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THE IRRADIATION OF THIN SHEETS

by

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Summary

The irradiation of a circular area and an infinite strip on an infinite sheet of thin material is discussed theoretically.

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Introduction

When a localised area of a large thin sheet is irradiated, heat is conducted from the irradiated area to the surrounding material, reducing the temperature rise of the heated area. This problem is discussed theoretically in this note for two shapes of irradiated area, a circle and an infinite strip. One of the applications of this work is the testing of the ease of ignition of thin fabrics.

Theory

Let Q be the heat flux per unit area/sec

K be the thermal conductivity of the material

ρ be its density

C its specific heat

k its diffusivity K/C

Δ its thickness

H be the cooling coefficient for the surface

h the ratio H/K

t the time

R the radius of the circular irradiated area

L the half width of the strip irradiated area.

We assume the sheet to be thin enough for it to be regarded as heated uniformly across the sheet. The validity of this assumption depends on conditions $h\Delta \ll 1$ and $\Delta/L \ll 1$. The heat flux Q is assumed to be a steady flux. If it were a pulse the theory may be used to calculate the maximum temperature reached at the end of the pulse.

Irradiated strip

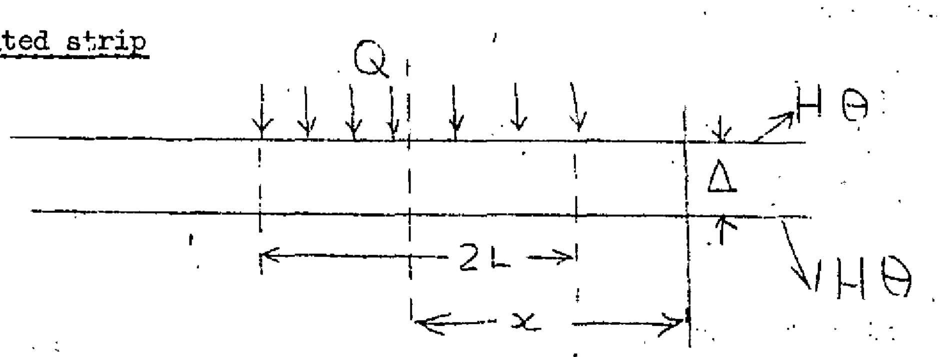

Figure 1

We have for the sheet in Figure 1

$$\frac{d^2\theta}{dx^2} = \frac{1}{k}\frac{d\theta}{dt} - \frac{Q}{K\Delta} + \frac{2H\theta}{K\Delta} \qquad (-L<x<L) \qquad \ldots\ldots(1)$$

Outside the range $-L < x < L$ the equation above is replaced by one in which Q is zero. We apply the Fourier Cosine Transform from which we have

$$-p^2\bar{\theta} = \frac{1}{k}\frac{d\bar{\theta}}{dt} - \frac{Q}{K\Delta}\frac{\sin pL}{L} + \frac{2h\bar{\theta}}{\Delta} \qquad \ldots\ldots(2)$$

where

$$\bar{\theta} = \int_0^\infty \theta \cos px \, . dx.$$

Since $\theta = 0$ at $t = o$ we have from (2)

$$\bar{\theta} = \frac{Q}{K\Delta}\frac{\sin pL}{p\left(p^2+\frac{2h}{\Delta}\right)}\left(1-e^{-k\left(p^2+\frac{2h}{\Delta}\right)t}\right) \ldots\ldots(3)$$

i.e. after inversion

$$\theta = \frac{2Q}{\pi K\Delta}\int_0^\infty \frac{\sin pL}{p\left(p^2+\frac{2h}{\Delta}\right)}\left(1-e^{-k\left(p^2+\frac{2h}{\Delta}\right)t}\right)\cos px\, dp \qquad \ldots\ldots(4)$$

Differentiating (4) we have

$$\frac{d\theta}{dt} = \frac{2Qk}{\pi K\Delta}\int_0^\infty \frac{\sin pL}{p} e^{-k\left(p^2+\frac{2h}{\Delta}\right)t}\cos px\, dp$$

The maximum temperature, θ_0, occurs at the centre of the strip at $x = o$.

Hence

$$\frac{d\theta_0}{dt} = \frac{2Q}{\pi\rho c\Delta}\int_0^\infty \frac{\sin pL}{p} e^{-k\left(p^2+\frac{2h}{\Delta}\right)t}\, . dp. \qquad \ldots\ldots(5)$$

$$= \frac{Q}{\rho c\Delta} e^{-\frac{2hkt}{\Delta}} \operatorname{erf}. \frac{L}{2\sqrt{kt}} \qquad \ldots\ldots(6)$$

Hence

$$\theta_0 = \frac{Q}{\rho c \Delta}\int_0^t e^{-\frac{2hk\lambda}{\Delta}} \operatorname{erf}\frac{L}{2\sqrt{k\lambda}}\, d\lambda \qquad \ldots\ldots(7)$$

If cooling at the surface is neglected

$$\theta_0 = \frac{QL^2}{2K\Delta}\left[\frac{\operatorname{erf} X}{2X^2} - \operatorname{erfc} X + \frac{1}{\sqrt{\pi}}\frac{e^{-X^2}}{X}\right] \ldots\ldots(8)$$

Where $X = \dfrac{L}{2\sqrt{kt}}$

Now the maximum temperature of the sheet calculated on the assumption of no cooling to the neighbouring material or from the surface is

$$\theta_{max} = \frac{Qt}{\rho c \Delta} \qquad \ldots\ldots(9)$$

Hence $$\theta_0/\theta_{max} = \operatorname{erf} X - 2X^2 \operatorname{erfc} X + \frac{2}{\sqrt{\pi}} X e^{-X^2} \qquad \ldots\ldots(10)$$

If θ_0 is not to be permitted to exceed 250°C H does not exceed 10^{-3} cal/cm²/s/°C.

For Aluminium	K =	0·36	C.G.S.	Units
	ρ =	2·8	"	"
	c =	0·23	"	"
	k =	0·56	"	"

and for a heating time of 10 seconds for a sheet of 50×10^{-3} in. thick

we have $$\frac{2Ht}{\Delta} = \frac{2 \times 10^{-3} \times 10}{50 \times 10^{-3} \times 2{\cdot}54} = 0{\cdot}15$$

From equation 7 such conditions as the surface cooling do not affect the temperatures reached by more than 17 per cent. Having calculated θ_{max} and having taken a permitted temperature rise θ_0 the maximum value of $L/2\sqrt{kt_H}$, where t_H is the duration of the pulse, can be calculated. Hence 2L the permitted width irradiated may be found. Figure 2 shows the relation between θ_0/θ_{max} and X given in equation 10.

Circular area

Instead of equation 1 we have for the circle

$$-\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}=\frac{1}{k}\frac{\partial\theta}{\partial t}-\frac{Q}{K\Delta}+\frac{2h\theta}{\Delta}\quad .....(11)$$

where $Q = 0 \quad r > R$

Hence $-p^2\bar{\theta} = \frac{1}{k}\frac{d\bar{\theta}}{dt} - \frac{QR}{K\Delta p}J_1(pR) + \frac{2h\bar{\theta}}{\Delta}$

where $\bar{\theta} = \int_0^\infty \theta\, r\, J_0(pr)\,dr$

Hence $\bar{\theta} = \frac{QR}{K\Delta}\frac{J_1(pR)}{p\left(p^2+\frac{2h}{\Delta}\right)}\left(1-e^{-k\left(p^2+\frac{2h}{\Delta}\right)t}\right)$

and $$\theta = \frac{QR}{K\Delta}\int_0^\infty \frac{J_1(pR)\,J_0(pr)}{\left(p^2+\frac{2h}{\Delta}\right)}\left(1-e^{-k\left(p^2+\frac{2h}{\Delta}\right)t}\right)dp \quad .....(12)$$

As before surface cooling will be negligible if $\frac{2H\,t}{\Delta} \ll 1$, and we consider only the centre temperature.

i.e. $$\theta_0 = \frac{QR}{K\Delta}\int_0^\infty \frac{J_1(pR)\left(1-e^{-p^2kt}\right)}{p^2}dp \quad .....(13)$$

Differentiating (13) we have

$$\frac{d\theta_0}{dt} = \frac{QR}{\rho c \Delta} \int_0^\infty J_1(pR)\, e^{-kp^2 t}\, dp$$

This may be shown to reduce to

$$\frac{d\theta_0}{dt} = \frac{Q}{\rho c \Delta} \left(1 - e^{-R^2/4kt} \right) \quad \ldots\ldots(14)$$

$$\therefore \frac{\theta_0}{\theta_{max}} = 1 - e^{-Y^2} - Y^2 E_i(-Y^2) \quad \ldots\ldots(15)$$

$$\text{where} \quad E_i(-x) = \int_{-\infty}^{-x} e^u \frac{du}{u}$$

where θ_{max} is defined as in equation 9

and where $Y = \frac{R}{2\sqrt{kt}}$

This relation (equation 15) between θ_0/θ_{max} and Y is also shown in Figure 2. As is expected the effect of cooling to the surrounding cool material is greater for the circle than for a strip of width equal to the circle diameter. From the data in Figure 1 the effect can be estimated for any size and heating time. It may be noted that the thickness of the material appears only in θ_{max}. For a given flux θ_{max} increases as Δ decreases and this alters the L or R necessary to restrict the actual temperature to a given value of θ_0. If surface cooling cannot be neglected, the relevant integrals cannot be reduced to a form involving simple tabulated functions and they must be computed for each case.

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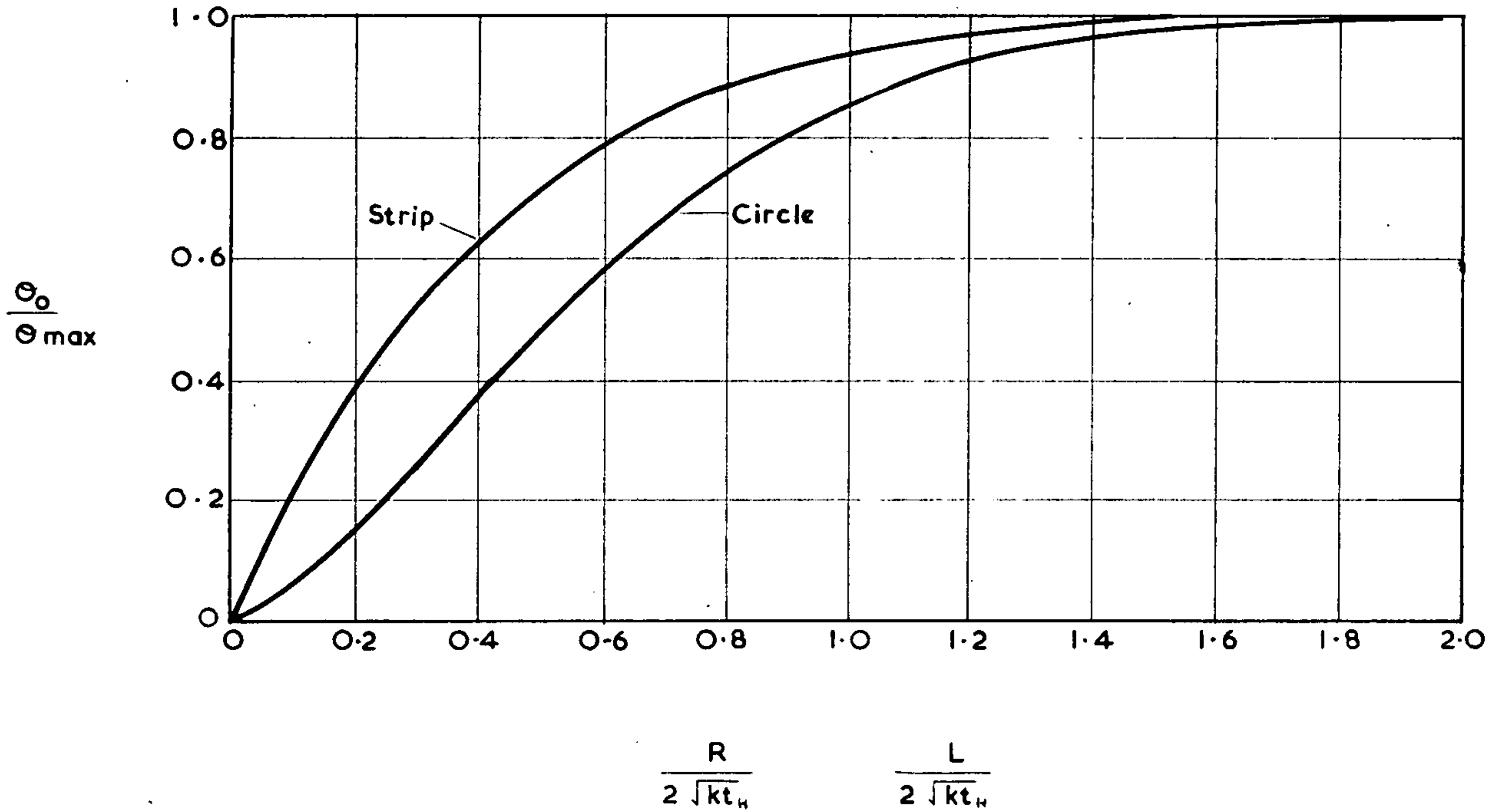

FIG. 2. THE EFFECT OF SIZE & HEATING TIME ON THE MAXIMUM TEMPERATURE OF IRRADIATED AREAS