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IGNITION DUE TO SELF-HEATING IN A PLANE SLAB WITH A CONSTANT HEAT FLUX ON ONE FACE

by

P C Bowes

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SUMMARY

This note gives a theoretical analysis of ignition due to self-heating in a plane slab with one face exposed to a constant heat flux and with the other face maintained at a constant temperature. The results are applicable to practical situations such as the ignition of panels by nearby flue pipes and the ignition of thick layers of combustible dust on enclosed electrical equipment.

KEY WORDS: Ignition, self-heating, heat flux, (thermal radiation), plane slab

Department of the Environment and Fire Offices' Committee
Joint Fire Research Organization

LIST OF SYMBOLS

a	=	$\sqrt{\dfrac{\Phi}{2D} + e^{-\theta_o}}$
A	=	pre-exponential factor of Arrhenius equation
D	=	δe^{θ_o}
E	=	apparent activation energy of heat generation reaction
F	=	incident flux
M	=	$e^{\theta_o/2}$
Q	=	heat of reaction per unit mass of slab
q(T)	=	rate of heat generation at temperature T
R	=	universal gas constant
r	=	half-thickness of slab
T	=	temperature (Kelvin), subscripts P and S refer to hot surface and cool surface of slab respectively
x	=	distance from hot face of slab
z	=	x/r
δ	=	Frank-Kamenetskii self-heating parameter, defined for equation (3)
δ_c	=	critical value of δ
$\delta_c(T_s)$	=	δ_c defined in terms of T_s instead of T_p
ϵ	=	surface emissivity; subscripts 1, 2, 3 refer to slab, surroundings and pipe respectively
θ	=	dimensionless temperature
θ_o	=	dimensionless temperature at cool surface of slab
θ_s	=	dimensionless temperature defined in terms of T_s instead of T_p
λ	=	thermal conductivity of slab
ρ	=	density of slab
Φ	=	dimensionless flux, defined for boundary condition on equation (3)
ψ	=	$\theta_o - \theta$

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INTRODUCTION

A number of practical examples of ignition due to self-heating occur in slabs or layers of material with one face exposed to a source of heat and with the other face losing heat to cooler surroundings. Commonly, the face exposed to the heat source will either receive a constant flux or be maintained at a constant elevated temperature.

Frank-Kamenetskii's 'stationary state' model of thermal explosion[1] has been extended to systems of the above kind although with greater attention being paid to those with a constant temperature at the hot face. For this particular boundary condition, theoretical results are available for plane slabs[2] and for thick cylindrical[3–5] and spherical shells[5] (with hot inner faces) and they have been applied to the analysis of experimental observations of the ignition of dust layers on hot surfaces[6] and of oil-soaked lagging on hot pipes[7].

Theoretical results for the slab exposed to a constant flux have been obtained by Clemmow & Huffington[8], both in forms capable of high accuracy but requiring considerable computation and in a more easily used form (their method 3) involving an approximation for the Arrhenius relationship akin to Frank-Kamenetskii's well-known approximation. However, their analysis is in terms of parameters which are not related in a simple way to those which have become conventional in thermal explosion theory and it has been pointed out[2] that, at least for the case of the plane slab with one face at a constant high temperature, their approximation gives considerably less accurate results than a straight-forward application of the Frank-Kamenetskii approximation with an appropriate choice of reference temperature.

This note presents the analysis for the constant flux problem using the Frank-Kamenetskii approximation and the conventional parameters. The results are related to those of associated problems and are used to predict ignition in a wood-based material under some practical conditions of exposure to a constant heat flux.

THEORETICAL

General

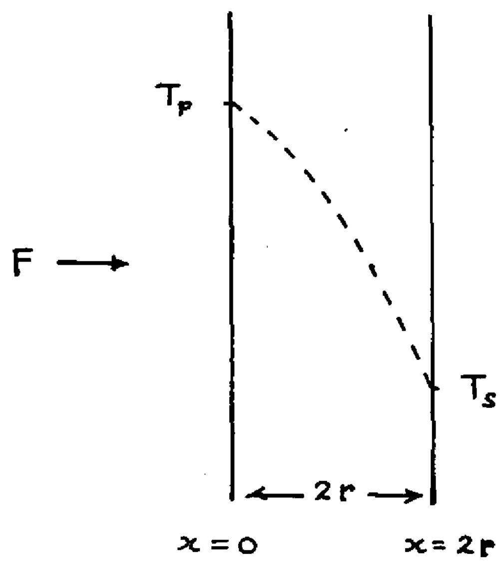

FIG. 1. PLANE SLAB EXPOSED TO CONSTANT HEAT FLUX

Consider a plane slab, as Fig. 1, of thickness 2r (for conformity with earlier analyses) with a heat flux F incident on one face at $x = 0$ and with the other face, at $x = 2r$, in perfect contact with a heat sink at a constant temperature T_s. Heat losses in the negative direction at $x = 0$ will be assumed initially to be zero.

Let heat be generated in the solid by a chemical reaction whose rate depends on temperature in accordance with the Arrhenius equation (rate $\propto \exp(E/RT)$) and does not depend on time. Then, if steady state temperature distributions can exist within the slab, they will be given, for heat transfer by conduction, by solutions of the following equation:

$$\lambda \frac{d^2T}{dx^2} = -q(T) \qquad (1)$$

where

T = temperature in plane at distance x from the exposed face

λ = thermal conductivity of solid

$q(T)$ = rate of heat generation per unit volume at temperature T and is given by:

$$q(T) = \rho Q A e^{-E/RT}$$

where

ρ = density of solid

Q = heat of reaction per unit mass

A = pre-exponential factor of Arrhenius equation

E = 'activation energy' in Arrhenius equation

R = universal gas constant.

The boundary conditions for steady states are:

at $x = 0$,

$$-\lambda \frac{dT}{dx} = F$$

and, at $x = 2r$,

$$T = T_s$$

The surface temperature T_p , at $x = 0$, is a dependent variable in this problem.

In deriving solutions for equation (1) it is convenient to introduce the Frank-Kamentskii approximation to the exponential in the heat generation term immediately, choosing the dependent variable T_p as reference temperature, and to defer justification until later.

The approximation is

$$-\frac{E}{RT} \equiv -\frac{E}{RT_p}\left(1 + \frac{T - T_p}{T_p}\right)^{-1} \approx -\frac{E}{RT_p} + \theta \qquad (2)$$

where

$$\theta = \frac{E}{RT_p^2}(T - T_p)$$

θ is the well-known dimensionless temperature but is not a physically meaningful dependent variable in this problem unless T and T_p are measured from a given temperature. We therefore introduce a new dependent variable, ψ , defined as

$$\psi = \theta_0 - \theta$$

where

$$\theta_0 = \frac{E}{RT_p^2}(T_s - T_p)$$

and, therefore,

$$\psi = \frac{E}{RT_p^2}(T_s - T)$$

With the above approximation, and putting $z = x/r$, equation (1) can be written in the dimensionless form

$$\frac{d^2\psi}{dz^2} = D e^{-\psi} \tag{3}$$

where

$$D = \delta e^{\theta_0}$$

and δ, the Frank-Kamenetskii self-heating parameter, has the usual form

$$\delta = \frac{E}{RT_p^2} \cdot r^2 \cdot \frac{\rho Q A}{\lambda} e^{-E/RT_p}$$

The boundary conditions become:

at $z = 0$,

$$\frac{d\psi}{dz} = \Phi \tag{4i}$$

where

$$\Phi = \frac{E}{RT_p^2} \cdot \frac{Fr}{\lambda},$$

and, at $z = 2$,

$$\psi = 0 \tag{4ii}$$

We also have, by definition, $\psi = \theta_0$ at $z = 0$.

In accordance with the Frank-Kamenetskii thermal explosion model we seek maximum values of the parameter D in solutions of equation (3) satisfying the boundary conditions (4i) and (4ii) and, thence, sets of values of the parameters δ, Φ and θ_0 which, by hypothesis, comprise limiting conditions for the existence of steady states and, a fortiori, critical conditions for explosion or ignition.

The solution of equation (3) subject to conditions (4i) and (4ii) is as follows:

$$e^{\psi/2} = \frac{1}{a} \cosh\left[\cosh^{-1} a + a\sqrt{2D}\left(\frac{z}{2} - 1\right)\right] \tag{5}$$

where

$$a^2 = \frac{\Phi^2}{2D} + e^{-\theta_0}$$

At $x = 0$, equation (5) gives

$$\sqrt{2D} = \frac{1}{a}\left[\cosh^{-1} a - \cosh^{-1}\left(a\, e^{\theta_0/2}\right)\right] \tag{6}$$

For a given value of Φ , D will be a maximum with respect to θ_0 when $dD/d\theta_0 = 0$. Whence, from equation (6) and the definition of a it is found that D is a maximum when

$$\sqrt{2D} = \frac{1}{\sqrt{a^2 - 1}} + M\sqrt{a^2 M^2 - 1} \tag{7}$$

where

$$M = e^{\theta_0/2}$$

It follows from the definition of D that

$$\frac{dD}{d\theta_0} = e^{\theta_0}\left(\delta + \frac{d\delta}{d\theta_0}\right)$$

Hence $dD/d\theta_0$ and $d\delta/d\theta_0$ cannot be zero together and, when D is a maximum, δ assumes a highest value which is not itself a maximum as in other problems where δ is defined in terms of an independent rather than a dependent temperature

Eliminating D between equations (6) and (7) and expressing the inverse hyperbolic functions in their logarithmic forms, we have

$$\ln \frac{a + \sqrt{a^2 - 1}}{aM + \sqrt{a^2 M^2 - 1}} = \frac{a}{\sqrt{a^2 - 1}} + aM\sqrt{a^2 M^2 - 1} \tag{8}$$

Equations (8) and (6) may be used to calculate maximum values of D and, thence, associated values of δ and θ_0 - both as functions of Φ . These values of δ are the required critical values (which will be denoted δ_c) and with the associated values of θ_0 correspond to the highest rates of heat generation and the highest temperature differences possible without ignition with fluxes corresponding to the associated values of Φ incident on slabs of thickness $2r$ and with the unexposed surfaces at temperature T_s .

Limiting cases

It follows from the definition of a that, when $\Phi = 0$, ie when the incident flux is zero,

$$a^2 = e^{-\theta_0} \quad \text{and} \quad a^2 M^2 = 1$$

Then, from equations (6) and (7),

$$a = \cosh \frac{a}{\sqrt{a^2 - 1}} = 1{\cdot}8102 \ldots \tag{9}$$

It follows, then, that

$$\theta_o = -1{\cdot}19 \quad \text{and} \quad 4D = 4\,\delta_c e^{\theta_o} = 0{\cdot}88$$

These are the values obtained by Frank-Kamenetskii[1] for the symmetrically heated slab, ie with a constant temperature on both faces and $dT/dx = 0$ at the mid-plane. Thus, at $\Phi = 0$, the ignition condition for the asymmetrically heated slab reduces to that for one half of the symmetrically heated slab - as is to be expected. The value of δ_c , as defined here in terms of T_p and with r equal to one quarter of the thickness of the symmetrically heated slab, is 0.72.

When the rate of heat generation approaches zero and, therefore, $\delta_c \to 0$, equation (6) approaches

$$-\theta_o = 2\Phi \tag{10}$$

or

$$F = \lambda \frac{T_p - T_s}{2r}$$

which is, of course, the equation for steady heat conduction in an inert slab.

It may be noted further that, when $\Phi = 0$ and $aM = 1$, equation (6) becomes

$$\cosh^{-1}\left(e^{-\theta_o/2}\right) = \sqrt{2D}\; e^{-\theta_o/2}$$

or

$$\ln\left(e^{-\theta_o/2} + \sqrt{e^{-\theta_o} - 1}\right) = \sqrt{2\delta_c}$$

For $e^{-\theta_o/2} \gg 1$ (θ_o being negative), this reduces to

$$\delta_c = \frac{(2\ln 2 - \theta_o)^2}{8} = \frac{(1{\cdot}4.. + |\theta_o|)^2}{8} \tag{11}$$

This is the result obtained elsewhere[2] for the ignition of plane slabs with one face maintained at a constant high temperature and with the condition for criticality taken as $d\theta/dx = 0$ at the hot face, which is an approximation valid for large values of $|\theta_o|$. However, equation (11) represents limiting steady states which, except for that where $\theta_o = -1.19$, are excluded by the criticality condition appropriate here, ie equation (7). These states, except the one, are obviously physically inaccessible in the constant flux problem when $\Phi = 0$.

Introducing θ_s defined as $E(T_p - T_s)/RT_s^2$, δ_c defined in terms of T_s instead of T_p (and denoted by $\delta_c(T_s)$) is given by

$$\delta_c(T_s) \approx \delta_c e^{\theta_s}$$

From equation (11), we then have, for $\theta_s \gg 1$,

$$\delta_c(T_s) \approx \theta_s^2 e^{-\theta_s} / 8 \tag{12}$$

Thus solutions of equation (11), which correspond to the upper branch of solutions of equation (12), are seen to be analogous to the known <u>unstable</u> steady states of the symmetrically heated slab which, for the Semenov model, are given accurately by the upper branch of the equation

$$\delta_c(T_s) = \theta_s e^{-\theta_s}$$

This analogy is of interest here but is not, of course, sufficient to establish the instability of states represented by solutions of equation (11) in this problem. This is outside the scope of the present paper.

<u>Numerical values</u>

Selected values of δ_c, θ_o and Φ calculated from equations (6) and (8) are given in Table 1 for ranges of practical interest and δ_c and θ_o are plotted in Fig.2 as functions of Φ

Although not readily deduced from the analysis, it will be seen from Fig.2 that δ_c and θ_o vary almost linearly with Φ over the ranges covered. Empirical equations calculated for the substantially linear ranges are

$$\delta_c = 0.614 + 0.489\,\Phi \tag{13}$$

Table 1

Selected values of critical parameters

Φ	δ_c	θ_o
0.4201	0.8892	- 2
1.444	1.339	- 4
2.459	1.813	- 6
3.469	2.298	- 8
5.479	3.282	-12
8.486	4.771	-18

for

$$1{\cdot}444 \leqslant \Phi \leqslant 8{\cdot}486$$

and

$$\theta_o = -(1{\cdot}120 + 1{\cdot}987\Phi) \tag{14}$$

for

$$0{\cdot}934 \leqslant \Phi \leqslant 8{\cdot}486$$

In the region where these linear relationships apply, we may write

$$\delta_c = 0{\cdot}338 - 0{\cdot}246\ \theta_o \tag{15}$$

Errors are less than 2% within the indicated ranges of Φ ; they are greatest at $\Phi = 0$, where the error is 18% for δ_c and 7% for θ_o .

Comparing equation (14) with equation (10) it will be seen that the critical temperature increase for the self-heating slab exposed to a constant flux is approximately equal to the sum of the critical temperature increase for the symmetrically heated slab ($\theta_o = -1.19$) and the temperature difference which would be established across the slab by the incident flux if the slab were inert ($\theta_o = -2\Phi$).

Comparison with Clemmow and Huffington and accuracy of results

The relationships between the parameters ϕ_o , l and h used by Clemmow and Huffington[8] and those used here are as follows:

$$\phi_o = -\frac{T_p}{T_s}\,\theta_o$$

$$l^2 = 8\,\frac{T_p}{T_s}\,\delta_c\,e^{-\phi_o}$$

$$h = 2\,\frac{T_p}{T_s}\cdot\frac{\Phi}{l}$$

When the critical temperature rise ($T_p - T_s$) is small, the ratio T_p/T_s may be taken as unity but, otherwise, the relationship between the numerical results of the two systems will depend on T_p/T_s and will vary from one problem to another. This situation arises from the different approximations used for the Arrhenius term in equation (1).

The accuracy of the different approximations has already been discussed[2] in connection with their application to the problem of the slab with one face at a constant high temperature. It is necessary here to cover some of this ground again because, as will be seen, the conclusions for the constant flux problem are slightly different.

As stated earlier, the approximation used here is Frank-Kamenetskii's, namely

$$\exp(-E/RT) \approx \exp(-E/RT_p)\,\exp\left[E(T-T_p)/RT_p^2\right] \qquad (16)$$

and, based as it is on truncation of a binomial expansion of the form $(1 + n)^{-1}$ where $n < 1$, it is valid strictly only for $(T - T_p)/T_p \ll 1$.

Physically, the use of this approximation in the problem of the plane slab with one face at a constant high temperature[2] was justified on the grounds that the approximation was good in the neighbourhood of T_p where the rate of heat generation was high and, in problems of practical interest for which values of RT_p/E tended to be small, the rate of heat generation decreased rapidly with decreasing temperature and became negligible towards the cool face of the slab where, of course, the approximation was most seriously in error. It was then possible to show, from series integration of the exact equation (1) for fixed temperature at the hot and cool faces, that the error in δ_c introduced by the approximation was less than about 9% for $RT_p/E = 0.04$ (a typical value in practical applications).

An error of this magnitude, which is only about twice that arising from the use of the Frank-Kamenetskii approximation for symmetrical heating, is quite acceptable - corresponding in practice to an error of about 1K in estimates of critical temperature and about 5% in critical size.

The approximation used by Clemmow and Huffington is given in the present notation by

$$\exp(-E/RT) \approx \exp(-E/RT_s)\,\exp[E(T-T_s)/RT_sT_p] \quad ,$$

which can be re-written as

$$\exp(-E/RT) \approx \exp(-E/RT_p)\,\exp[E(T-T_p)/RT_sT_p] \qquad (17)$$

With this modified Clemmow and Huffington approximation, δ_c becomes

$$\delta_c = \frac{E}{RT_sT_p} \cdot r^2 \cdot \frac{\rho Q A}{\lambda} e^{-E/RT_p}$$

Approximation (17) is exact at both T_p and T_s. It follows from equation (15) that

$$\delta_c \propto \theta_o \quad \text{for} \quad \theta_o \gg 1$$

and accordingly, approximations (16) and (17) tend here to be almost equally satisfactory in practical applications. This is unlike the case of the plane slab with one face at a constant high temperature (equation 11) where we have

$$\delta_c \propto \theta_o^2 \quad \text{for} \quad \theta_o \gg 1$$

In this case, as has been pointed out elsewhere[2], estimates of, for example, heat production from the model using approximation (17) will tend to be too large by a factor of T_p/T_s. Estimates of critical size will tend to be too large by $\sqrt{T_p/T_s}$. In practical cases, T_p/T_s can be as large as 2.

Because, again, for the case of constant flux, δ_c tends to depend on θ_o rather than on θ_o^2 we may expect the error in δ_c arising from the use of the

Frank-Kamentskii approximation to tend towards half the value of 9% estimated for the case of constant temperature[2].

The model is otherwise subject to limitations arising from the usual assumptions of inexhaustible reactant and of physical constants independent of temperature. With large temperature gradients in the slab at criticality, these limitations may be more severe than for the case with symmetrical heating but, at present, there is no available analysis of their magnitude. For the time being their effect will be ignored – as elsewhere[2-7].

APPLICATIONS

General

In most real situations, the above model will represent only the kernel of the problem. Generally there will be heat losses at a finite rate by radiation and convection from both surfaces of the slab to cool surroundings. Use of the model then requires the internal and external heat transfer rates to be matched at the surfaces of the slab. This can readily be effected by trial and error.

Given finite heat losses from the cool face of a slab, two different practical situations can be envisaged. The first, with zero heat loss from the heated face and, the second, with finite heat loss from the heated face. The first will correspond to a continuous source of heat in an enclosure covered by a combustible layer on the outside, such as a large electric motor (with a smooth surface) covered with a thick layer of sawdust in a sawmill. The second will apply to examples such as a combustible panel exposed to a hot flue pipe. Some calculations for typical cases are given below using data obtained elsewhere[6,4] for ignition of wood fibre insulating board – which has been chosen as a wood-based material having self-heating properties of practical significance at elevated temperatures.

Data

The constants governing heat generation in the wood-fibre insulating board, and values for thermal conductivity and density are as follows[4]:

$$\rho Q A = 1.45 \times 10^{15} \ \mathrm{W/m^3}$$

$$E/R = 12650 \ \mathrm{K}$$

$$\lambda = 5.0 \times 10^{-2} \ \mathrm{W/mK}$$

$$\rho = 270 \ \mathrm{kg/m^3}$$

whence, from the definition of δ_c and θ_o, we have

$$\delta_c = 3{\cdot}65 \times 10^{20} \frac{r^2}{T_p^2} e^{-12650/T_p} \tag{18}$$

$$\theta_o = \frac{12650}{T_p^2} (T_s - T_p) \tag{19}$$

Criticality with zero heat loss at heated face of slab

The solution of problems in this class consists of equating the internal heat flux at the cool surface of the slab at criticality with the heat loss to the surroundings. Using equation (5) to determine $d\psi/dx$ at $x = 2$, the critical internal flux at the cool face is given by the following equation:

$$-\lambda \frac{dT}{dx} = \frac{\lambda}{r} \cdot \frac{RT_p^2}{E} \sqrt{\Phi^2 + 2\delta_c(1 - e^{\theta_o})} \tag{20}$$

This equation can be simplified in the obvious way when $|\theta_o| \gg 1$.

For comparison with experimental results obtained for ignition of slabs of wood-fibre insulating board on a horizontal hot surface at constant temperature[6], we use the appropriate expression for the rate of heat loss by natural convection and radiation from a horizontal surface facing upwards in air[9]. Whence we have, at the cool surface (SI units):

$$-\lambda \frac{dT}{dx} = 1{\cdot}32 \frac{(T_s - T_o)^{1\cdot25}}{L^{0\cdot25}} + 5{\cdot}75 \times 10^{-8} \epsilon_1 \epsilon_2 (T_s^4 - T_o^4) \tag{21}$$

where

L = diameter of slab (m)
T_o = temperature of surroundings (K) (ambient temperature)
ϵ_1, ϵ_2 = emissivities of surface of slab and surroundings respectively.

Application to a given problem then involves solution of the set of simultaneous equations (18), (19), (20), (21), (13) and (14) or, where Φ is too small for equations (13) and (14) to be used, equations (6) and (8).

A convenient procedure for use with a desk calculator, given a slab of thickness 2r and a given ambient temperature, is to assume a value for T_s and find a value for T_p by trial which will allow equations (15), (18) and (19) to be satisfied simultaneously. Then, with the corresponding value for Φ obtained from either of equations (13) and (14), the chosen value of T_s is tested in equations (20) and (21). This process is repeated until equations (20) and (21) can be satisfied simultaneously.

Solutions for slabs of wood-fibre insulating board of two thicknesses and an ambient temperature of 25°C are given in Table 2. The emissivity of the wood fibre insulating board has been taken as 0.9 and that of the surroundings as unity.

Table 2

Computed critical data for ignition of horizontal plane slabs of wood-fibre insulating board exposed to a constant heat flux on the underside and to surroundings at 25°C

Thickness of slab mm	Ambient temperature °C	θ_o	δ_c	Φ	Critical flux W/m^2	Hot surface temperature °C	Cool surface temperature °C
14	25	- 8.05	2.32	3.49	547	253	76
41	25	- 8.91	2.53	3.92	176	208	44

Figure 3 compares the critical temperature distribution (sketched) for ignition of the 41 mm slab of wood-fibre insulating board exposed to a constant flux with the temperature distribution for ignition under conditions of constant temperature at the hot face - for which the indicated hot face temperature is the value found experimentally.

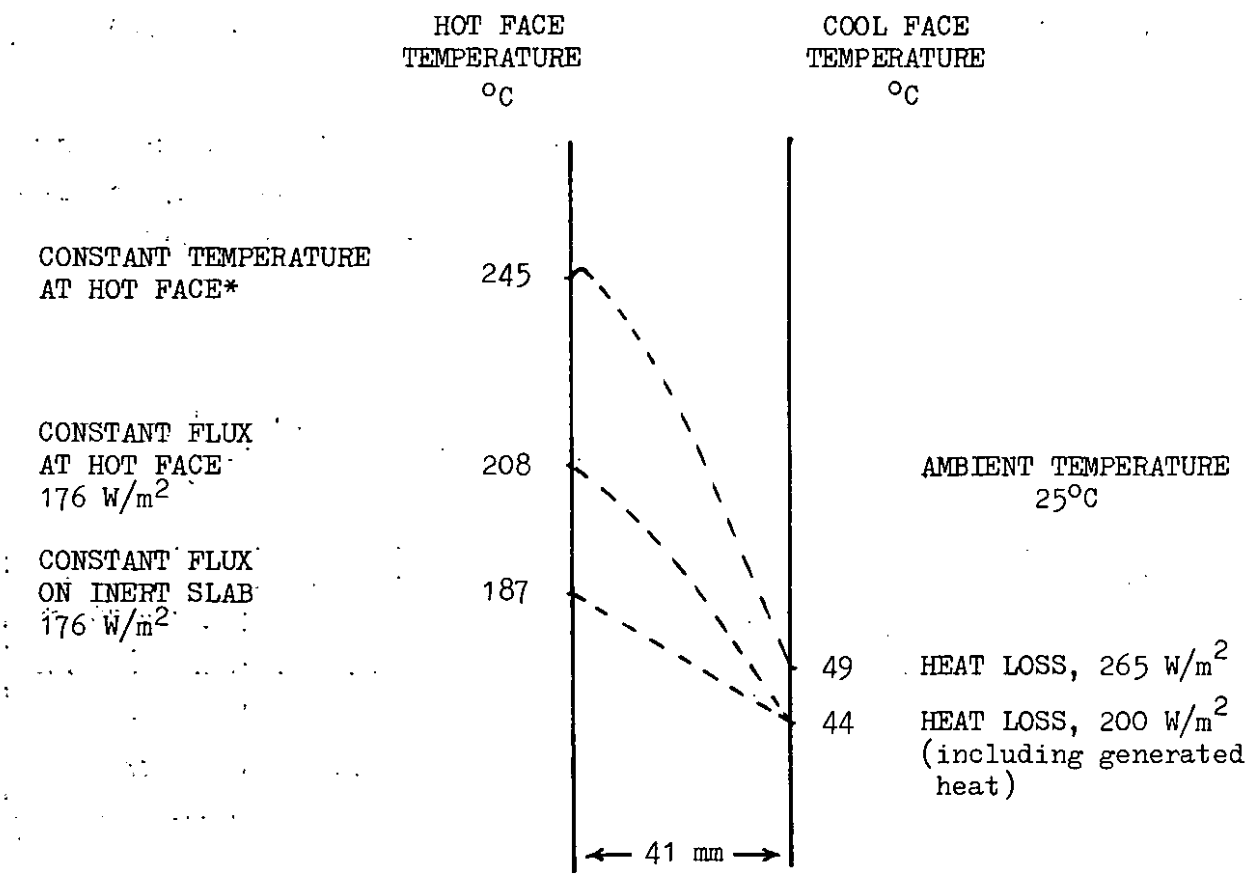

FIG.3. COMPARISON OF CRITICAL TEMPERATURE DISTRIBUTION IN 41 MM SLAB OF WOOD-FIBRE INSULATING BOARD FOR TWO MODES OF SELF-IGNITION

*Note that temperature gradient here is actually positive at the hot face and a small temperature maximum occurs close to the hot face when θ_o is large.

The critical temperature rise under constant flux conditions is 21°C higher than for the inert slab and, as indicated earlier, is comparable with the critical temperature rise for a symmetrically heated slab. Under constant temperature conditions at the hot face where, effectively, the system is stabilised by contact with an infinite isothermal reservoir, this critical temperature rise can be, and indeed is, considerably higher.

Criticality with finite heat loss from the heated face of the slab

Lawson et al[10] have evaluated configuration factors for the radiant heating of vertical plane panels exposed to hot vertical flue pipes of circular section and any length. It is convenient to use this arrangement as an example, but it will be necessary to assume that the lack of uniformity of temperature in the plane of the panel can be ignored and that criticality can be expressed in terms of the thermal equilibrium along the line of nearest approach of the panel to the cylindrical pipe.

Assuming that the panel is large enough for turbulent convective heat transfer[9] at the hot face, the thermal equilibrium at criticality can be written as

$$-\lambda \frac{dT}{dx} = 5.75 \times 10^{-8}\left[\phi\, \epsilon_1 \epsilon_3 (T_f^4 - T_p^4) - (1-\phi)\epsilon_1\epsilon_2 (T_p^4 - T_0^4)\right] - 1.31 (T_p - T_0)^{1.33} \quad W/m^2 \qquad (22)$$

where ϕ = configuration factor at panel along line of nearest approach to pipe

T_f = temperature of flue pipe surface (K)

ϵ_3 = emissivity of flue pipe surface

The convective heat loss from the cool face of a large vertical panel differs by only about 18% from the convective loss calculated above for the small horizontal slabs. Therefore for illustrative purposes, the data in Table 2 may be matched directly to equation (22) with little error.

Taking the emissivity of the pipe surface as unity and, as an example, a value of 0.5 for the configuration factor, values of the pipe temperature, T_f, necessary to give nett fluxes on the panel equal to those in Table 2 may be calculated as 424°C for the 14 mm panel and 343°C for the 41 mm panel. The actual incident fluxes will then be 4.1 kW/m^2 for the 14 mm and 2.3 kW/m^2 for the 41 mm panel. The excess over the values in Table 2 corresponds to the convective and radiative heat losses from the hot face of the panel.

A configuration factor of 0.5 corresponds, for a long pipe, to a pipe with its axis at a distance of one diameter from the panel; in particular, for a pipe of 100 mm diameter, the shortest distance between the surface of the pipe and the panel for safety will be 50 mm.

DISCUSSION

Applied to the problem of thermal explosion in a plane slab with one face exposed to a constant flux, a modified Clemmow and Huffington approximation and the Frank-Kamenetskii approximation are almost equally accurate. This is contrary to the result obtained when these approximations are applied to the 'constant temperature' problem - where the latter approximation is markedly superior. However, use of the Frank-Kamenetskii approximation* is preferable in that it yields solutions to the 'constant flux' problems which are of considerable simplicity in a region of practical utility and, at the same time, provides deeper insight into the physical significance of these solutions.

The numerical estimates of critical ignition conditions for wood-fibre insulating board need experimental confirmation but, for the time being, may be accepted as reasonably sound for practical purposes - especially as they are based on experimental data obtained under asymmetrical conditions of heating.

The 'steady state' model, of course, provides no estimates of times to ignition. However, experimental observations indicate that for the range of dimensions considered these times will be of the order of from $\frac{1}{2}$ h to several hours.

The incident flux required for ignition of wood-fibre insulating board by self-heating, and in the presence of heat losses from the exposed face, is at least an order of magnitude lower than the minimum intensities, about 40 kW/m^2, observed by Simms for spontaneous ignition under similar conditions[11-13] with the production of flames within short periods (25 s or so). This spontaneous ignition occurred in the volatile thermal decomposition products. Under these conditions of relatively high flux and rapid heating, experimental results were correlated adequately by a theoretical model which regarded the exposed material as a semi-infinite solid and related ignition times to the attainment of a certain surface temperature - namely, about 500°C for the spontaneous production of flame. It was possible to ignore the contribution of self-heating within the material, if it occurred at all under these conditions of exposure. Clearly there will be a region, between the extremes represented by the pure self-heating case considered in the present paper and the rapid heating case studied by Simms, where a more comprehensive model will be needed. Such a model will be considerably more complex than either since it will need to take account of the kinetics and thermal effects of pyrolysis, including diffusion and changes of properties, in addition to oxidative self-heating in both the solid and gas phases.

It may be estimated that, for the ignition of wood-based materials, the heat generation is only a small fraction of the incident flux required for ignition and,

* Together with the conventional variables

in any experimental study, will be difficult to measure. Thus, the heat due to reaction in the slab which leaves the cool face at criticality is the difference between the heat loss to the surroundings at the cool face and the nett flux absorbed at the exposed face. For the 41 mm slab, this is 200-176 = 24 W/m^2, which is 13% of the critical incident flux for ignition with zero heat loss from the exposed face and only 0.1% of the critical incident flux with heat losses from the exposed face.

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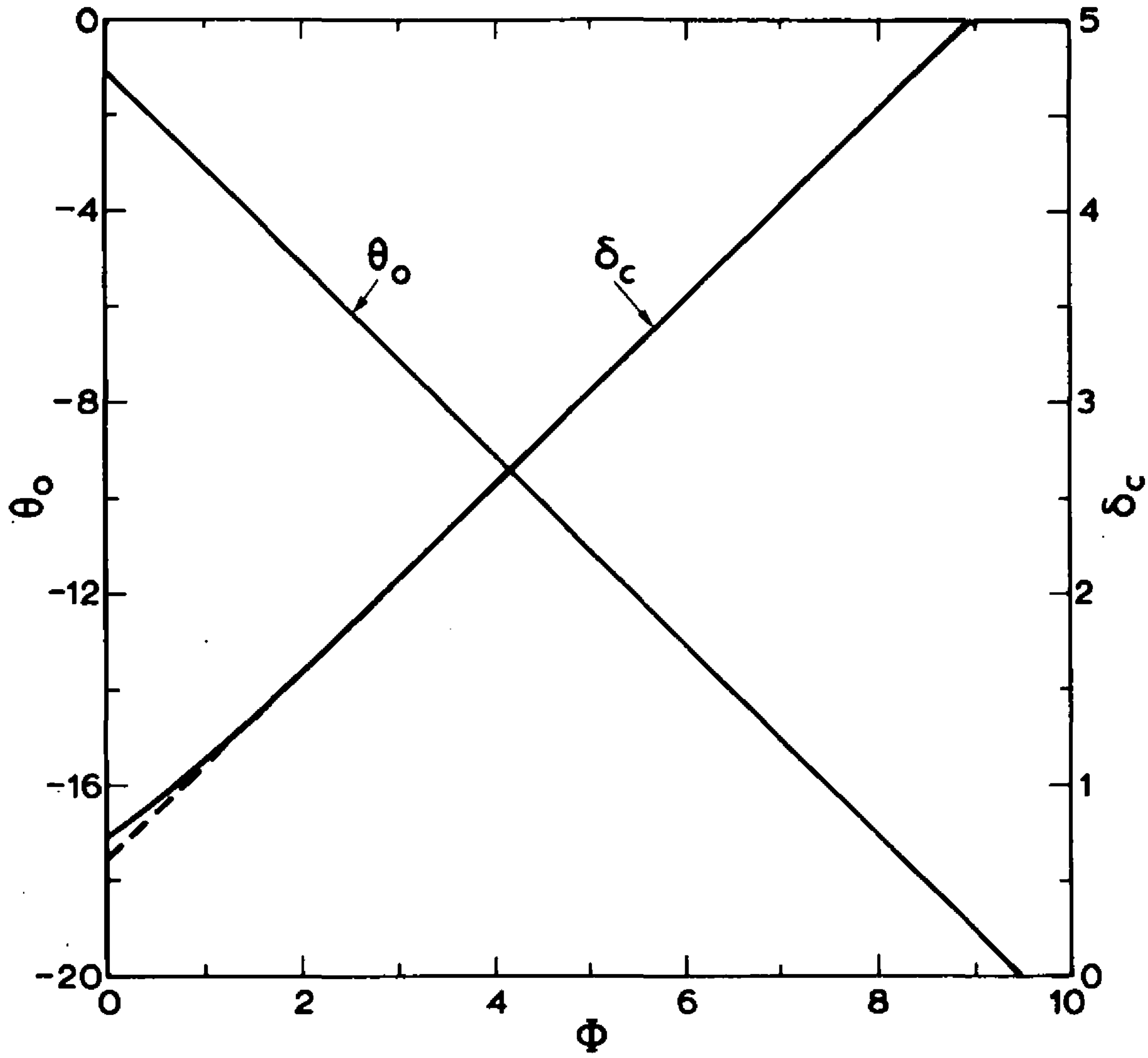

Figure 2 Ignition of plane slab exposed to heat radiation on one face— critical parameters

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