Fire Research Note No 1076

THE FLOW OF BUOYANT FIRE GASES BENEATH CORRIDOR CEILINGS: A THEORY

by

Howard P Morgan

August 1977

FIRE RESEARCH STATION

Fire Research Station
BOREHAMWOOD
Hertfordshire WD6 2BL

Tel: 01 953 6177

Fire Research Note No. 1076
August 1977

THE FLOW OF BUOYANT FIRE GASES BENEATH CORRIDOR CEILINGS: A THEORY

by

Howard P Morgan

SUMMARY

Formulae for the mass flux and velocity beneath corridor ceilings of buoyant fire gases are derived theoretically. The theory is based on assumptions applying to simple corridors, of the type often occurring in practice, and on the application of the Steady Flow Energy Equation to the flowing layer. Practical applications of these formulae in calculating flows in corridors, tunnels and malls are discussed.

Department of the Environment Fire Research Station of the
Building Research Establishment

SYMBOLS

C_p	Specific heat of air at constant pressure
C_v	Coefficient of discharge
d_1	Layer depth below ceiling
d_s	Depth of corridor exit
g	Acceleration due to gravity
h	Corridor height
k	Velocity-pressure Coefficient
M	Mass flux in the layer
p	Gas/air pressure
Q	Heat flux in the layer
T	Absolute temperature of air and/or fire gases
ΔT	Temperature of fire gases above ambient temperature
v	Velocity of layer gases
$\bar{v}$	Mean velocity of the layer
w	Width of corridor exit
W	Width of corridor
x	Height above the corridor floor
y	Height above the corridor floor (used in integration)
β	A constant of proportionality
$\mathcal{K}$	"Profile correction factor"
Λ	Denotes a location in the corridor
ρ	Air/gas density

SUBSCRIPTS

a	Ambient value of a variable
c	Value of a variable at the ceiling
n	Value of a variable at obstacle n
v	Value of a variable at the vena contracta
Λ	Value of a variable at position Λ

THE FLOW OF BUOYANT FIRE GASES BENEATH CORRIDOR CEILINGS: A THEORY

by

H P MORGAN

1. INTRODUCTION

The flow of fire gases beneath a horizontal ceiling is of interest in a number of situations. Leaving aside the particularly intractable problem of three-dimensional radial outflow from a fire-plume impinging on a ceiling, the simpler two-dimensional problem of flow beneath a corridor ceiling is important for tunnels and shopping malls, as well as for corridors. Knowledge of such flows is particularly important in the design of smoke-extraction systems for shopping malls and for tunnels. In the remainder of this note the term "corridor" should be taken to include both tunnels and malls.

In a corridor, a flowing stream of buoyant gases is usually characterised by a free lower surface with ambient temperature air occupying the lower part of the corridor.

The advance of the leading edge of such a stream has been described by Hinkley.[1] The behaviour of an established horizontal flow, regarded as a steady-state process, was briefly discussed in the same note. An approach based on the Froude number of the flow, led to an expression for the mean velocity of the gas layer in the corridor:

$$\bar{v} = 0\cdot8\left(\frac{g\,Q\,T_c}{C_p\,\rho_a\,T_a^2\,W}\right)^{1/3} \qquad (1)$$

where 0.8 is an empirical constant.

A regression analysis of the data presented in Ref 1 gave a value for this empirical constant of 0.9 $\pm$ 0.1.

The related problem of the flow out of a compartment, under a roof-screen, was examined in Appendix 1 of Ref 2. The approach used applied Bernouilli's equation to the gas flow, with the assumption of zero initial velocity for the gases in the compartment. Equation 77 of Ref 2 followed, but this too, involves an empirical constant. A little manipulation of this equation gives an expression for the mean velocity of outflow of the gas:

$$\bar{v} = 0{\cdot}69 \left(\frac{g Q T_c}{C_p \rho_a T_a^2 W} \right)^{1/3} \qquad (2)$$

where 0.69 contains the empirical constant and a discharge coefficient.

The difference between equations (1) and (2) is not really surprising, since the assumption of zero initial velocity is rarely applicable to corridor flows.

The method of applying Bernouilli's Equation, as used in ref 2, is similar to the approach of many Fluid Mechanics text books[3] to fluid flows - for example rivers or pipe flows.

Despite this, there appears to be no direct application of these principles to the particular case of steady-state flows of buoyant gases beneath corridor ceilings.

The velocity-pressure coefficient[3,4], k, can be used to allow for the presence of obstacles to flow in a pipe. With certain simplifying assumptions described below, the theory that follows attempts to apply Bernouilli's equation to a corridor, using the concept of the velocity-pressure coefficient. Equations are derived for some of the more important flow parameters of buoyant streams in simple corridors, a case of considerable practical importance.

2. THEORY

2.1 Assumptions Incorporated into the Theory

a) The buoyant layer exhibits tranquil flow[3] (ie. its Froude Number <1.0).

b) Flow processes in the layer are turbulent (ie. the Reynolds Number is large).

c) There is no mixing across the interface between the air below and the horizontally-flowing gas layer above (ie. the Richardson Number >0.8). Experiments[5,6] suggest that this is a reasonable approximation for fully-developed flows in a corridor.

d) Flow is uniform across the width of the corridor.

e) Energy losses from the layer due to internal turbulence, wall friction and heat conduction are small compared to the energy flux along the corridor. The major source of heat loss from the ceiling layer is heat radiation downwards from the layer. For a short corridor (eg. the great majority of malls), radiative heat losses will be a small proportion of the original heat flux in the layer. Such losses can be incorporated into a more detailed theory applicable to long tunnels, but would lead to unwieldy final equations. This paper will consider the case of short corridors only, for which case all the above mentioned energy losses can be assumed to be absent.

f) There are no horizontal pressure gradients in the air below the layer ie. the air is everywhere at ambient pressure (or the air velocity $\ll$ the layer velocity).

g) The base of the layer (the interface) is at a constant height throughout the flow. In most corridors of practical interest, the major obstacle to flow occurs at the exit, and this assumption is then observed to be, at least approximately, true (for example in shopping malls). It is probably not valid for very long tunnels or corridors, but would still serve as a useful first approximation.

In practice, assumption e) above will not be strictly true, even for short to medium length corridors. The kinetic energy loss per unit length due to friction etc. must be balanced by a change in potential energy due to buoyancy as the gases travel along the corridor. Since heat is again assumed to be conserved the layer depth will change. Any flow obstacle leading to a loss of energy will also cause a change in layer depth. Therefore this assumption should more properly be that such changes in layer depth are small with respect to the total layer depth.

h) Fire gases rising into the hot corridor layer from the fire, are already heavily diluted with entrained air. Hence smoky gases in the layer can be regarded as having the same density as their major constituent, air, would have at the same temperature.

i) The vertical profiles of buoyancy at all positions along the corridor have the same shape. Also the vertical velocity profiles have a common shape.

j) The hot gases can be regarded as incompressible.

2.2 The Effective velocity-pressure coefficient at a point in a Corridor

The definition and use of the velocity-pressure coefficient is discussed in detail elsewhere[3,4]. To summarise the salient points, an obstacle in the way of a fully-developed turbulent flow (usually in a pipe or duct) causes an energy loss, proportional to the incident kinetic energy. There is a pressure drop across the obstacle, such that for a steady flow

$$\Delta p = k \cdot \frac{1}{2} \rho v^2 \tag{3}$$

where k is the velocity-pressure coefficient.

Equation 3 can be incorporated into the steady-flow energy equation for an individual flow-line.

Let us consider the case of a buoyant flow in a corridor, where the flow passes a number of obstacles before escaping to the exterior atmosphere (See Fig 1). Bernouilli's equation can be written for a flowline at height x, comparing the total energy at position Λ and at a point outside the corridor, when the flow is dispersed. It is here assumed that the flowline returns to height x after passing each obstacle.

$$p_\Lambda(x) + \frac{1}{2}\rho_\Lambda(x)\, v_\Lambda^2(x) - \sum_{n=1}^{N} k_n \cdot \frac{1}{2} \rho_n v_n^2(x) = p_a(x) \tag{4}$$

but

$$\left.\begin{aligned} p_\Lambda(x) &= p_\Lambda(x=0) - \int_0^x \rho_\Lambda(y)\, g\, dy \\ \text{and} \quad p_a(x) &= p_a(x=0) - \int_0^x \rho_a(y)\, g\, dy \end{aligned}\right\} \tag{5}$$

From assumptions f) and g), and since the plane x = o is below the layer

$$p_\Lambda(x=0) = p_a(x=0)$$

So equation (4) becomes

$$\int_0^x (\rho_a - \rho_\Lambda)(y) g \, dy = \sum_{n=1}^{N} k_n \frac{1}{2} \rho_n(x) v_n^2(x) - \frac{1}{2} \rho_\Lambda v_\Lambda^2(x) \quad (6)$$

For obstacle n, we can write

$$\rho_n(x) v_n^2(x) = \beta_n \rho_\Lambda(x) v_\Lambda^2(x)$$

Equation (6) becomes

$$\int_0^x \Delta\rho_\Lambda(y) g \, dy = \left(\sum_{n=1}^{N} k_n \beta_n - 1\right) \cdot \frac{1}{2} \rho_\Lambda v_\Lambda^2(x)$$

or
$$\int_0^x \left(\rho_a \frac{\Delta T}{T}\right)(y) g \, dy = k' \cdot \frac{1}{2} \rho_\Lambda(x) v_\Lambda^2(x) \quad (7)$$

where

$$k' = \left(\sum_{n=1}^{N} k_n \beta_n - 1\right)$$

is the effective velocity-pressure coefficient between position Λ and the external air. The dependence of k' on the corridor exit is considered in more detail in Appendix 1.

2.3 Mass Flux and Velocity in the hot layer

Equation (7) can be used to calculate the horizontal mass flux in the layer. First, equation (7) is rearranged using assumption h).

$$v_\Lambda(x) = \left(\frac{2g}{k'} \frac{T(x)}{T_a} \left[\int_0^x \left(\frac{\Delta T}{T}\right)(y) \, dy \right] \right)^{1/2} \quad (8)$$

The mass flux through an element dx at height x is

$$dM_\Lambda(x) = \rho(x) W v_\Lambda(x) dx = \rho_a \frac{T_a}{T(x)} \cdot W v_\Lambda(x) \, dx \quad (9)$$

for a corridor of width W.

Combining equations (8) and (9), and integrating, the total horizontal mass flux is given by

$$M_\Lambda = \int_0^{\ell} \left(\frac{2g}{k'} \frac{T_a}{T(x)} \right)^{1/2} \rho_a W \left(\int_0^x \left(\frac{\Delta T}{T}\right)(y) \, dy \right)^{1/2} dx \quad (10)$$

To take this further, the form of $\left(\frac{\Delta T}{T}\right)(x)$ must be specified. A typical experimental profile is shown in fig 2. Following ref 2, this can be approximated by a rectangular profile (Fig 3).

Other idealised profiles are possible (if less likely) and one, a "triangular" profile, is discussed in Appendix 2.

For a rectangular profile,

$$T(x) = T_c \quad \text{for} \quad (\ell - d_l) < x < \ell$$

$$T(x) = T_a \quad \text{for} \quad x < (\ell - d_l)$$

$$\text{ie.} \quad \left(\frac{\Delta T}{T}\right)(x) = 0 \quad \text{for} \quad x < (\ell - d_l)$$

equation (10) becomes

$$M_\Lambda = \int_{(\ell-d_l)}^{\ell} \left(\frac{2 g T_a}{k' T_c}\right)^{1/2} \rho_a W \left(\frac{\Delta T_c}{T_c}\right)^{1/2} \left(\int_{(\ell-d_l)}^{\ell} dy\right)^{1/2} dx$$

which gives, on completing the integration

$$M_\Lambda = \frac{2}{3} \left(\frac{2 g T_a \Delta T_c}{k'}\right)^{1/2} \frac{\rho_a W}{T_c} d_l^{3/2} \tag{11}$$

The horizontal heat flux $Q_\Lambda = M_\Lambda C_p \Delta T_c$.

so equation (11) can also be written

$$M_\Lambda = \left(\frac{2}{3}\right)^{2/3} 2^{1/3} \left(\frac{g\, Q\, T_a}{C_p\, k'} \left[\frac{\rho_a W}{T_c}\right]^2\right)^{1/3} d_l \tag{12}$$

Since the uncertainty in d_l is often large, and Q can be measured more accurately than ΔT_c, equation (12) should be more useful in applying experimental results. Either could be used for design purposes.

A mean velocity for the layer can be defined, such that

$$\bar{v}_\Lambda = \frac{M_\Lambda}{\rho_c W d_l} \tag{13}$$

using equations (11),(12) and (13)

$$\bar{v}_\Lambda = \left(\frac{2}{3}\right)^{2/3} 2^{1/3} \left(\frac{g\, Q\, T_c}{k' C_p \rho_a T_a^{\,2} W}\right)^{1/3} \tag{14}$$

This can be compared with equations (1) and (2). Note the dependence on k', representing the influence on the flow of obstacles downstream.

3. PRACTICAL APPLICATIONS

It is thought that this work might be of particular use in calculating flows in corridors, tunnels or malls where the exit is more restricted than the mall itself (as is usually the case). This can also apply to ceiling vents, when these vents are sufficiently large to remove all the smoky gases appearing beneath them. In this case, w is the total vent perimeter. If the smoke is flowing radially towards a circular vent, ie. the flow is normal to the perimeter at all points (at least in the immediate vicinity of the vent), then k' would in this case be given by equation (A9).

For the design of smoke control systems, one would usually obtain values for d_l, Q, ΔT_c and M_Λ from other sources (eg ref 8). One would then use equations (11) or (12) to calculate k', and hence the necessary dimensions of the smoke extraction "exits" following Appendix 1. Alternatively one could start with the existing geometry of a corridor and calculate one of the other variables. For design work, it is important to remember that this theory applies to steady flows only.

Fairly gentle obstacles to flow (eg. bends in the corridor) could be included in the analysis by using the appropriate values of k_n. Various values of k_n are known[4] for pipe flows, and should be applicable to corridors. Also internal energy losses could be incorporated into k' as a "k per unit length." The statements in this paragraph are all subject to assumption g).

4. CONCLUSION

Equations describing the mass flux and velocity of a layer of fire gases flowing beneath the ceiling of a corridor have been derived. The equations include the effects of the geometry of the exit, and of gentle obstacles to flow in the corridor. The resulting formulae are based on assumptions that are thought to be reasonable for simple corridors, such as are often found in practice.

5. REFERENCES

1. HINKLEY, P L. The flow of hot gases along an enclosed shopping mall. A tentative theory. JFRO Fire Research Note 807 (1970).

2. THOMAS, P H. et al. Investigations into the flow of hot gases in roof venting. JFRO Fire Research Technical Paper No 7 London, 1963, HMSO.

3. MASSEY, B S. Mechanics of Fluids. Van Rostrand, London, 1968.

4. IHVE Guide, Volume C. 1970.

5. HESELDEN, A J M, WRAIGHT, H G H, and WATTS P R. Fire Problems of Pedestrian Precincts Part 2. Large Scale Experiments with a shaft vent. JFRO Fire Research Note 954 (1972).

6. HESELDEN, A J M and HINKLEY, P L. Smoke Travel in Shopping Malls. Experiments in Co-operation with Glasgow Fire Brigade - Part 1. JFRO Fire Research Note 832 (1970).

7. MORGAN, H P and BOLTON, M. To be published.

8. HINKLEY, P L. Work by the Fire Research Station on the control of smoke in covered shopping centres. BRE Current Paper 83/75.

APPENDIX 1

THE EFFECTIVE VELOCITY-PRESSURE COEFFICIENT FOR A CORRIDOR HAVING A RESTRICTED EXIT

The corridor is represented in fig 4, showing the shape of the exit, and in fig 5, showing the flow through the exit. If $w \gg d_s$, the vena contracta formed just outside the exit should have width w and an effective depth $C_v d_s$.

If we compare the vena contracta with position Λ , assumptions g) and i) imply the same relative heights for flow lines, and hence

$$x_v = \frac{C_v d_s}{d_1} x_s \tag{A1}$$

similarly,

$$v_v(x_v) = \frac{W d_1}{w C_v d_s} v_\Lambda(x_\Lambda) \tag{A2}$$

Remembering that the exit is obstacle N, we can apply Bernouilli's equation, with equation (3), to a flow line

$$p_\Lambda(x_\Lambda) + \frac{\rho_\Lambda}{2}(x_\Lambda)\, v_\Lambda^2(x_\Lambda) - \sum_{n=1}^{(N-1)} k_n \frac{\rho_n}{2} v_n^2(x_\Lambda) + \int_{x_v}^{x_\Lambda} \rho_\Lambda(x)\, g\, dx$$
$$= p_v(x_v) + \frac{\rho_v}{2}(x_v)\, v_v^2(x_v) \tag{A3}$$

At the vena contracta, however,

$$p_v(x_v) = p_a(x_v) \tag{A4}$$

Assuming the rectangular profile of fig 3, equations (A3) and (A4) give

$$\Delta\rho_c\, g\, x_v = \frac{\rho_c}{2}\left(\sum_{n=1}^{(N-1)} k_n v_n^2(x_\Lambda) + V_v^2(x_v) - V_\Lambda^2(x_\Lambda)\right) \tag{A5}$$

Using

$$v_n^2(x_\Lambda) = \beta_n\, V_\Lambda^2(x_\Lambda)$$

and equations (A1) and (A2), equation (A5) becomes

$$\begin{aligned}\Delta\rho_c\, g\, x_\Lambda &= \int_0^{x_\Lambda} \rho_a\, g \left(\frac{\Delta T}{T}\right)(y)\, dy \\ &= \frac{\rho_c}{2} V_\Lambda^2(x_\Lambda)\, \frac{d_l}{C_v d_s}\left(\sum_{n=1}^{(N-1)} k_n\beta_n + \left[\frac{W d_l}{w C_v d_s}\right]^2 - 1\right)\end{aligned} \tag{A6}$$

Comparing (A6) with (7), we see that, at position Λ

$$k' = \frac{d_l}{C_v d_s}\left(\sum_{n=1}^{(N-1)} k_n\beta_n + \left[\frac{W d_l}{w C_v d_s}\right]^2 - 1\right) \tag{A7}$$

NOTE: In most practical applications, all k_n for $n \neq N$ will approach zero, so

$$k' \simeq \frac{d_l}{C_v d_s}\left(\left[\frac{W d_l}{w C_v d_s}\right]^2 - 1\right) \tag{A8}$$

For the special case where the exit has no roof screen, the vena contracta coincides with the exit, and $d_l = C_v\, d_s$. Hence for this case

$$k' = \left(\left(\frac{W}{w}\right)^2 - 1\right) \tag{A9}$$

APPENDIX 2

MASS FLUX AND VELOCITY IN A LAYER HAVING A TRIANGULAR PROFILE OF BUOYANCY

A (hypothetical) triangular layer profile is shown in fig 6. It can be seen that it has the same buoyancy at the ceiling $\left(\rho_a \frac{\Delta T_c}{T_c} g d_l\right)$ as the rectangular layer.

For $x > (\ell - 2d_l)$ the triangular profile is described by the equation

$$\left(\frac{\Delta T}{T}\right)(x) = \frac{\Delta T_c}{2 d_l T_c} \left(x - \ell + 2 d_l\right) \qquad \text{(A10)}$$

Using this in equation (10), one can integrate (with some difficulty), giving

$$M_\Lambda = \frac{2}{3} \left(\frac{2 g T_a \Delta T_c}{k'}\right)^{1/2} \frac{\rho_a W}{T_c} d_l^{3/2} \mathcal{K} \qquad \text{(A11)}$$

where $\mathcal{K}$ is a "profile correction factor" to equation (11), and is

$$\mathcal{K} = \left(\frac{6 T_a^2}{5 (\Delta T_c)^2} - \frac{2 T_a T_c}{(\Delta T_c)^2} + \frac{4 T_c^{5/2}}{5 (\Delta T_c)^2 T_a^{1/2}}\right) \qquad \text{(A12)}$$

For $T_a = 290^\circ K$, K takes the values of

1.51	for	ΔT_c	=	$10^\circ K$
1.58	for	ΔT_c	=	$100^\circ K$
2.16	for	ΔT_c	=	$1000^\circ K$

Clearly therefore, the more the actual buoyancy profile departs from a rectangle, the greater will be the errors resulting from the use of equations in the main body of this note. Such errors may not be too important for design purposes, and $\mathcal{K}$ is unlikely to exceed 1.2 for most real profiles.

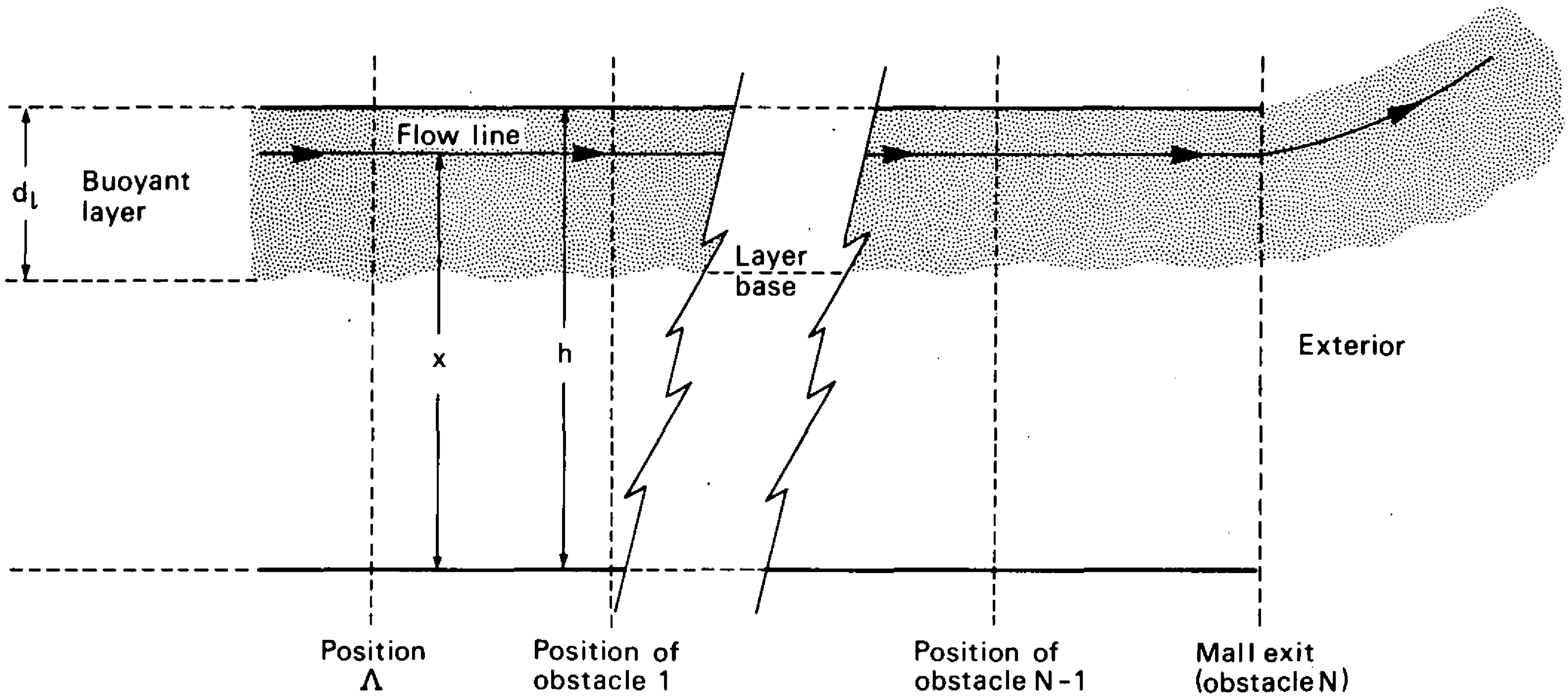

Figure 1 Horizontal flow along a corridor of width W

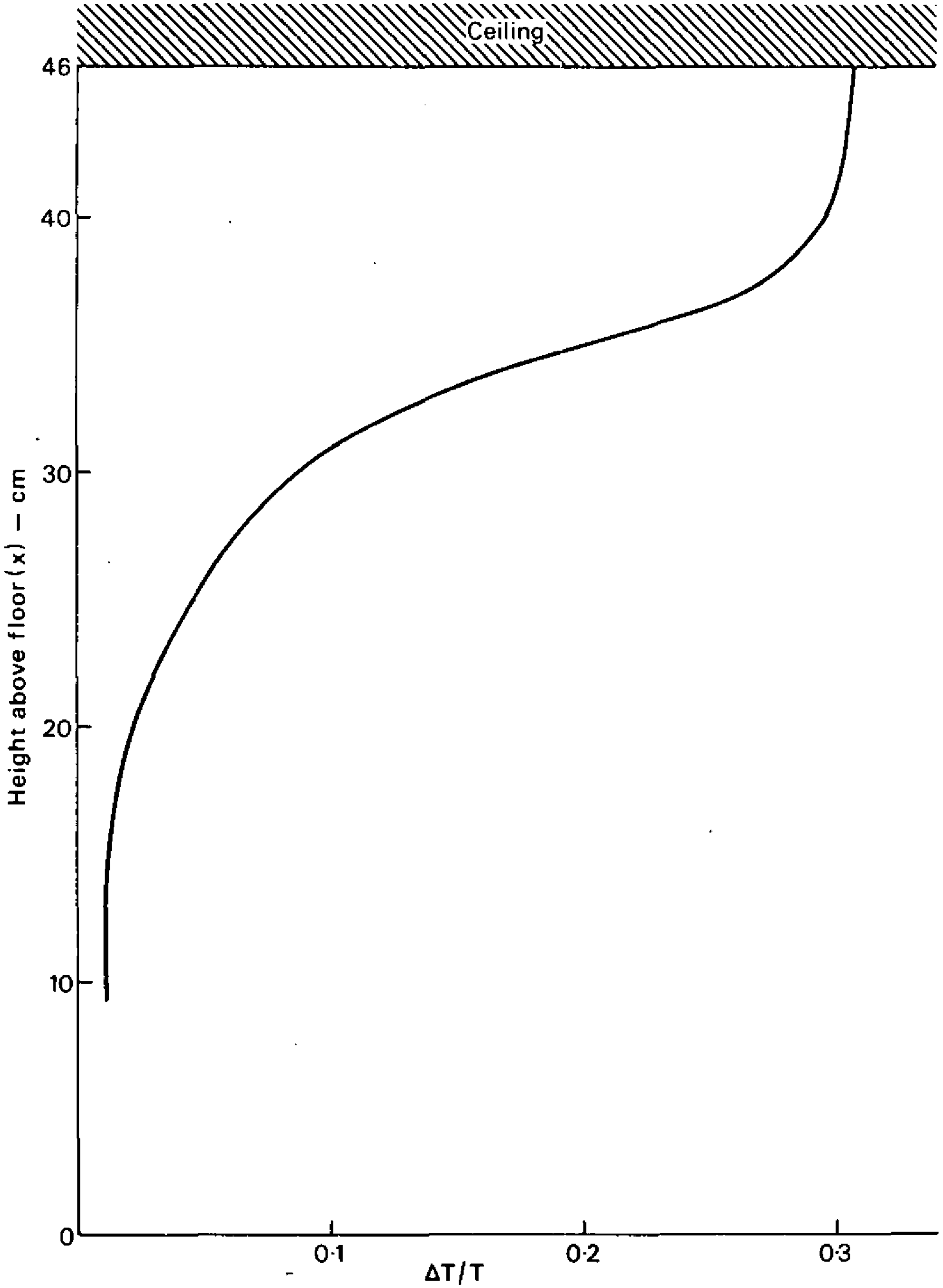

Figure 2 An experimental buoyancy (ΔT/T) profile from a flow in a model corridor

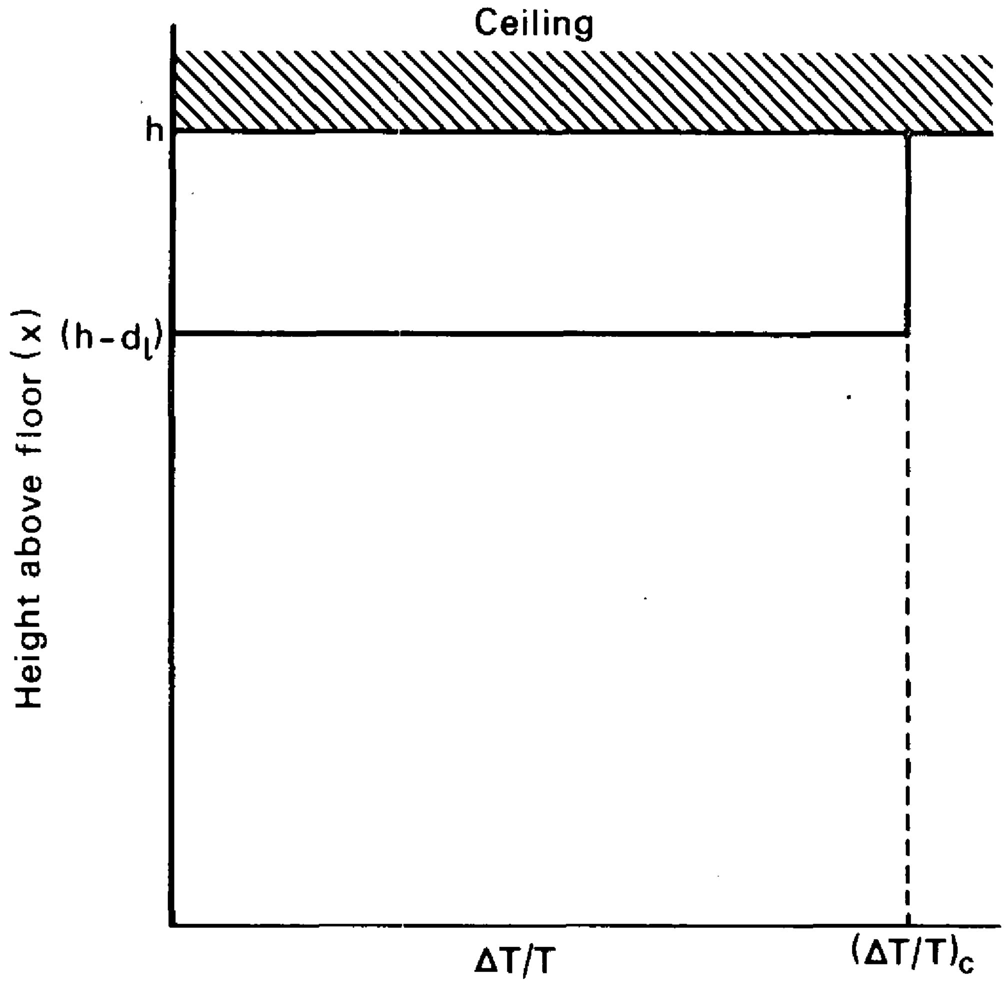

Figure 3 A rectangular layer profile

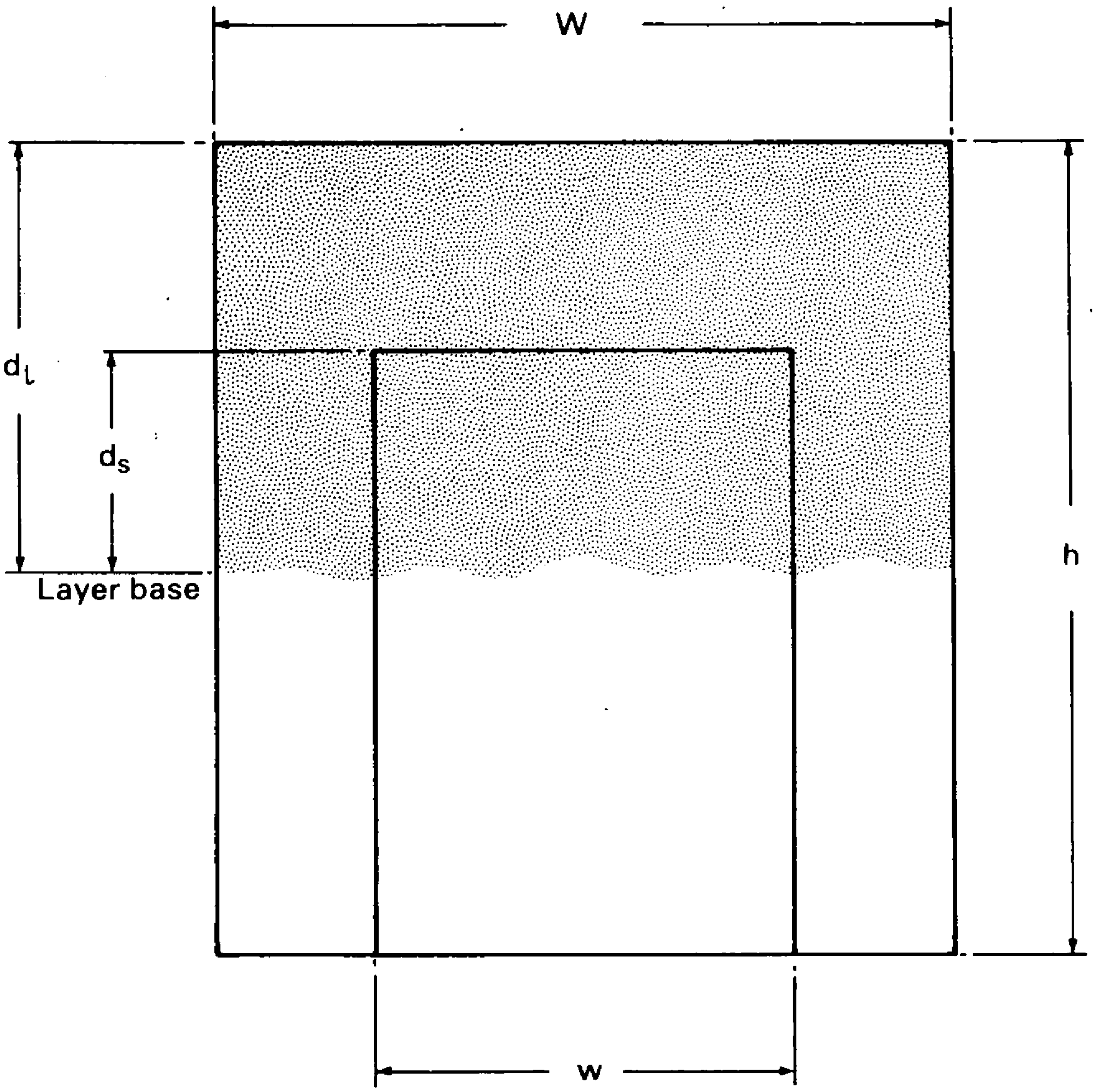

Figure 4 Front elevation of restricted exit at right angles to the flow

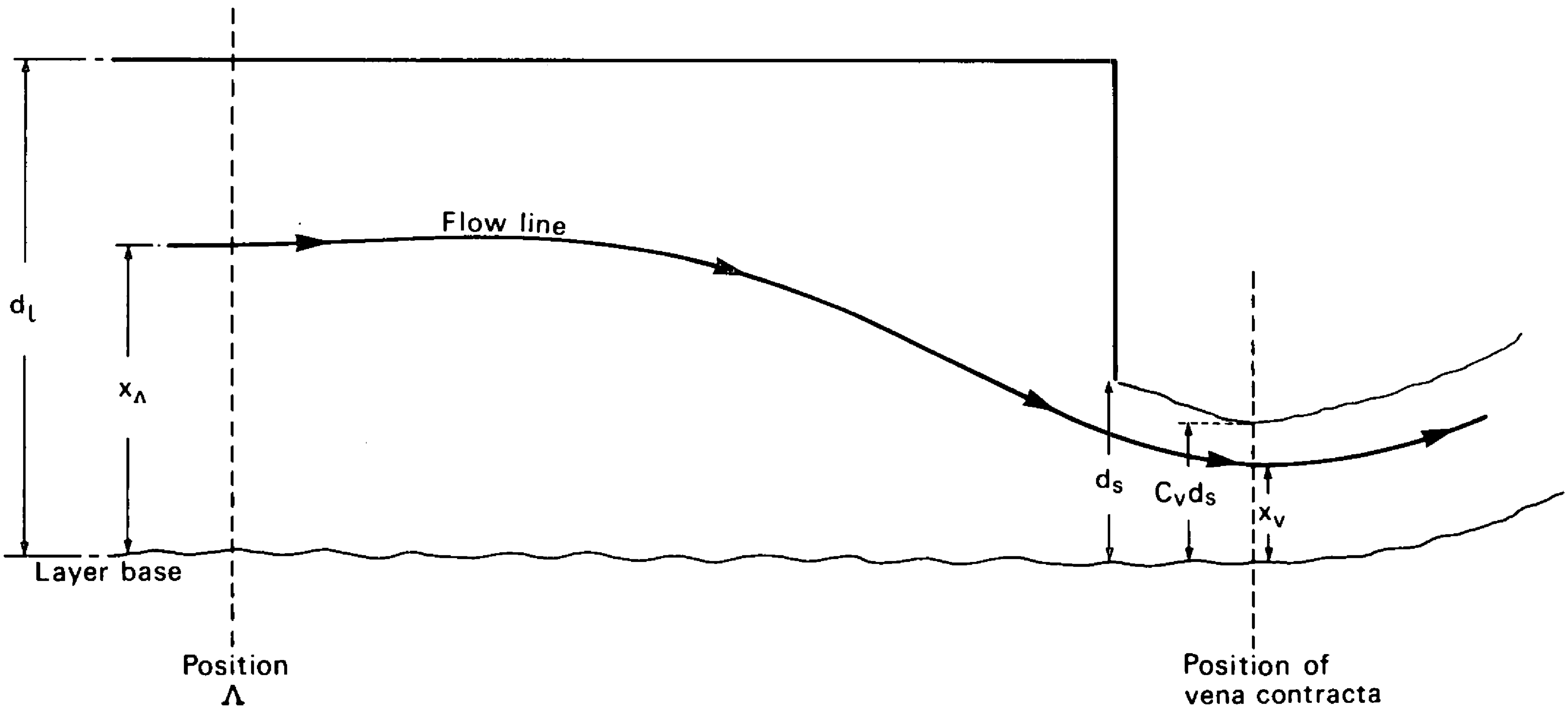

Figure 5 Side view of flow through a restricted exit, showing a flow line

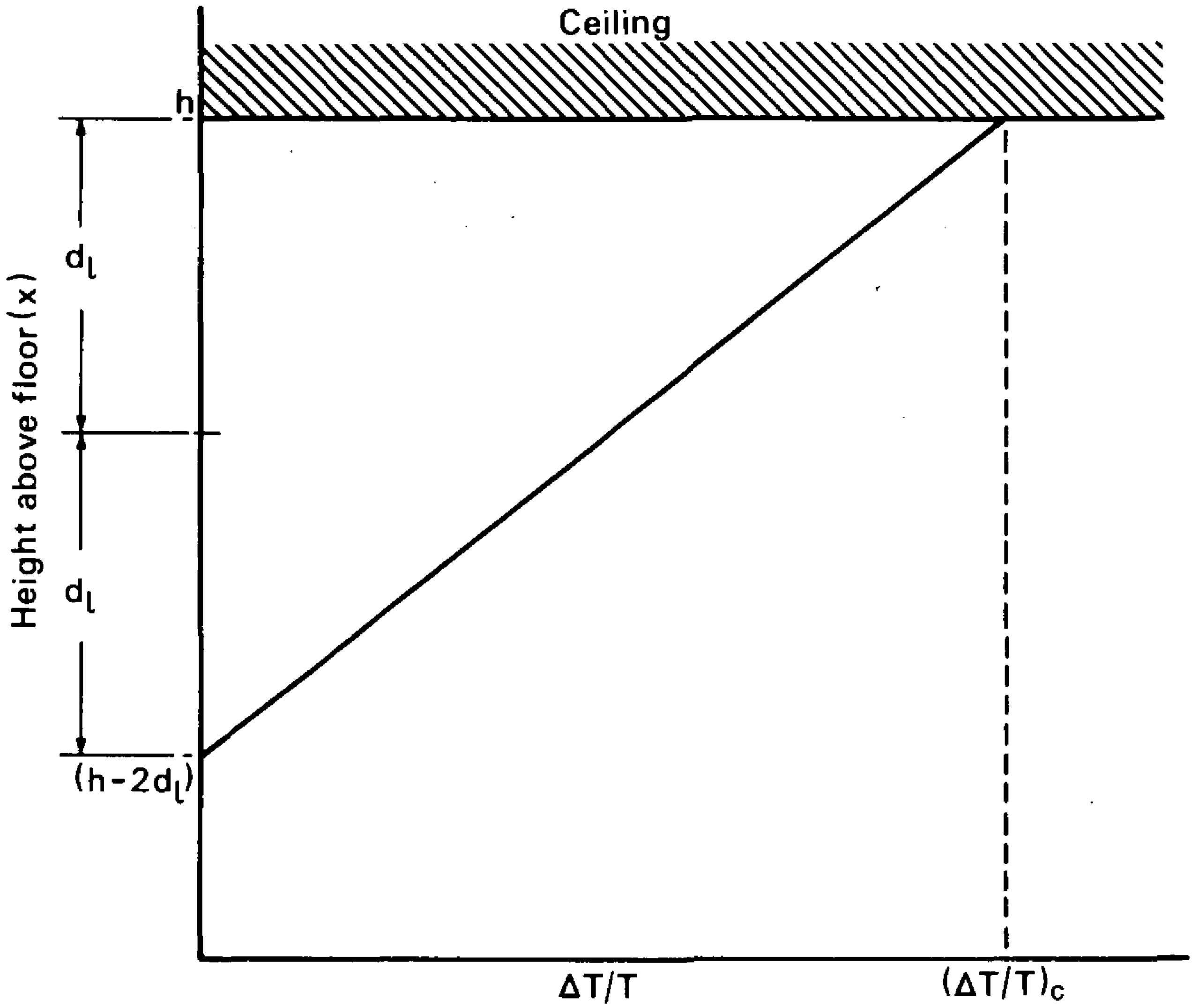

Figure 6 A triangular layer profile