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COEFFICIENTS OF HEAT TRANSFER BY NATURAL CONVECTION

by

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Summary

This note is a brief introduction to the discussion of free convection given in the standard text books. The differences between spheres, disks and long cylinders at small Grashof numbers, and the limitations on the use in transient conditions of data valid for quasi-stationary conditions are discussed.

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Introduction

Experimental and theoretical data are often correlated by dimensionless variables. For example, in heat conduction the time variable 't' may be expressed non-dimensionally as kt/L^2 where k is the thermal diffusivity and L is a characteristic length.

These non-dimensional variables, which are simply numbers without units, can be derived in two ways. The first is based on a study of the differential equations and the boundary conditions and the second on the theory of dimensions. These two methods, particularly the former, show how the various terms in the equation would be combined together in any solution to the equations. In conduction problems the equations can often be solved if only approximately; in convection problems, rarely, and then only for laminar flow.

Dimensionless groups are the basis of all scaling or model theory and their relevance to convection is very fully discussed in "Heat Transfer" (1) by Jakob.

Application of theory to free convection

In free convection the driving force for the flow is the buoyancy term g $(\rho_0 - \rho)$ where "g" is the gravitational constant, ρ the local density of the convective fluid, and ρ_0 is the density of the fluid 'far away' from the heated or cooled surface. For a perfect gas this term equals $g\theta\rho/T_0$

where is the difference between the absolute temperatures T and T_0 of the gases locally and 'far away' from the surface.

The dimensionless form of this term is $\frac{gL^3\theta}{\nu^2 T_0}$ where L is a characteristic linear dimension, and ν is the kinematic viscosity i.e. the ratio of dynamic viscosity μ to the density ρ. This number is known as the Grashof Number, and it is the fundamental independent variable of free convection. There is a second such variable, the Prandtl Number ν/k, but this is almost invariant and for air is 0.71 over a wide range of temperature. The dependent variable is the heat transfer coefficient itself, H, and the dimensionless form of this is the Nusselt Number $\frac{HL}{K}$ where K is the thermal conductivity of the fluid. Data for steady-state free convection are then correlated by expressing the Nusselt No., N_{Nu}, as a function of the Grashof and Prandtl Nos. N_{Gr} and N_{Pr}.

$$\text{i.e.} \quad \frac{HL}{K} = \phi\left(\frac{gL^3\theta}{\nu^2 T_0}, \frac{\nu}{k} \right) \qquad \ldots\ldots(1)$$

The function ϕ depends only on the shape of the convective system. For shapes such as short cylinders a single ϕ could be obtained which would be a function of the Grashof and Prandtl Nos. and a third number, the ratio of the diameter to the length of the cylinder. For vertical surfaces L is the height, and for spheres and cylinders L is the diameter. Worked examples are given by Fishenden and Saunders (2).

In general it has been found practical to use power functions for ϕ over a considerable range in values and in free convection this has some theoretical justification (1), (2). For laminar flow over vertical surfaces the relation is of the form

$$\frac{H\,L}{K} \propto \left(\frac{g L^3 \theta}{\nu^2 T_0} \cdot \frac{\nu}{k} \right)^{1/4} \quad \ldots\ldots.(2)$$

A correlation given by Jakob (1) (p.530)

$$N_{Nu} = 0.55 \left(N_{Gr} \cdot N_{Pr} \right)^{1/4} \quad \ldots\ldots.(3)$$

This, however, does not hold for very high or for very low values of the Grashof No. and in both directions estimates obtained by extrapolating the formula give values of convective transfer which are too low.

It should be noticed that the index $\frac{1}{4}$ introduces a scale factor of $L^{\frac{1}{4}}$ so that the mean heat transfer per unit area from small plates is larger than from big plates. As the Grashof No. increases beyond 10^8, the index $\frac{1}{4}$ increases to $\frac{1}{3}$ so that the term L disappears from the equation. This increase in the index corresponds to a change from a primarily laminar flow to a primarily turbulent one. Data on convective heat transfer are given by Jakob (1). Fishenden and Saunders (2). Eckert (3) McAdams (4).

For forced convection the "buoyancy" term may be neglected, and if this is done the Grashof No. is replaced as an independent variable by the Reynolds No. appropriate to the velocity in the bulk stream of the fluid.

A correlation given by Jakob (1)(p.473) for forced laminar convection parallel to a plane is

$$N_{Nu} = 0.66 \; N_{Pr}^{1/3} \; N_{Re}^{1/2} \quad \ldots\ldots.(4)$$

The characteristic dimension is the distance along the plane.

Relation between free and forced convection

It is of interest to relate free convection (equation (3)) to forced convection (equation (4)) by replacing the velocity of the main stream which appears in the Reynolds Number by the velocity due to the buoyancy of of heated gases.

In a stream of heated gas of temperature θ above ambient and of uniform cross section the difference is pressure Δp at the level of the bottom of the stream can be equalled to the difference between the weights of the heated gas and unheated gas

$$\text{i.e.} \quad \Delta p = L(\rho_0 - \rho) \quad \ldots\ldots.(5)$$

$$\frac{v^2}{2g} = \frac{\Delta p}{\rho} = L\left(\frac{\rho_0 - \rho}{\rho}\right)$$

$$v = \sqrt{2g \frac{\theta}{T_0} L} \quad \ldots\ldots.(6)$$

In a general discussion this can be taken as representative of the velocity of the gases near a heated plate.

If we substitute V as given by equation (6) into equation (4) we obtain

$$N_{Nu} = 0.66 \left(\frac{L}{\nu}\right)^{1/2} \left(\frac{2gL\theta}{T_0}\right)^{1/4} N_{Pr}^{1/3} \qquad \ldots\ldots.(7)$$

$$= 0.78\ N_{Pr}^{1/3}\ N_{Gr}^{1/4} \qquad \ldots\ldots.(8)$$

The index of N_{Pr} does not agree with that given by equation (3) but for air equation (8) gives values of N_{Nu} only about 20 per cent in excess of that given by the equation for free laminar convection, and in view of the approximation made this is reasonably close.

In systems intermediate between free and forced convection both Reynolds and Grashof Nos. must be considered as independent variables. Although the Reynolds No. cannot enter into the equation for the heat transfer coefficient of free convection it is the same at corresponding points in space in similar systems of free convection and would be a function of the Grashof No. or the Nusselt No. for dissimilar systems.

The Nusselt No. for small Grashof Nos.

When the Grashof No. is small compared with unity the problem may be regarded primarily as one of conduction. It is therefore possible to calculate limiting values for the Nusselt Nos. for different bodies. For a sphere, simple theory shows this limiting value to be 2. It will be seen that Jakob ("Heat Transfer" Vol. I. Jakob, 1951, p.525 and 529), gives a correlation of data which implies that the Nusselt No. for sphere, plates and cylinders can be correlated for small Grashof Nos. as they can for large ones. In fact the only data quoted for low Grashof Nos. refers to cylinders and these underestimate the Nusselt No. for spheres which must exceed 2.

Less well known is the limiting Nusselt No. for a disk. The problem of conduction from a disk at uniform potential to a surrounding infinite medium is a classical problem in electrical theory, and the result in thermal units leads to a value of the Nusselt No. in terms of the mean heat flow from the disk of $8/\pi$ or 2.56. The corresponding problem in terms of the mean temperature for a uniform flow yields a Nusselt No. of $3\pi/4$ or 2.25.

The limiting Nusselt No. for an infinite cylinder in an infinite medium, unlike that for finite heat sources, does not approach a finite limiting value but tends to zero. In practice, however, the cylinder is finite and in a finite medium, in which circumstances the limiting Nusselt No. depends on the ratio of the diameter to the length of the cylinder and on the ratio of the cylinder diameter to the distance from the cylinder to the surrounding surfaces at constant temperature. Much of the value of experimental data obtained at low Grashof Nos. may be lost if these relevant items are not quoted (see the contribution of Le Fevre in the "General Discussion of Heat Transfer") (5).

Data on the dependence of the Nusselt No. on the Grashof No. for spheres and disks, are not given in the textbooks for small values of the Grashof No. but for forced convection Ranz and Marshall (6) showed that experimental data could be correlated in the range $2 < N_{Re} < 10^5$ by simply adding the heat transfer by conduction without convection to that for forced convection. Thus

$$N_{Nu} = 2 + 0.6\ N_{Pr}^{1/3}\ N_{Re}^{1/2} \qquad \ldots\ldots.(9)$$

In view of the relationship between free and forced convection Ranz and Marshall suggested a similar treatment for free convection. Adding 2 to the Nusselt No. given by equation (3) gives

$$N_{Nu} = 2 + 0.55\, N_{Pr}^{\frac{1}{4}}\, N_{Gr}^{\frac{1}{4}} \quad (N_{Gr} < 10^8) \qquad \ldots\ldots.(10)$$

Some experimental work on the convective loss of heat from metal spheres has been done by Lyakhovskii (7) who expressed his results as

$$N_{Nu} = 2.29\, N_{Gr}^{\,0.03} \qquad (1 < N_{Gr} < 35)$$

$$N_{Nu} = 1.4\, N_{Gr}^{\,0.167} \qquad (35 < N_{Gr} < 2500).$$

This corresponds closely to values about 10 per cent less than those given by equation (10) throughout the range $1 < N_{Gr} < 2500$.

One would also expect the Nusselt No. for a disk to be given by equation (10) with the constant 2.5 instead of 2. Hence the error in using equation (3) for spheres and disks in air may be estimated as about 40 per cent and 70 per cent at a Grashof No. of 10^4 and 10^3 respectively.

Transient convection

The convection formulae generally quoted are based on steady state conditions and can only be used for transient conditions, such as the heating of a surface to ignition, if the time for any significant change in surface temperature is long compared with the time constant 't_c' of the convective system. By time-constant in this discussion we shall mean a time, of a certain order of magnitude, in which the surface heat transfer has become of the same order of magnitude as in the steady state. Dimensional analysis shows that for a given Grashof No. the time constant is proportional to L^2. We shall for convenience consider here only systems in which the linear dimensions in the plane of heat flow are of the same order of magnitude, so that no ambiguity arises as to the particular dimension concerned.

For small Grashof numbers, the time constant must tend to that for simple diffusion i.e. kt_c/L^2 is of order unity and with

$k = 0.187$ cm^2/sec for air and L, 3 cms, t_c is of order 1 minute. If the motion of the gases plays a significant part in the heat transfer t_c is different and a lower limit for its order of magnitude is given by the time t_o for an element of gas to pass by the solid.

If v is a suitable mean gas velocity we have

$$t_o \doteq \frac{L}{v}$$

An estimate of v has been obtained above in equation (1). Hence we have

$$t_o = \sqrt{\frac{T_0 L}{2g\,\theta}} \qquad \ldots\ldots.(11)$$

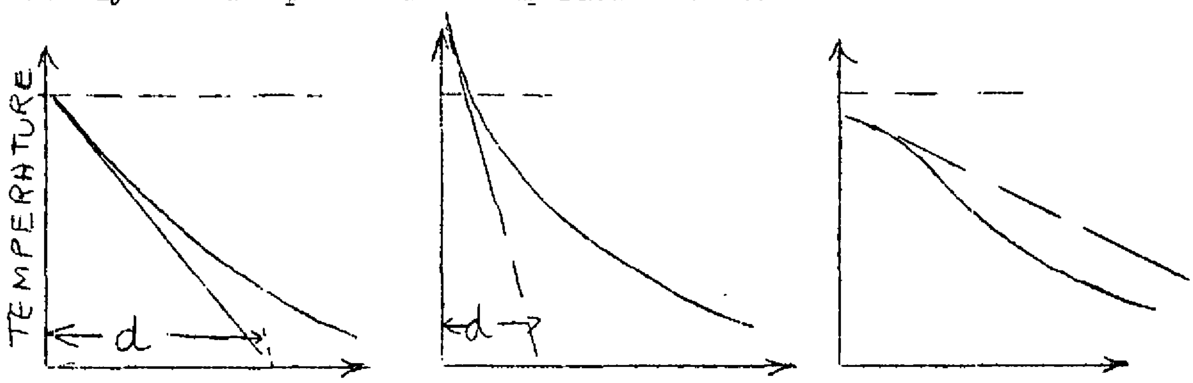
$$\doteq \frac{L^2}{\nu}\left(2\, N_{Gr}\right)^{-\frac{1}{2}} \qquad \ldots\ldots.(12)$$

$$= \frac{\nu^{1/3}}{\sqrt{2}}\left(\frac{T_0}{g\,\theta}\right)^{1/3} N_{Gr}^{\,1/6} \qquad \ldots\ldots.(13)$$

Equations (12) and (13) show that for a given Grashof number ' t_0 ' is proportional to L^2, or inversely proportional to $\Theta^{\frac{1}{3}}$, and for L, 3 cms. t_0 is of the order of 10^{-1} second. It may be concluded that the time constant of the system will be intermediate between tenths of a second and minutes and will tend towards the upper end of this range for small Grashof numbers. When transient changes of surface temperature or of flux to a surface occur, the heat loss will differ from that in the steady state. Consider the curve in Figure 1a which represents the temperature distribution near a heated surface. Since the heat transfer is proportional to the temperature gradient we can write

$$N_{Nu} \propto \frac{1}{d}$$

If the temperature of the surface increases at a rate faster than the thermal gradients can follow "d" is decreased see Figure 1b, and similarly with a rapid fall in temperature "d" is increased



Distance from heated surface

(a) (b) (c)

Figure (1)

Hence a rapid heating produces a higher Nusselt No. and rapid cooling a lower one than obtains in the steady state [7].

The case of rapid heating is of interest in experiments in ignition. Consider a solid of height L initially cold, before being subjected to an instantaneous rise in surface temperature of V. For short times, the behaviour will be similar to that of a semi-infinite solid and we have the surface temperature given by[8]

$$\Theta = V \operatorname{erfc} x/2\sqrt{kt} \qquad \text{.......(14)}$$

Hence the heat flux "q" is given by

$$q = -K\left(\frac{d\theta}{dx}\right)_0 = VK/\sqrt{\pi k t}$$

and

$$N_{Nu} = \left(\frac{L^2}{\pi k t}\right)^{1/2} \qquad \text{.......(15)}$$

For short times this result will hold for any solid. The result only holds good while $(kt)^{1/2}$ is much less than any of the dimensions of the solid.

It follows that for a rapid increase in surface temperature the instantaneous Nusselt No. becomes very large and then falls before attaining its steady value for the convective system. It is interesting to consider this in the light of Figure (2).

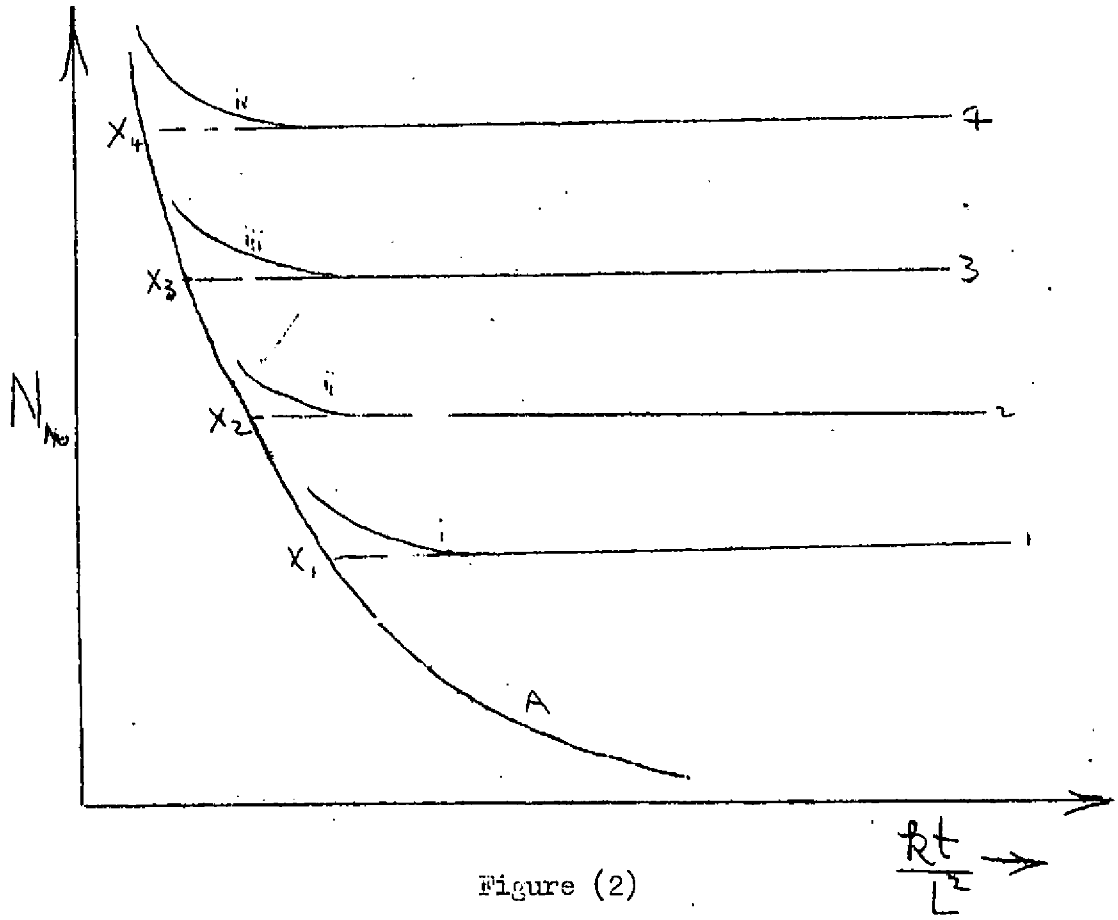


Figure (2)

Line A represents equation (15) and the horizontal lines correspond to different steady state values of N_{Nu} for different Grashof Nos. Lines i, ii and iii will therefore be the relation between N_{Nu} and t for different Grashof Nos.

It is interesting to note that the points X_i etc. are related to the Grashof No. for a given Prandtl No. by

$$t_x \propto \frac{L^2}{k} N_{Gr}^{-1/2} \qquad \text{.......(18)}$$

This suggests that the time constant decreases with increasing Grashof No. The fact that equation (18) is similar to equation (12) in the index of the Grashof No. is a consequence of the index of the Grashof No. in equation (3) being $\frac{1}{4}$.

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