F.R. Note No. 118/1954
Research Programme
Objective I/1/1(P)

DEPARTMENT OF SCIENTIFIC AND INDUSTRIAL RESEARCH AND FIRE OFFICES' COMMITTEE
JOINT FIRE RESEARCH ORGANIZATION

A COPPER BLOCK ABSOLUTE RADIOMETER

by

D. I. Lawson, D. L. Simms, M. Law, and R. W. Pickard

Summary

A copper block has been used as an absolute radiometer by measuring its transient temperature rise on being exposed to radiation. Two methods have been derived and used for calculating the intensity of radiation; the values obtained agree satisfactorily with those obtained using other instruments operating in the same range of intensities (1, 2).

November, 1954.

File No.

Fire Research Station,
Boreham Wood,
Herts.

A COPPER BLOCK ABSOLUTE RADIOMETER

by

D. I. Lawson, D. L. Simms, M. Law and R. W. Pickard

1. Introduction

During recent studies of the ignition of materials, intensities of radiation of up to 50 W/cm^2 have been used. A copper block absolute radiometer has been developed so that this range of radiant intensities may be measured. This paper describes the development of the radiometer and compares the results obtained from it with those of other instruments (1, 2).

2. Description of apparatus and experimental method

The apparatus (Figure 1, Plate 1) consists of a square-faced copper block set flush with a water-cooled guard ring; this prevents radiation from striking the edges of the block and also acts as a frame to hold it. The surface of the copper is coated with a black paint having an absorptivity of 98 per cent, measured by comparison with carbon black having an absorptivity of 95 per cent (2). The temperature of the block is measured by a thermocouple consisting of a constantan wire attached to the front face and a copper wire attached to the black face.

The radiometer is exposed to the radiation and the rise in temperature of the block is recorded; it is then shielded from the radiation and a cooling curve is taken. A thermopile (3) is used to ensure that the intensity of radiation from the source remains constant during each experiment.

3. Calculation of the intensity of the radiation

Preliminary calculations and experiments were made to find whether there was a temperature gradient across the block during its exposure to radiation (Appendix I). The difference in temperature between the front and back surfaces of the block was negligible for temperature rises of the order 100°C.

The calculation of the temperature distribution across the face of the block is difficult. It was therefore determined experimentally by coating the face of the block with a temperature indicating paint which, when the block was exposed to radiation, melted at the same time over the whole surface, showing that the face was at a uniform temperature.

The law of cooling from the block was then determined. By plotting the rate of fall of temperature, $\frac{d\theta_c}{dt}$ corresponding to an excess temperature θ_c , the law was found to be Newtonian in form (Figure 2), the slope of the line being

$$\frac{1}{\theta_c}\frac{d\theta_c}{dt} = 0.0075 \pm 0.00028 \qquad \text{......(1)}$$

The rise in temperature, θ_H of the copper block time t after the beginning of the experiment is shown to be (Appendix II)

$$\theta_H = a\left(1 - e^{-bt}\right) \qquad \text{......(2)}$$

and the temperature θ_c any time after the radiation has been cut off is given by

$$\theta_c = -\frac{1}{b}\frac{d\theta_c}{dt}$$

49.

where $a = It/\Phi$ and $b = \Phi/Ms$

M is the mass of the copper block

s " " specific heat
A " " area exposed to radiation
I " " intensity of the radiation
and Φ " " cooling constant

The values of these constants together with the dimensions of the block are given in Appendix III.

Two methods of calculating the intensity of radiation have been derived (Appendix IV).

Method A

If the intensity of radiation I, be cut off at time τ, $\theta = \theta_H$ and the tangent to the cooling curve at this point cuts the temperature axis at θ_0 (Figure 3).

then

$$I = \frac{Ms}{A\tau}\left(\frac{\theta_0 + \theta_H}{2}\right) \qquad \ldots\ldots (3)$$

Method B

Expression (2) may be rearranged as an explicit function of the intensity of radiation I.

$$I = \frac{Ms}{A}\, \theta_H\, b\, \frac{1}{1 - e^{-b t}} \qquad \ldots\ldots (4)$$

As the mean value of b is known from expression (1), the values of the intensity of radiation corresponding to various values of temperature rise and time can be calculated (Figure 4).

4. Accuracy of the instrument

Method A

There are two main sources of error:

(1) The omission of the second and higher order terms in $b\tau$ from expression (2) in Appendix IV; this results in a calculated value of intensity of radiation which is 3 per cent higher than the true value when τ is 100 seconds. Provided that this same heating time is used in a series of experiments, the intensity of radiation can be deduced by reducing the calculated intensity by 3 per cent. For experiments carried out using different heating periods, the individual readings can be corrected by a factor $97\tau^2 \times 10^{-4}$

(2) The value of b is dependent on the construction of the tangent to the cooling curve at the instant at which heating ceases. By constructing large numbers (about 150) of tangents to the cooling curve, it was found that b was constant to within 3 per cent for confidence limits of 95 per cent.

The total error in the observed reading will therefore be between + 6 per cent and the observed reading for confidence limits of 95 per cent.

Method B

A more convenient form than expression (4) from which to calculate the possible errors of method B is

$$I = \frac{Ms}{A} \cdot \frac{\theta_H}{t}\left(1 + \frac{bt}{2} - \frac{7b^2t^2}{24} \cdots\right) \ldots\ldots (5)$$

The error due to the terms inside the bracket of expression (5) is less than $\frac{1}{2}$ per cent since the error in bt is small compared with unity. The error in measuring radiation intensities is therefore not likely to exceed the error in the term $\frac{MS}{A} \cdot \frac{\theta_H}{t}$ which is of the order 3 per cent.

The accuracy of any value of the intensity of radiation may then be checked by comparing the calculated heating curve with the corresponding experimental one. These are compared in Figure 5 for an intensity of radiation of 8·8 W/cm^2. Providing that a heating period of two minutes and a temperature rise of 120°C is not exceeded, the curves coincide.

Both methods have been used to calculate intensities of radiation; the results are given in terms of the calibration of a thermopile (Figures 6, 7). Both give substantially the same calibration.

This calibration of the thermopile has been compared with one made in the range 0 - 12·5 W/cm^2 using a water-flow calorimeter [2] and with one made in the range 0 - 2 W/cm^2 using Guild's standard scale [1] (Figure 8). Guild's radiometer is stated to be accurate to one part in a thousand [1]. The readings of the water flow calorimeter have a root mean square deviation of 2·8 per cent and the probable error of any reading is 1·8 per cent. The three methods agree therefore to within the estimated limits of error.

5. Conclusions

The copper block radiometer which is simple to use can be used to measure intensities of radiation up to 12·5 W/cm^2. Two methods of calculating the intensity of radiation falling on the block have been described. The accuracy of the better method is three per cent and the agreement between this instrument and other instruments working in the same range is well within the estimated limits of maximum error.

References

1. GUILD, J. "Investigations in absolute radiometry, a new temperature-drift radiometer". Proc. Roy. Soc. A1937, 161.

2. LAWSON, D. I. and McGUIRE, J. H. "A radiation calorimeter for the absolute measurement of radiation intensities between 0·4 and 12·5 watts/cm^2". F.R. Note No. 37/1953, March, 1952.

3. SIMMS, D. L. and PICKARD, R. W. "Thermopiles for measuring high intensity radiation". F.R. Note No. 83/1953, November, 1953.

4. CARSLAW, H. S. and JAEGER, J. C. "The conduction of heat in solids". Chap. III p. 43, (VI) p. 105, 1948.

5. Loc. Cit. Appendix IV, Table II, p. 378.

6. KAYE and LABY. Tables of physical constants, Longmans Green and Co., 1948, p. 72.

APPENDIX I

UNIFORMITY IN TEMPERATURE OF THE COPPER BLOCK

The temperature rise θ of a parallel sided slab being irradiated from a medium at temperature T at the surface $x = l$, losing heat from both its surfaces is [4]

$$\theta = \frac{h T x}{2(1+hl)} - hT \sum_{n=1}^{\infty} e^{-k\beta_n^2 t} \frac{\sin \beta_n x}{[h + l(\beta_n^2 + h^2)]\sin\beta_n l}$$

$$+ \frac{1}{2} T - hT \sum_{n=1}^{\infty} e^{-k\alpha_n^2 t} \frac{\cos \alpha_n x}{[h + l(\alpha_n^2 + h^2)]\cos\alpha_n l} \qquad (1)$$

where $h = \Psi / K$

Ψ = Newtonian cooling constant

k = thermal diffusivity $= K/\rho s$

K = thermal conductivity and ρ = density of copper and the other symbols have the same meaning as in the main text and α_n, $n = 1, 2, 3$ are the positive roots of

$$\alpha \tan \alpha l = h$$

and β_n, $n = 1, 2, 3$ are the positive roots of

$$\beta \cot \beta l = -h$$

The boundary condition at the irradiated surface, $x = l$ is [4]

$$K\frac{\partial\theta}{\partial x} = \Psi(\theta - T) \qquad t > 0$$

If the slab is exposed to radiation of intensity I the boundary condition becomes

$$K\frac{\partial\theta}{\partial x} = \Psi\left(\theta - \frac{I}{\Psi}\right) \quad t > 0$$

Using this boundary condition

$$T \equiv \frac{I}{\Psi}$$

and equation (1) may be rewritten

$$\theta = \frac{I x}{2K(1+hl)} - \frac{I}{K} \sum_{n=1}^{\infty} e^{-k\beta_n^2 t} \frac{\sin \beta_n x}{[h + l(\beta_n^2 + h^2)]\sin\beta_n l}$$

$$+ \frac{1}{2}\frac{I}{\Psi} - \frac{I}{K} \sum_{n=1}^{\infty} e^{-k\alpha_n^2 t} \frac{\cos \alpha_n x}{[h + l(\alpha_n^2 + h^2)]\cos\alpha_n l}$$

The temperature of the front surface θ_F, $x = l$ is

$$\theta_F = \frac{I l}{2K(1+hl)} - \frac{I}{K}\sum_{n=1}^{\infty} e^{-k\beta_n^2 t} \frac{\sin\beta_n l}{[h + l(\beta_n^2+h^2)]\sin\beta_n l}$$

$$+ \frac{1}{2}\frac{I}{\psi} - \frac{I}{K}\sum_{n=1}^{\infty} e^{-k\alpha_n^2 t} \frac{\cos\alpha_n l}{[h + l(\alpha_n^2+h^2)]\cos\alpha_n l}$$

The temperature of the back surface θ_B, $x = -l$ is

$$\theta_B = -\frac{I l}{2K(1+hl)} + \frac{I}{K}\sum_{n=1}^{\infty} e^{-k\beta_n^2 t} \frac{\sin\beta_n l}{[h + l(\beta_n^2+h^2)]\sin\beta_n l}$$

$$+ \frac{1}{2}\frac{I}{\psi} - \frac{I}{K}\sum_{n=1}^{\infty} e^{-k\alpha_n^2 t} \frac{\cos\alpha_n l}{[h + l(\alpha_n^2+h^2)]\cos\alpha_n l}$$

∴ the temperature difference between the front and back surface,

$$\theta_F - \theta_B = \frac{I l}{K(1+hl)} - 2\frac{I}{K}\sum_{n=1}^{\infty} e^{-k\beta_n^2 t} \frac{1}{[h + l(\beta_n^2+h^2)]}$$

For the present apparatus

ψ = 0·0103 watts/cm^2/°C = 0·0025 cal/cm^2/sec/°C

K = 3·88 W/cm/°C (0·93 cal/cm/sec/°C)
l = 0·72 cm
k = 1·14 cm^2/sec

∴ $$hl = \frac{\psi l}{K} = 1\cdot 9 \times 10^{-3}$$

∴ $$\frac{I l}{K(1+hl)} = \frac{I l}{K}$$

also $$\beta \cot \beta l = -h$$

∴ $$\beta l \cot \beta l = -hl = -1\cdot 9 \times 10^{-3}$$

The first three solutions of this equation (5) are:-

$$\beta_1 \ell = 1{\cdot}57, \quad \beta_2 \ell = 4{\cdot}71, \quad \beta_3 \ell = 7{\cdot}85$$

With these values, only the first term of the series need be considered. Equation (2) may be written

$$\Theta_F - \Theta_B = \frac{I\ell}{K}\left(1 - \frac{2}{\ell}\, e^{-k\beta_n^2 t}\right)$$

The rise in temperature of the front and back surfaces with time are sketched in Figure 9a. The change in temperature difference is sketched in Figure 9b. After 0·25 seconds, the temperature difference between the front and back surfaces is established and within 10 per cent of its equilibrium value,

that is,

$$\Theta_F - \Theta_B = \frac{I\ell}{K} \tag{3}$$

The uniformity in temperature of the block was then examined experimentally. Using short exposure times giving small temperature rises, the heating curves showed a sharp rise in temperature of the block at the start of the experiment followed by a discontinuity in the curve when the radiation was cut off (Figure 10). The thermocouple thus followed the rise in temperature of the front surface (Figure 9a). By increasing the exposure time to about 100 seconds, the mean temperature rise was increased so that the percentage error due to the difference in temperature between the front and back surface was negligible. A typical curve is shown in Figure 11.

APPENDIX II

DERIVATION OF AN EQUATION FOR THE INTENSITY OF RADIATION FALLING ON THE COPPER BLOCK IN TERMS OF THE TRANSIENT TEMPERATURE RISE

The following expressions are derived on the assumptions that

(1) the temperature throughout the block is uniform (Appendix I)
(2) the heat losses from all the surfaces of the block obey the same law which has been found, by experiment, to be Newtonian (Figure 2).

The transient heating equation is

$$\frac{d\theta}{dt} + b\theta = ab$$

$$\text{where} \quad a = \frac{Ik}{\Phi} \qquad \text{and} \quad b = \frac{\Phi}{MS}$$

which may be integrated for the boundary condition $\theta = o$, $t = o$ to give

$$\theta_H = a\left(1 - e^{-bt}\right)$$

The transient cooling equation is

$$\frac{d\theta}{dt} + b\theta = 0$$

APPENDIX III

CONSTANTS USED IN THE CALCULATIONS AND DIMENSIONS OF THE APPARATUS

mass of copper block	=	175 gm
* specific heat of copper block	=	0·092 cal/gm/°C
	=	(0·385 joules/gm/°C)
side of square block	=	3·71 cm
thickness of block	=	1·42 cm
area of block receiving radiation	=	13·8 cm^2
Φ	=	0·50 $\pm$ 0·019 W/°C (0·12 $\pm$ 0·0046 cal/sec/°C)

* A mean value for the range 0° - 100°C

APPENDIX IV

COOLING CORRECTIONS FOR THE COPPER BLOCK

Method A

The transient heating curve is

$$\theta_H = a\left(1 - e^{-bt}\right) \qquad \text{(Appendix II)}$$

where $a = \frac{IA}{\Phi}$, $b = \Phi/_{MS}$

If the radiation be cut off at time τ , the temperature of the block is

$$(\theta_H)_\tau = a\left(1 - e^{-b\tau}\right)$$

Since the cooling is Newtonian the form of the curve is given by

$$\frac{d\theta_c}{dt} = -b\,\theta_c$$

At the moment the radiation is cut off $\theta_c = (\theta_H)_\tau$

∴ the slope of the tangent to the cooling curve at this point is

$$\left(\frac{d\theta_c}{dt}\right)_\tau = -ba\left(1 - e^{-b\tau}\right)$$

This cuts the temperature axis at θ_0 (Figure 3).

∴ $$\theta_0 = (\theta_H)_\tau - \tau\left(\frac{d\theta_c}{dt}\right)_\tau \quad \ldots\ldots (1)$$

Substituting from Appendix II and rearranging

∴ $$\frac{\theta_0 + (\theta_H)_\tau}{2} = a\left(1 - e^{-b\tau}\right)\left(1 + \frac{b\tau}{2}\right)$$

$$= ab\tau\left(-\frac{b\tau^2}{2} + \frac{b^3\tau^3}{24}\ldots\right) \ldots\ldots (2)$$

Φ = 0·12 ± 0·0046 cal/sec/°C (Appendix III)

M_S = 16·3 cal/°C (Appendix III)

for τ = 100 seconds, the error in assuming that

$$\frac{\theta_0 + (\theta_H)_\tau}{2} = ab\tau$$ is three per cent

and $$I = \frac{Ms}{AT}\left[\frac{\theta_c + (\theta_H)_T}{2}\right] \qquad \ldots\ldots (3)$$

Method B

The transient rise in temperature is (Appendix II)

$$\theta_H = a\left(1 - e^{-bt}\right) \qquad \ldots\ldots (4)$$

Rearranging (4) and substituting for a and b

where $a = \frac{IA}{\Phi}$ and $b = \Phi/_{Ms} = \frac{1}{\theta_c}\frac{d\theta_c}{dt}$

gives $$I = \theta_H \frac{Ms}{A}\left(\frac{1}{\theta_c}\frac{d\theta_c}{dt}\right)\frac{1}{\left[1 - e^{-\left(\frac{1}{\theta_c}\left(\frac{d\theta_c}{dt}\right)\right)t}\right]}$$

Since b is known, I, the intensity of radiation may be calculated from corresponding values of θ_H and t.

PC.

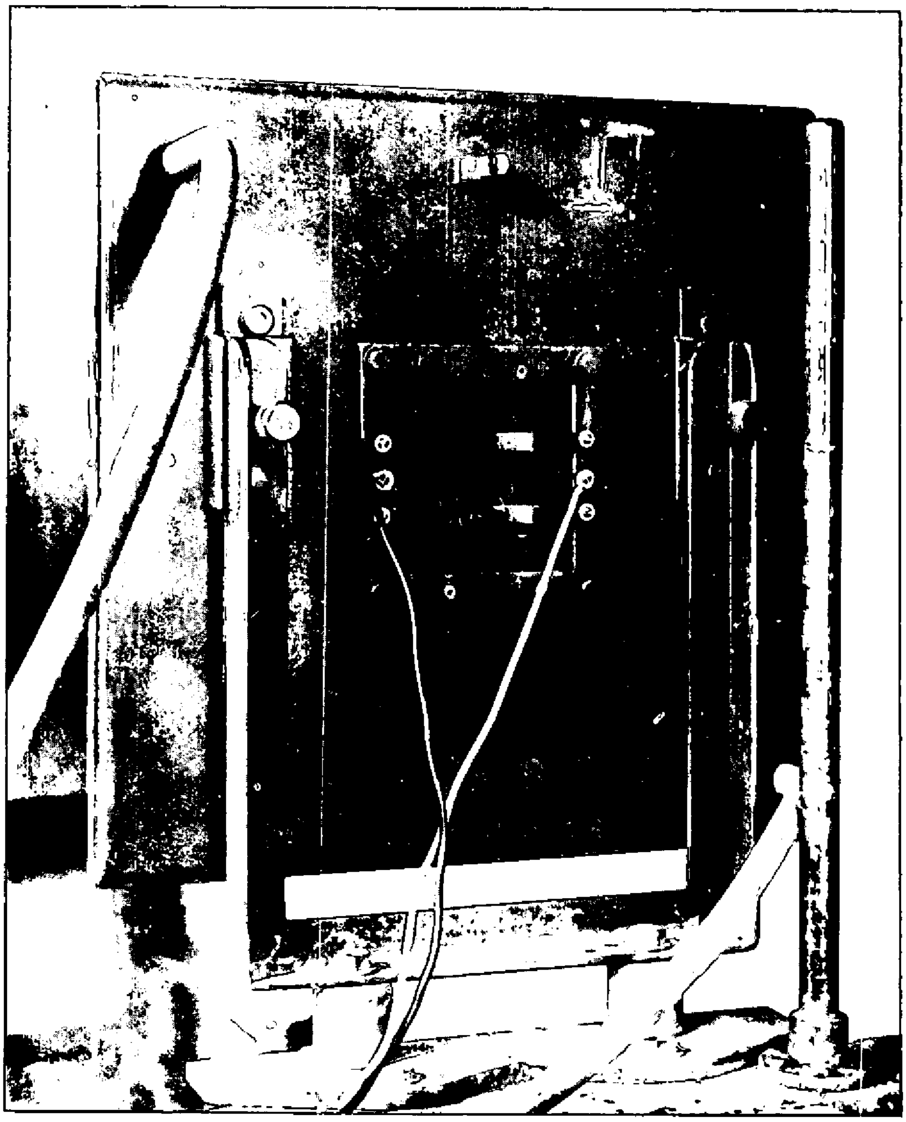
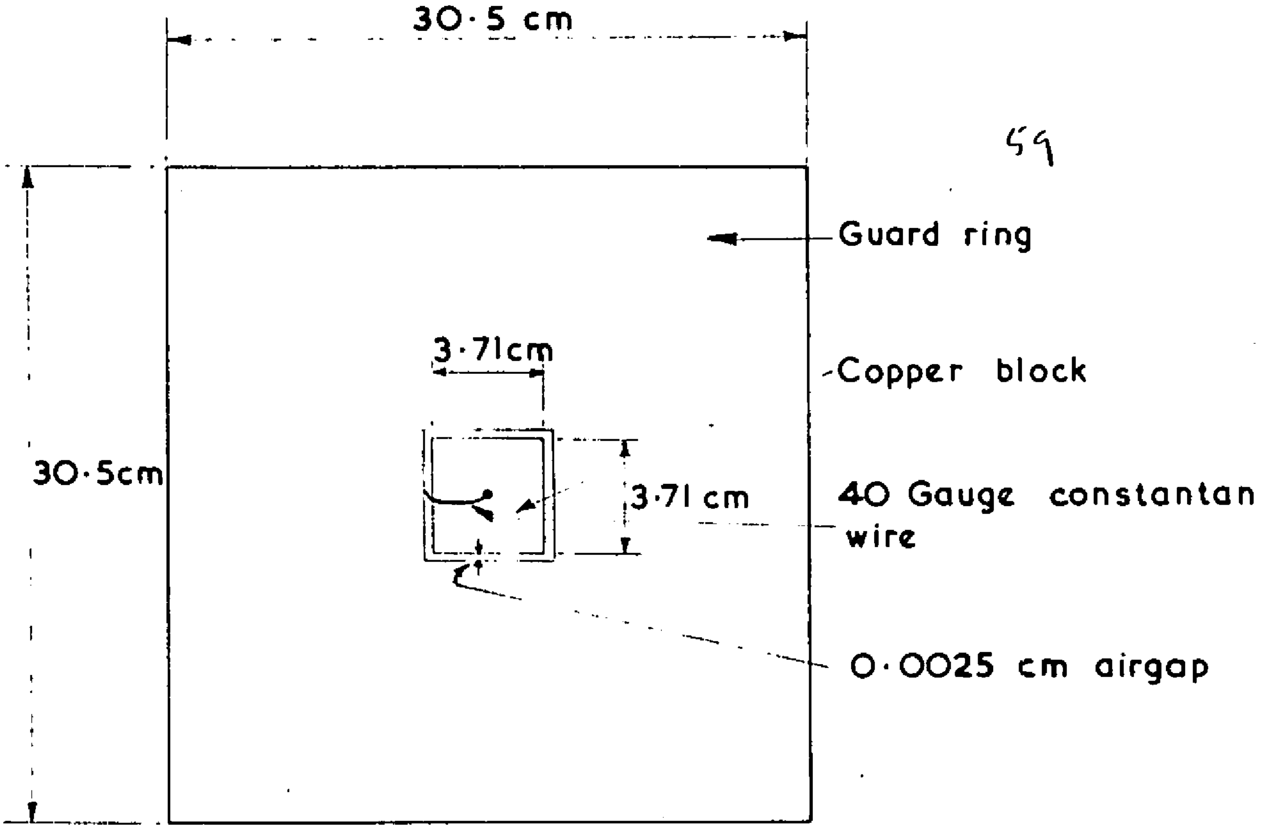


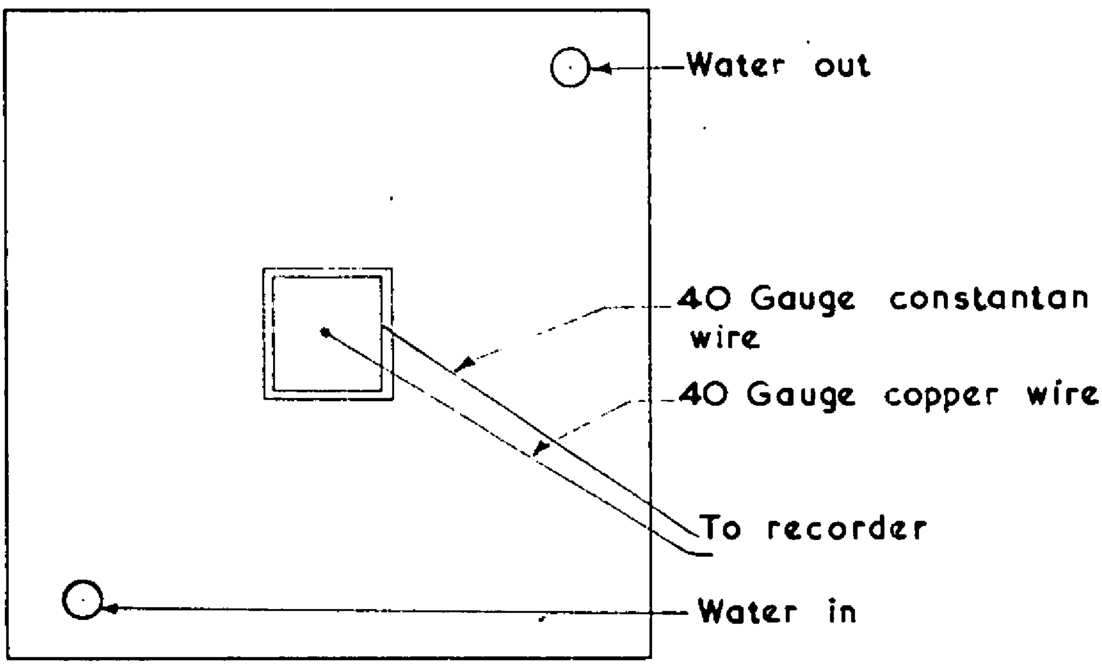
FIG. I. REAR VIEW OF COPPER BLOCK RADIOMETER

PLATE. I.

DS 83414/1/115 80 12/54 CL



FRONT FACE



BACK FACE

FIG.1. DIAGRAM OF APPARATUS

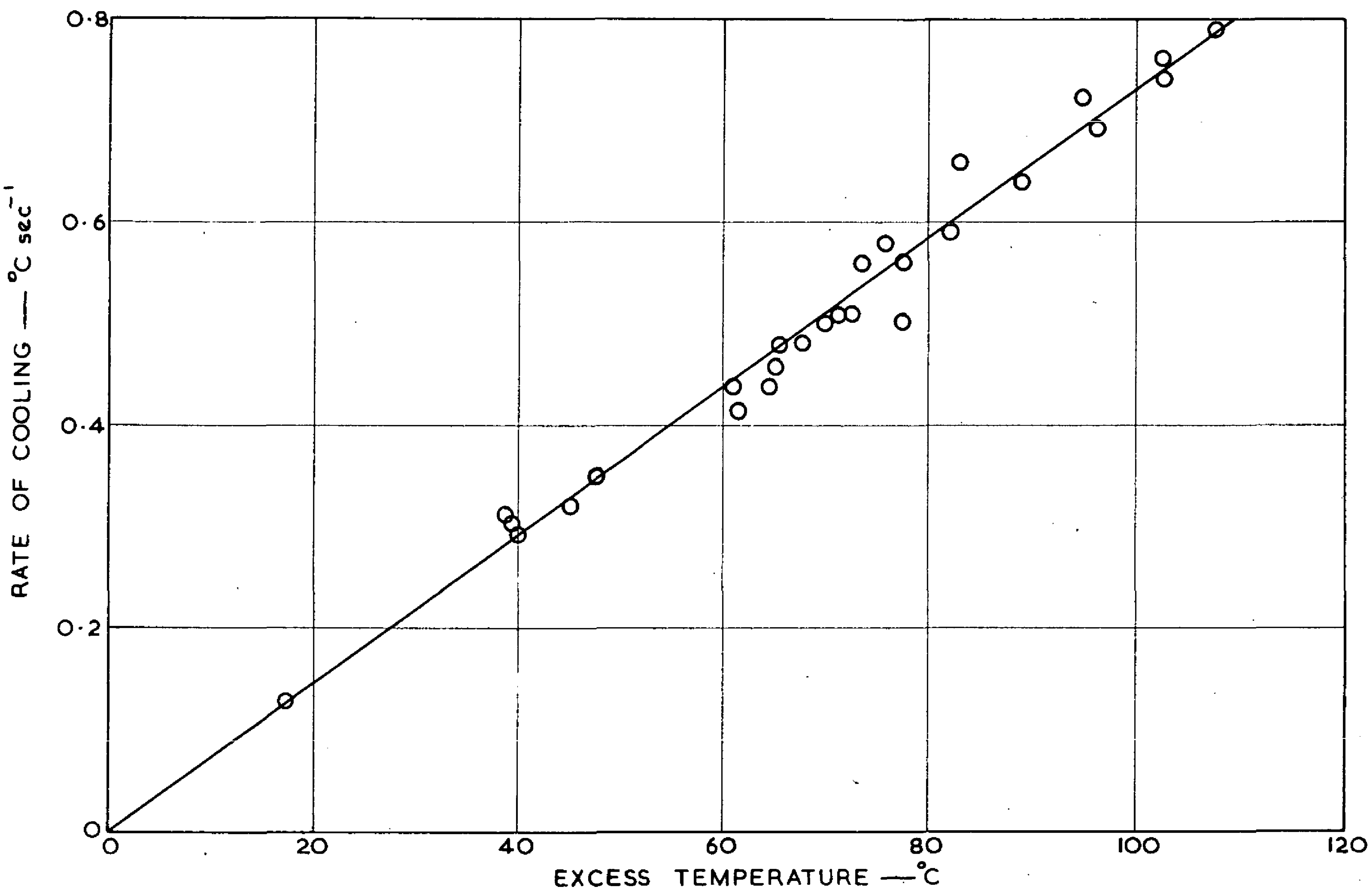


FIG. 2. THE RELATION BETWEEN RATE OF COOLING & EXCESS TEMPERATURE OF THE COPPER BLOCK

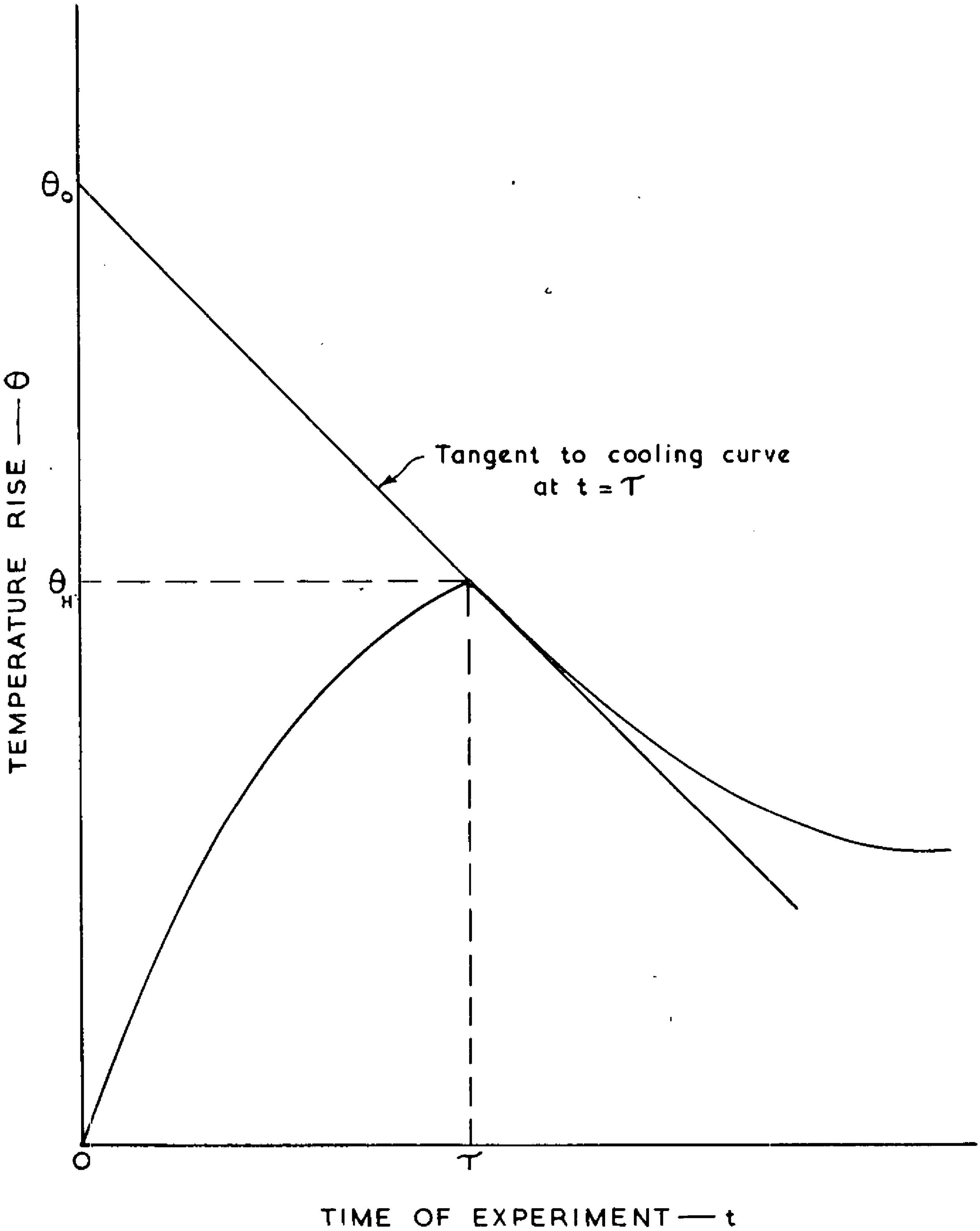


FIG. 3. DIAGRAM SHOWING SYMBOLS USED IN CALCULATING INTENSITY OF RADIATION FROM THE HEATING AND COOLING CURVES OF THE COPPER BLOCK

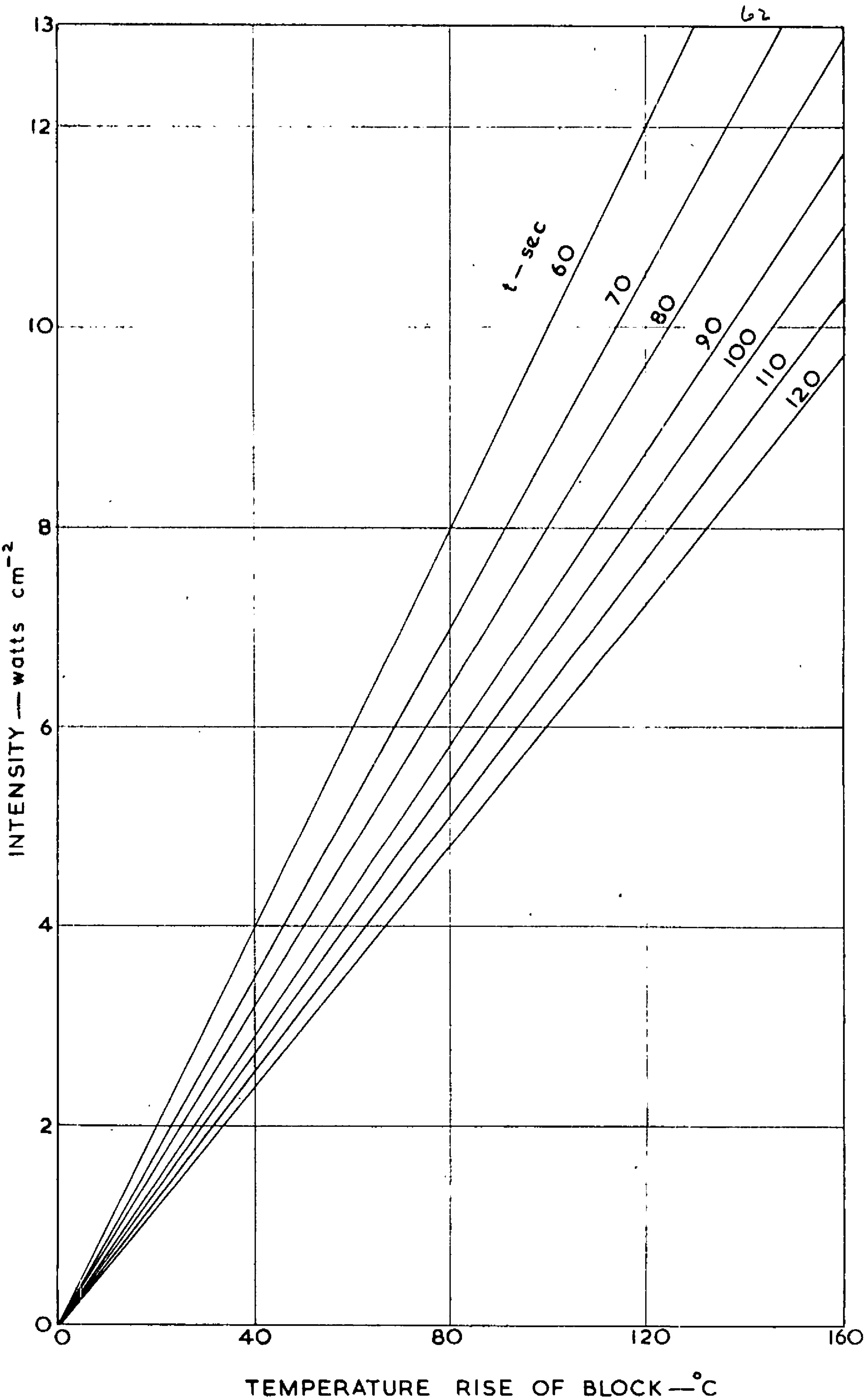


FIG. 4. RELATION BETWEEN TEMPERATURE RISE AND INTENSITY OF RADIATION FOR DIFFERENT EXPOSURE TIMES

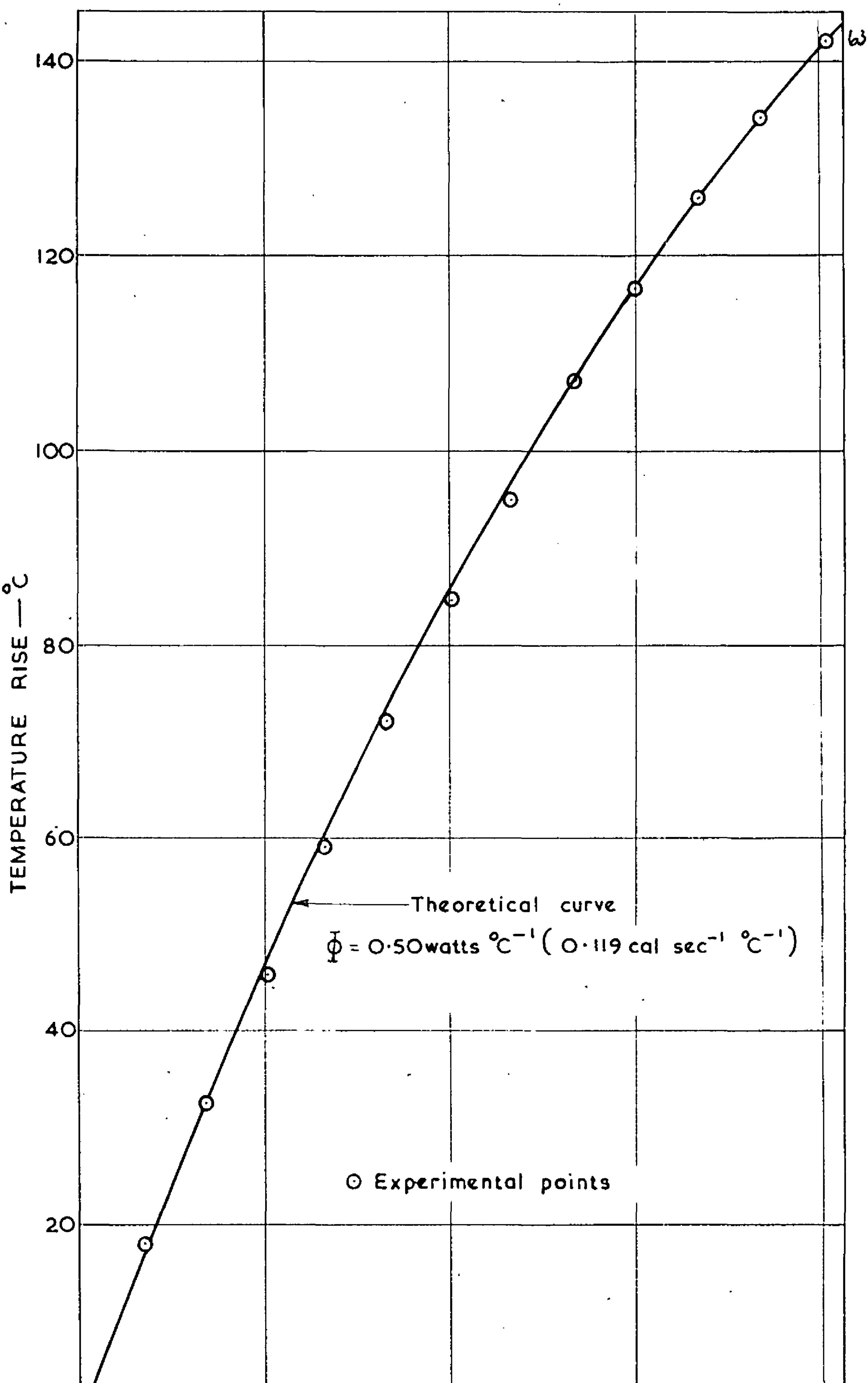


FIG.5. TIME TEMPERATURE CURVE FOR THE COPPER BLOCK EXPOSED TO AN INTENSITY OF 8·8 watts cm^{-2} (2·1 cal cm^{-2} sec^{-1})

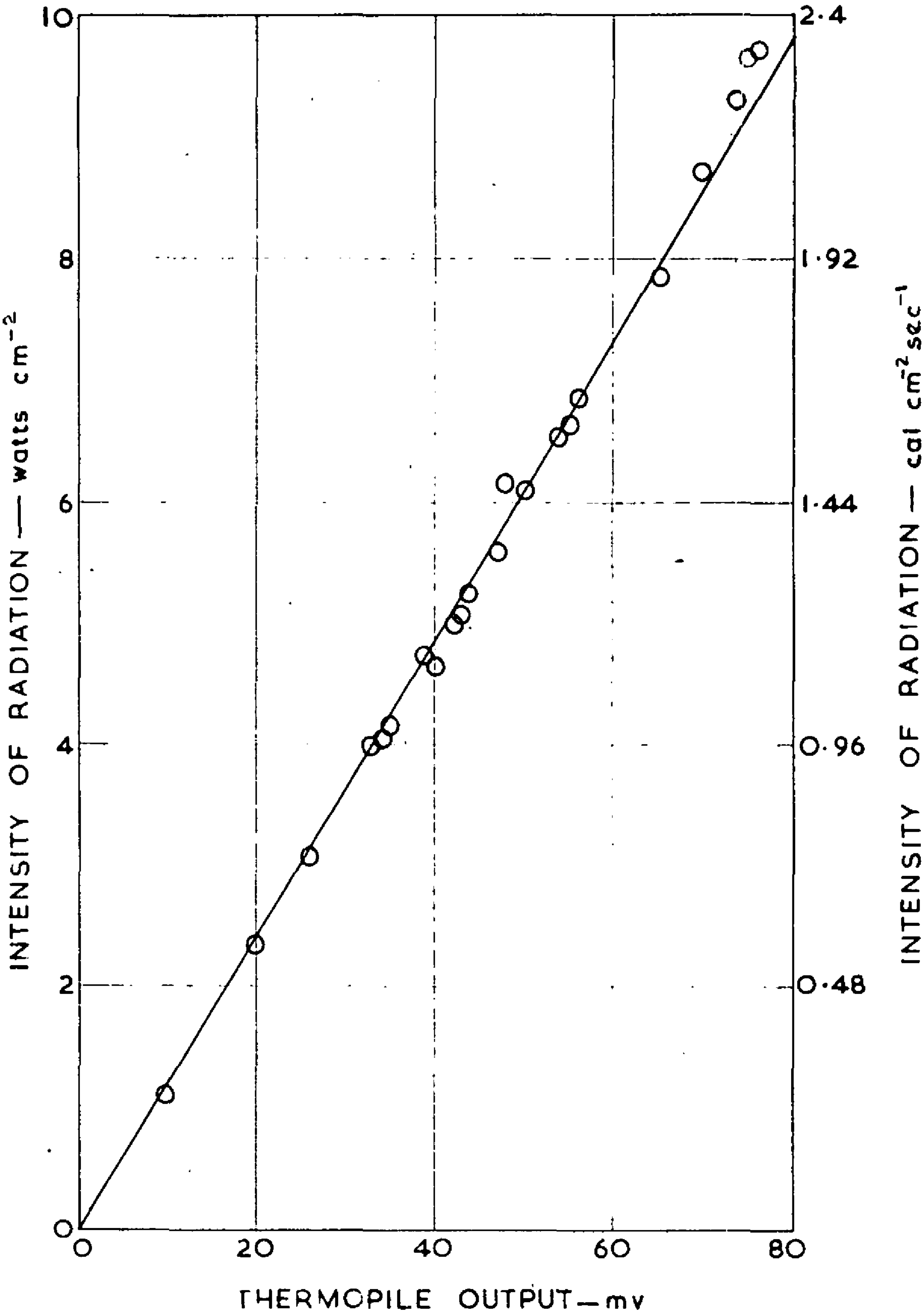


FIG. 6. CALIBRATION OF RADIOMETER A USING METHOD A TO CALCULATE THE INTENSITY OF RADIATION

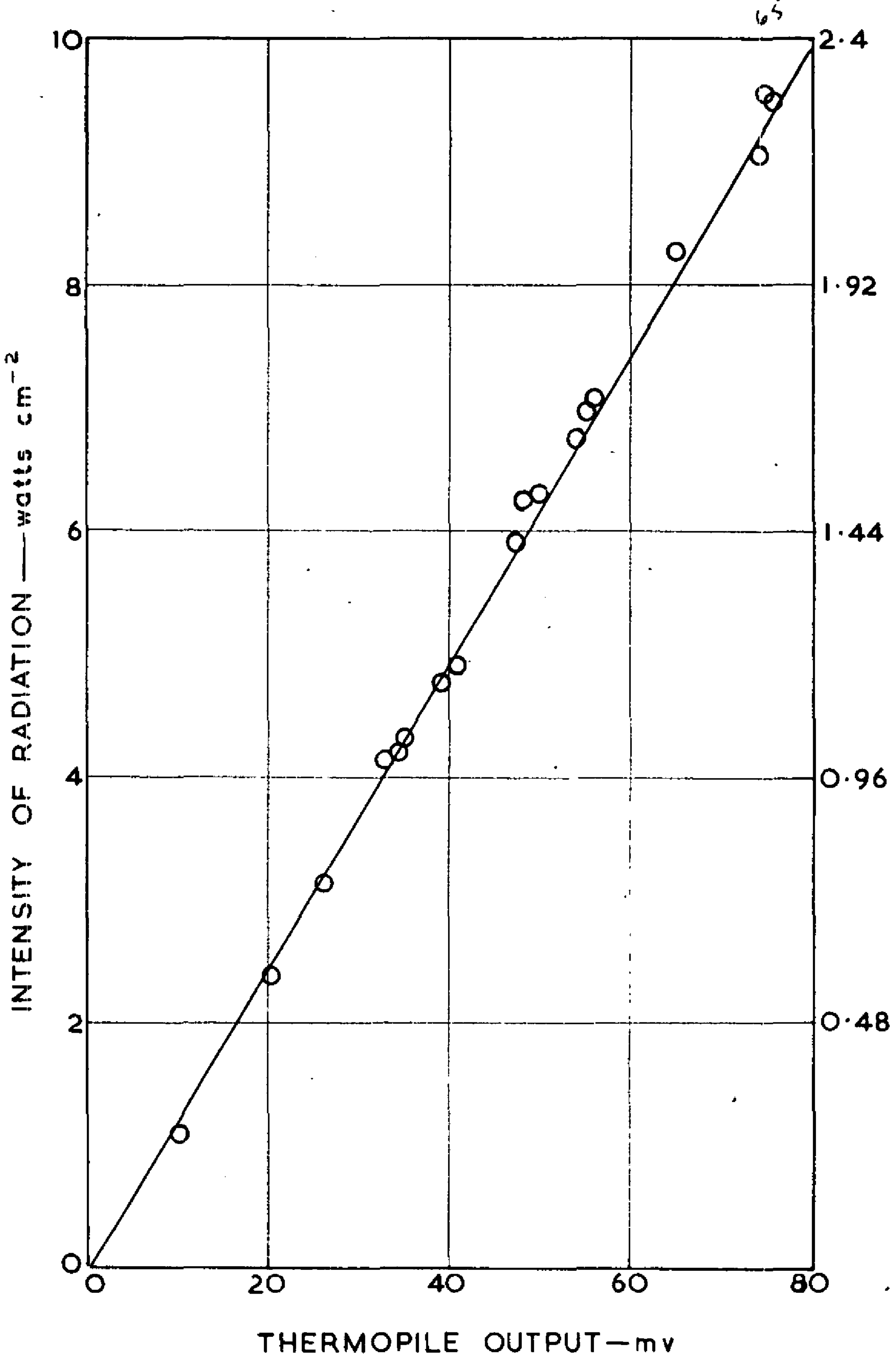


FIG.7. CALIBRATION OF RADIOMETER A USING METHOD B TO CALCULATE THE INTENSITY OF RADIATION

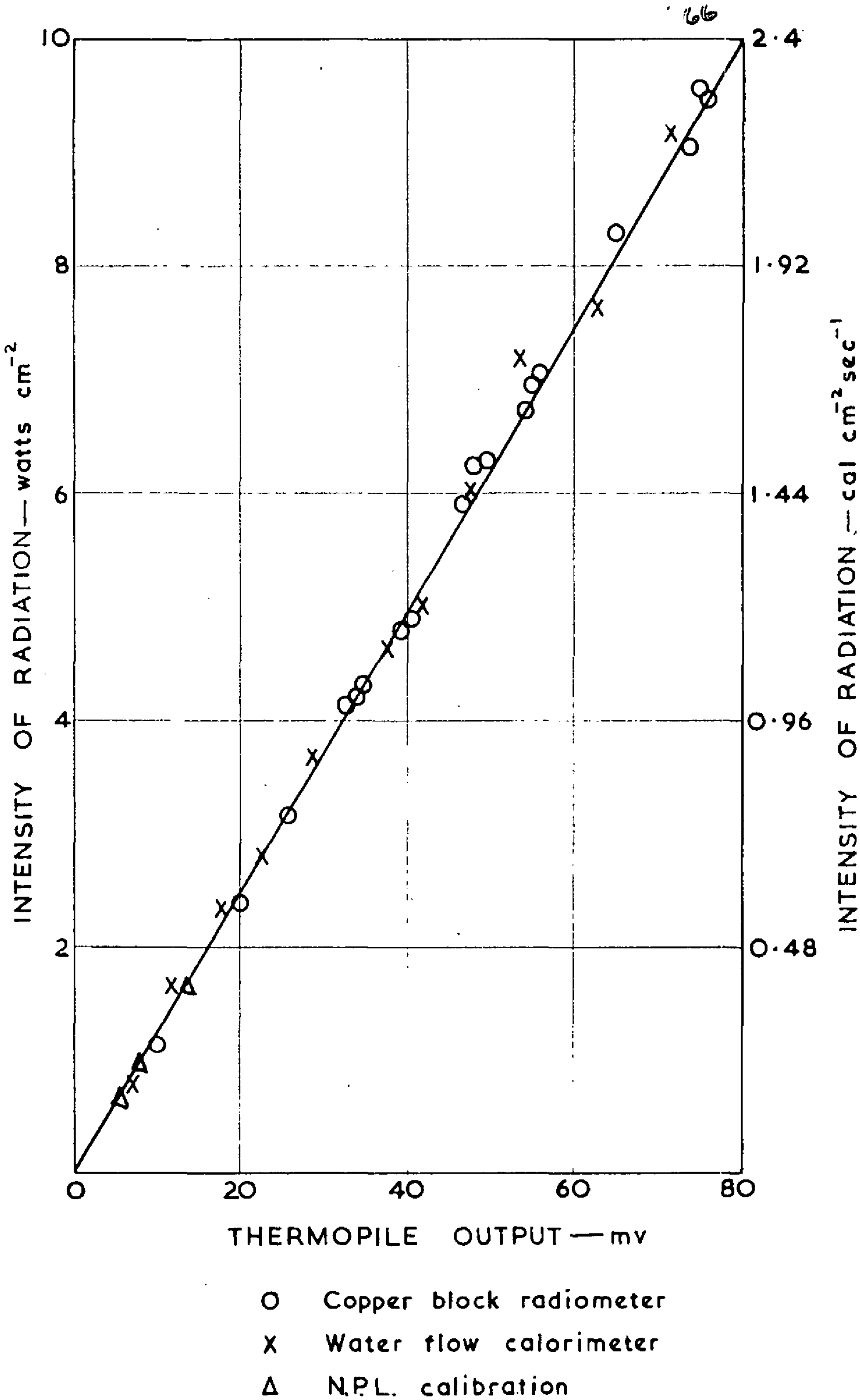


FIG. 8. COMPARATIVE CALIBRATIONS OF RADIOMETER A

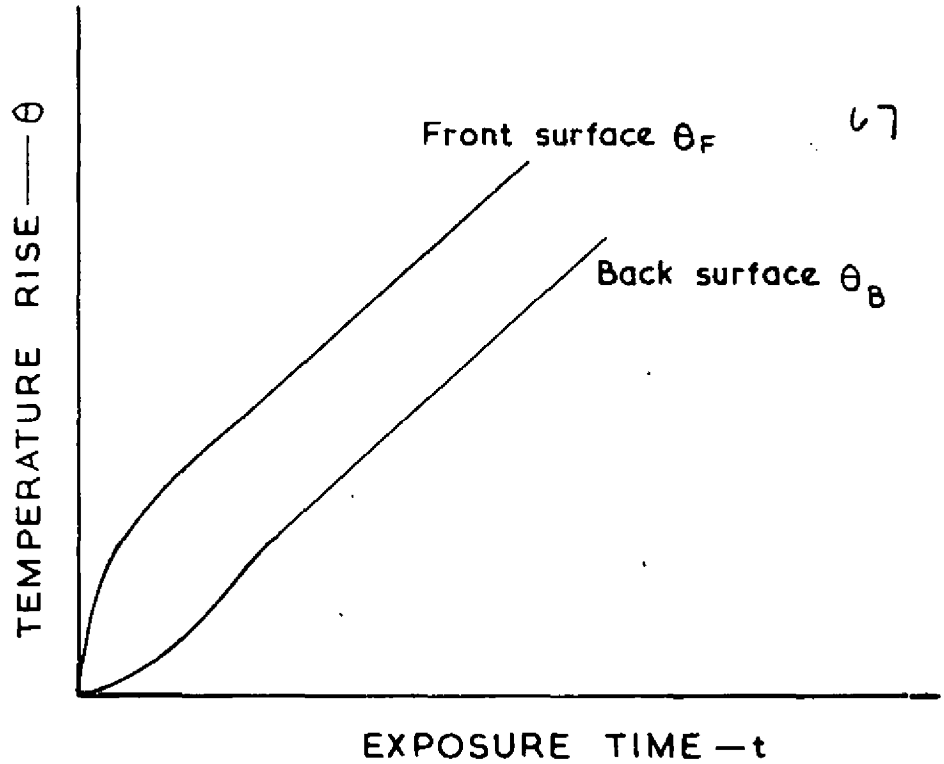


FIG. 9a. SKETCH OF RISE IN TEMPERATURE OF FRONT AND BACK SURFACE OF COPPER BLOCK EXPOSED TO A CONSTANT INTENSITY OF RADIATION

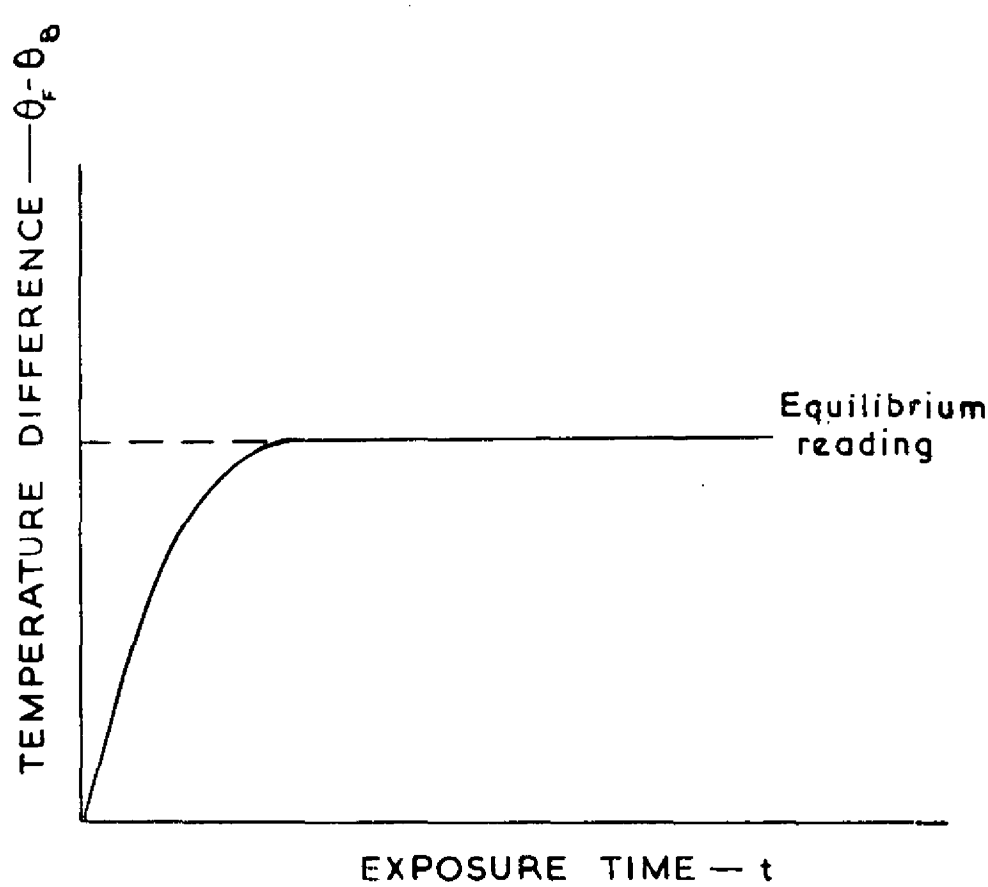


FIG. 9b. SKETCH OF TEMPERATURE DIFFERENCE BETWEEN FRONT AND BACK SURFACES OF COPPER BLOCK EXPOSED TO A CONSTANT INTENSITY OF RADIATION

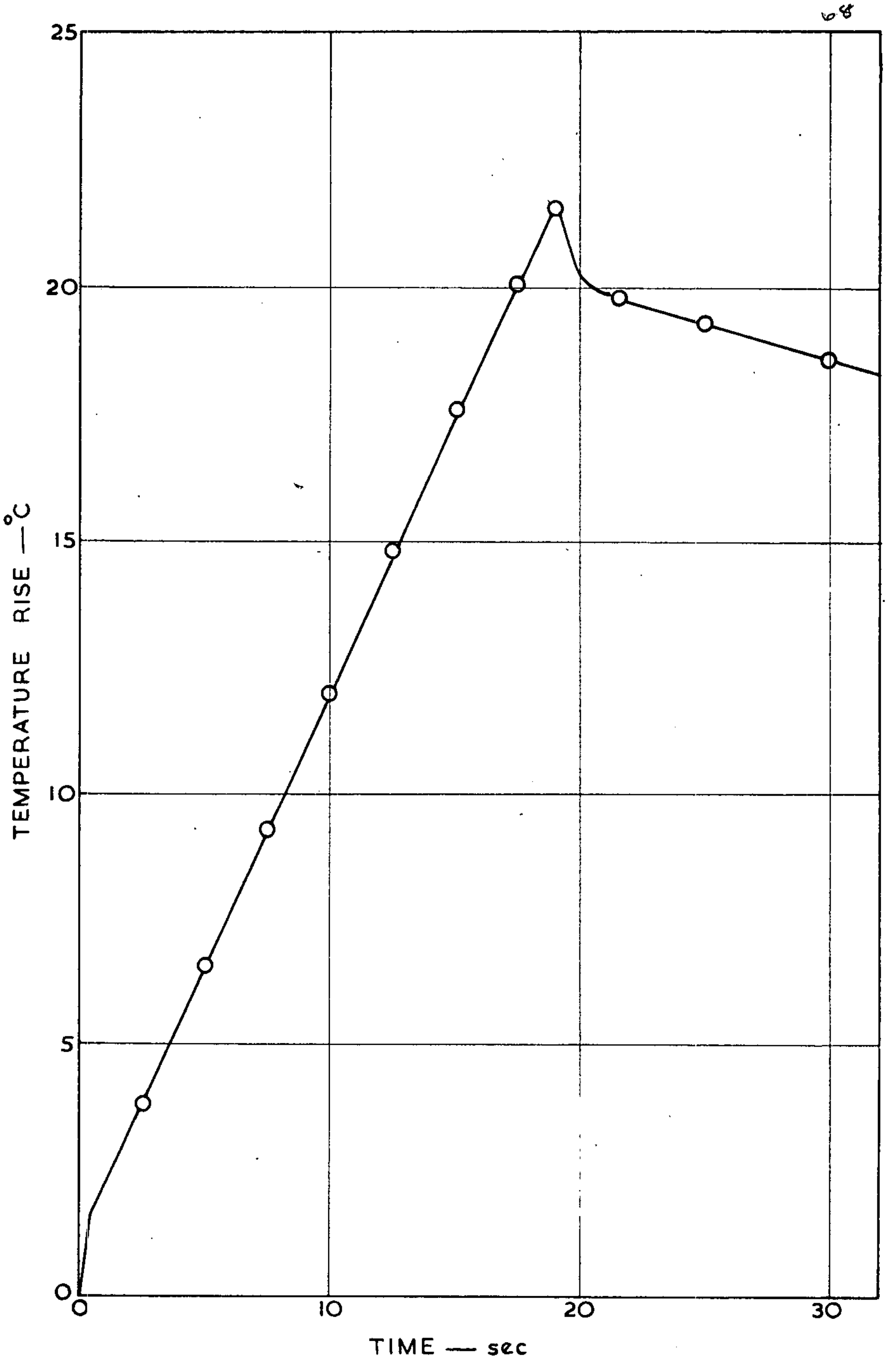


FIG. 10. TIME - TEMPERATURE CURVE OF THE BLOCK WHEN EXPOSED TO AN INTENSITY OF 6·1 WATTS cm^{-2} (1·45 cal $cm^{-2} sec^{-1}$) FOR 19 SECONDS

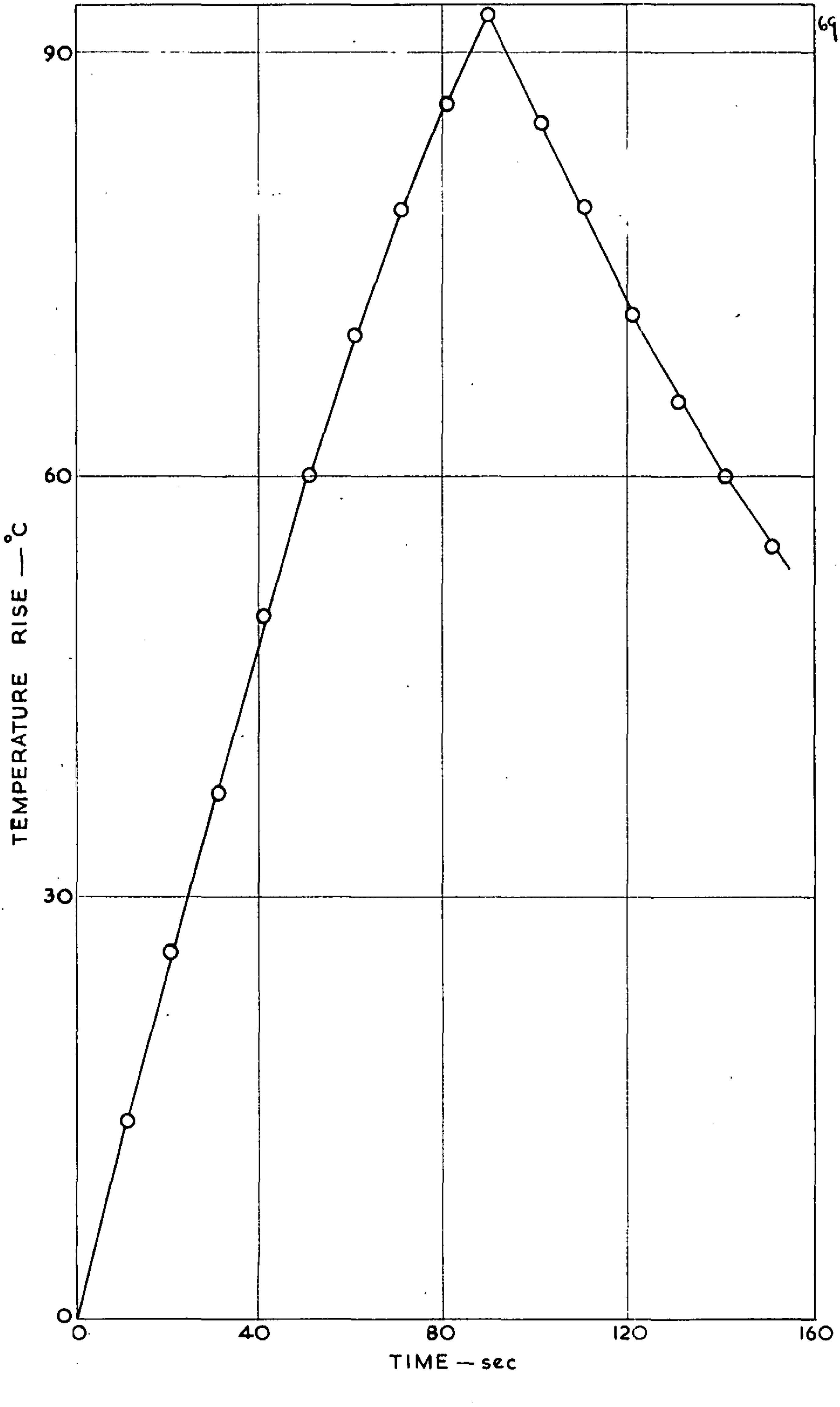


FIG. 11. TIME–TEMPERATURE CURVE OF THE BLOCK WHEN EXPOSED TO AN INTENSITY OF 6·9 WATTS cm^{-2} (1·66 cal cm^{-2} sec^{-1}) FOR 90 SECONDS

F.R. NOTE 117

NOT ISSUED