 f^2} \quad , \quad \tau = \frac{at}{\beta} \quad , \quad \Theta = \frac{T}{T_0 - T_\infty}$$

The assumptions in deriving the above equations are:

1. Planar flow defined by:

$$v_x = v_x(x, y, t), v_y = v_y(x, y, t), v_\theta = 0, T = T(x, y, t), Y_i = Y_i(x, y, t)$$

2. $\nabla \cdot \mathbf{v} = 0$, $\rho = \rho(x, y)$, $\mu = \mu(t)$, $P \neq P(t)$, $\mathbf{f}_i = 0$, $\mathbf{V}_i = (0, V_{y,i}, 0)$, $\mathbf{q}_R = (0, q_{Ry}, 0)$

3. Ideal gas:

$$\left(\frac{\partial \ln(\hat{V})}{\partial \ln(T)} \right)_{P, Y_j} = 1$$

4. $N_{Re} \gg 1$

5. Fixed wall configuration i.e. $\kappa^{**} \rightarrow 0$ for $N_{Re} \gg 1$

6. Soret and Dufour effects and pressure-gradient diffusion are neglected

7. The binary diffusion coefficients of all pairs of species are equal i.e. $D \equiv D_{ij}$

8. The strain rate, a , is constant

9. $N_{Br}/N_{Pr} \ll 1$

The assumption of planar flow (assumption (1)), the diffusion velocity vector given by $\mathbf{V}_i = (0, V_{y,i}, 0)$ and the radiant heat-flux vector given by $\mathbf{q}_R = (0, q_{Ry}, 0)$ are derived from geometrical considerations. The assumption that $\rho = \rho(x, y) \neq \rho(t)$ allows the introduction of a stream function ψ via Equation 50 such that the differential mass balance is automatically satisfied. The assumption that $\mu = \mu(t) \neq \mu(x, y)$ allows significant simplification of the stress tensor \mathbf{T} , defined by Equation 9.

In deriving the differential conservation equations it was assumed that the divergence of the velocity field, $\nabla \cdot \mathbf{v}$, was equal to zero. This is a mathematical assumption to simplify the derivation, not a physical assumption. The fact that $\nabla \cdot \mathbf{v} = 0$ does not imply that the flow is incompressible. From the differential mass conservation equation (Equation 5), if incompressible flow is assumed, the density, ρ , is constant, resulting in the differential mass conservation equation reducing to $\nabla \cdot \mathbf{v} = 0$. Thus,

constant density implies $\nabla \cdot \mathbf{v} = 0$, but initially assuming $\nabla \cdot \mathbf{v} = 0$ does not imply that the density is constant. Hence the *mathematical* assumption that $\nabla \cdot \mathbf{v} = 0$ does not imply that incompressible flow is assumed.

The assumption that the ideal-gas law is valid (assumption (3)) allows certain variables to be related to the dependent variables in the differential conservation equations via the ideal-gas equation of state:

$$P = \rho RT \sum_{i=1}^N \frac{Y_i}{W_i}$$

and also allows vast simplification of the differential energy conservation equations since:

$$\left(\frac{\partial \ln(\hat{V})}{\partial \ln(T)} \right)_{P, Y_j} = 1$$

Justification for the use of the ideal-gas law follows from the assumption of $\nabla \cdot \mathbf{v} = 0$ and $\mu = \mu(t) \neq \mu(x, y)$, which results in the stress tensor reducing to:

$$\mathbf{T} = -P\mathbf{I} + 2\mu\mathbf{D}$$

Performing the trace operation to both sides of the stress tensor gives the following expression:

$$P = -\frac{1}{3} \text{tr}(\mathbf{T})$$

From this equation it follows that, essentially by definition, the ideal-gas law is valid even for non-equilibrium systems²⁰.

Assumption (4) is simply a mathematical statement of the boundary layer approximation, and assumption (5) follows from the fact that only fixed boundaries are considered.

From Equation 41 it can be seen that Fick's Law is valid when assumptions (2) (specifically when $\mathbf{f}_i = 0$), (6) and (7) are satisfied. Because of the complex form of the multicomponent diffusion equation (Equation 39), in many combustion problems Fick's Law is utilised in order to make the governing equations tractable. The use of Fick's Law in combustion theory is partially justified by the fact that thermal diffusion (Soret effects) are negligible since the dimensionless ratio $D_{T,i} / \rho D_{ij} \ll 1$ for all pairs of gaseous species i and j , in general $\mathbf{f}_i = 0$, and pressure-gradient diffusion is negligible since $\nabla P / P \ll 1$. Thus concentration-gradient diffusion is the dominant part of the multicomponent diffusion equation.

In general, the assumption that the binary diffusion coefficients of all pairs of species are equal is the most difficult assumption to justify in combustion problems²⁰. If it is not assumed that the binary diffusion coefficients of all pairs of species are equal, the multicomponent diffusion equation with the remaining assumptions reduces to Equation 40, which is known as the Stefan-Maxwell Equation. This Equation is more complicated to employ in combustion theory than Fick's Law, since it is not readily solvable for the diffusion velocities \mathbf{V}_i in terms of concentration gradients – it instead provides concentration gradients in terms of diffusion velocities²⁰.

From Equation 16 the Dufour heat flux is given by:

$$RT \sum_{i=1}^N \sum_{j=1}^N \left(\frac{X_j D_{T,i}}{W_i D_{ij}} \right) (\mathbf{v}_i - \mathbf{v}_j)$$

In nearly all cases (not just combustion theory) the Dufour effect is so small that it is negligible, even when thermal diffusion (the Soret effect) is not negligible²⁰.

The assumption of constant strain rate, a (assumption (8)), is justified by the fact that the present study is concerned with stagnation flow and the analysis is restricted in the neighbourhood of a stagnation point. The assumption allows the use of Bernoulli's Law for the pressure distribution in the boundary layer (Equation 44) and results in a vast simplification of the transformation into the continuation equations by defining:

$$\frac{dv_{xe}}{dx} \equiv a$$

From the definitions of the Brinkman and Prandtl numbers it can be seen that the ratio $N_{Br}/N_{Pr} \ll 1$ is given by:

$$\frac{N_{Br}}{N_{Pr}} = \frac{\mu v_0^2}{\lambda(T_0 - T_\infty)} \times \frac{\lambda}{\hat{c}_p \mu} = \frac{v_0^2}{\hat{c}_p (T_0 - T_\infty)}$$

In general $v_0 \sim O(10^3)$ and $T \sim O(10^2)$ so that $v_0^2/\hat{c}_p(T_0 - T_\infty) \ll 1$, hence it may be assumed that $N_{Br}/N_{Pr} \ll 1$ in combustion problems. Note, however, that to retain the term involving the radiant heat flux in the y -direction, q_{Ry} , in Equation 31, this assumption is applied following the transformation of the differential conservation equations into the continuation equations. In other words, the assumption that $N_{Br}/N_{Pr} \ll 1$ is not a boundary-layer assumption like assuming $N_{Re} \gg 1$, but is a statement of considerations found in combustion problems.

The above assumptions results in a simplified transport model which has been applied previously (with modification) in many combustion studies^{5,10,12,23,25-38}. Many levels of accuracy of transport in multicomponent mixtures have been addressed in the literature^{36,39,40} and comparison of simplified transport models with more detailed ones for methane-air flames has shown that transport is not crucial for accurate modelling of flame structures and velocities⁴¹.

Finally, this paper should be considered a work in progress. A sound understanding of the equations that will be used to predict the extinction limits of flames is necessary before further work and new results are presented. The aim of this paper was to provide this sound understanding, and will be an important reference to later publications by the authors.

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NOMENCLATURE

A	strain rate
B_k	constant in the frequency factor for the k^{th} reaction
\hat{c}_p	specific heat capacity per unit mass at constant pressure
D	rate of deformation tensor
D	global binary diffusion coefficient
D_{ij}	binary diffusion coefficient for species i and j
$D_{T,i}$	thermal diffusion coefficient for species i
E_k	activation energy for the k^{th} reaction
f	function defined by $r = f(z)$
f^*	dimensionless function defined by $f^* = f/L_0$
\mathbf{f}_i	external/mutual force per unit mass on species i
F	function defined by $F(s, \eta, \tau) = \psi(x, y, t) / \sqrt{2s}$
g_{ij}	defined by $g_{ij} = \hat{\mathbf{g}}_i \cdot \hat{\mathbf{g}}_j$
$\hat{\mathbf{g}}_i$	orthonormal spatial vector field
h	defined by $h \equiv f[1 + f'^2]^{\frac{1}{2}} + y$
\bar{H}_i	partial mass specific enthalpy of species i
\bar{H}_i^*	dimensionless enthalpy defined by $\bar{H}_i^* = \bar{H}_i / \hat{c}_p (T_0 - T_\infty)$
I	unit tensor
\mathbf{j}_i	mass flux vector relative to \mathbf{v} for species i
L_0	characteristic length
M	total number of chemical reactions occurring
N	total number of chemical species present
N_{Br}	Brinkman number $N_{Br} = \mu v_0^2 / \lambda (T_0 - T_\infty)$
N_{Pr}	Prandtl number $N_{Pr} = \hat{c}_p \mu / \lambda$
N_{Re}	Reynolds number $N_{Re} = \rho v_0 L_0 / \mu$
N_{Ru}	Ruark number $N_{Ru} = \rho v_0^2 / P_0$

N_{Sc}	Schmidt number $N_{Sc} = \mu / \rho D$
N_{St}	Strouhal number $N_{St} = t_0 v_0 / L_0$
P	modified mean hydrostatic/thermodynamic pressure
P^*	dimensionless pressure defined by $P^* = P / P_0$
P_0	characteristic mean hydrostatic/thermodynamic pressure
q_{Ry}	y-component of radiant heat flux vector
q_{Ry}^*	dimensionless y-component of radiant heat flux vector defined by $q_{Ry}^* = q_{Ry} / \rho v_0^3$
q_{Ry}^{**}	dimensionless y-component of radiant heat flux vector defined by $q_{Ry}^{**} = \sqrt{N_{Re}} q_{Ry}^* = \sqrt{N_{Re}} q_{Ry} / \rho v_0^3$
\mathbf{q}	energy flux vector
\mathbf{q}_R	radiant heat flux vector
r	radial coordinate
R	universal gas law constant
s	dimensionless coordinate defined by $s = \int_0^x \rho_e \mu_e v_{xe} f^2 dx = s(x)$
\mathbf{S}	extra stress or viscous portion of the stress tensor
t	time
t_0	characteristic time
t^*	dimensionless time defined by $t^* = t / t_0$
T	temperature
T^*	dimensionless temperature defined by $T^* = (T - T_\infty) / (T_0 - T_\infty)$
T_0	reference temperature
T_∞	ambient temperature
\mathbf{T}	stress tensor
\hat{U}	internal energy per unit mass for the gas mixture
$v_{\langle i \rangle}$	component of \mathbf{v} in direction x^i
v_x	component of velocity defined by $v_x \equiv v_{\langle 1 \rangle}$
v_y	component of velocity defined by $v_y \equiv v_{\langle 2 \rangle}$

v_y^{**}	dimensionless y -component of velocity defined by $v_y^{**} = \sqrt{N_{\text{Re}}} v_y^* = \sqrt{N_{\text{Re}}} v_y / v_0$
v_θ	component of velocity defined by $v_\theta \equiv v_{(3)}$
v_0	characteristic magnitude of velocity
v_{xe}	free stream velocity
\mathbf{v}	mass-average velocity of the gas mixture
\mathbf{v}^*	dimensionless velocity defined by $\mathbf{v}^* = \mathbf{v} / v_0$
\mathbf{v}_i	velocity of species i
\mathbf{V}_i	diffusion velocity of species i
\mathbf{V}_i^*	dimensionless diffusion velocity of species i defined by $\mathbf{V}_i^* = \mathbf{V}_i / v_0$
$V_{y,i}$	y -component of \mathbf{V}_i
$V_{y,i}^{**}$	dimensionless y -component of \mathbf{V}_i defined by $V_{y,i}^{**} = \sqrt{N_{\text{Re}}} V_{y,i}^* = \sqrt{N_{\text{Re}}} V_{y,i} / v_0$
\hat{V}	volume per unit mass
W_i	molecular weight of species i
w_i	rate of production of species i by chemical reactions (mass per unit volume per unit time)
w_i^*	dimensionless reaction rate defined by $w_i^* = w_i t_0 / \rho$
x	coordinate defined by $x \equiv x^1$
x^*	dimensionless coordinate defined by $x^* = x / L_0$
x^i	orthogonal curvilinear coordinate
X_i	mole fraction of species i
y	coordinate defined by $y \equiv x^2$
y^*	dimensionless coordinate defined by $y^* = y / L_0$
y^{**}	dimensionless coordinate defined by $y^{**} = \sqrt{N_{\text{Re}}} y^* = \sqrt{N_{\text{Re}}} y / L_0$
Y_i	mass fraction of species i
z	axial coordinate
z^*	dimensionless axial coordinate defined by $z^* = z / L_0$

Greek Symbols

α_k	exponent determining the temperature dependence of the frequency factor for the k^{th} reaction
β	dimensionless constant defined by $\beta = 2as/\rho_e \mu_e (v_{xe})^2 f^2$
η	dimensionless distance defined by $\eta = (v_{xe} f / \sqrt{2s}) \int_0^y \rho(x, y) dy = \eta(x, y)$
κ	curvature/bulk viscosity coefficient
κ^*	dimensionless curvature defined by $\kappa^* = \kappa L_0$
κ^{**}	dimensionless curvature defined by $\kappa^{**} = 1/\sqrt{N_{Re}} \kappa^* = L_0/\sqrt{N_{Re}} \kappa$
λ	thermal conductivity
μ	coefficient of (shear) viscosity
μ_e	free stream viscosity
$v'_{i,k}$	stoichiometric coefficient for species i appearing as a reactant in reaction k
$v''_{i,k}$	stoichiometric coefficient for species i appearing as a product in reaction k
θ	coordinate defined by $\theta \equiv x^3$
Θ	dimensionless temperature defined by $\Theta = T/(T_0 - T_\infty)$
ρ	mass density
ρ_e	free stream density
ρ_i	mass density of species i
τ	dimensionless time defined by $\tau = at/\beta$
ψ	modified stream function

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