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DEPARTMENT OF SCIENTIFIC AND INDUSTRIAL RESEARCH AND FIRE OFFICES' COMMITTEE
JOINT FIRE RESEARCH ORGANIZATION

A PROBLEM IN THE HEATING OF SMALL AREAS ON A LARGE SOLID

by

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Summary

The transient temperature at the centre of a heated circular area on a semi-infinite solid with cooling to the atmosphere has been computed.

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Introduction

In F.R. Note 163/1955 (1) Blok's [2] analysis for the transient heating of a circular area without cooling was given. Steady state temperatures with cooling were also derived. The present note gives the temperature at the centre of circular area at various times with different degrees of cooling. The work is relevant to the use of small irradiated areas to represent large areas and allows some estimate to be made of the time for which the small area behaves as a large one.

Theory

Let θ be the temperature
- I the incident flux
- H the cooling coefficient
- K the thermal conductivity
- h the quotient H/K
- ρ the density
- c the specific heat
- k the thermal diffusity $K/\rho c$
- r, z cylindrical coordinates
- x y z cartesion coordination
- t the time
- and R the radius of heated area

We make use of Green's function for a point some at x' y' z' in a semi-infinite solid ($x>0$) with radiation at the boundary $x = o$ to a medium at zero temperature. This function is given by Carslaw and Jaegar in "The Conduction of heat in solids" (3). If u is the temperature due to an instantaneous point source of unit strength at x', y', z', the temperature at x, y, z after a time 't' is

$$u = \frac{1}{8(\pi k t)^{3/2}}\left\{e^{-\frac{(x+x')^2}{4kt}} + e^{-\frac{(x-x')^2}{4kt}}\right\} e^{-\frac{(y-y')^2+(z-z')^2}{4kt}} - \frac{h}{4\pi k t}\,\mathrm{erfc}\left(\frac{x+x'}{2\sqrt{kt}} + h\sqrt{kt}\right) \times \exp\left[h(x+x') + h^2 kt - \frac{(y-y')^2+(z-z')^2}{4kt}\right] \quad \ldots\ldots (1)$$

For a continuous source of strength $\frac{I}{\rho c}$ (over $r<R$) at $x' = o$, and $\sqrt{x'^2+y'^2}$ equal to r the temperature at $x = y = z = o$ at time 't' is, from (1).

$$\theta = \frac{I}{\rho c}\int_0^{2\pi} d\psi \int_0^t d\lambda \int_0^R r\,dr\, e^{-r^2/4k\lambda}\left\{\frac{1}{4(\pi k\lambda)^{3/2}} - \frac{h}{4\pi k\lambda} e^{h^2 k\lambda}\,\mathrm{erfc}\, h\sqrt{k\lambda}\right\} \quad \ldots\ldots (2)$$

$$= \frac{I}{\rho c}\int_0^t d\lambda\left(1 - e^{-R^2/4k\lambda}\right)\left(\frac{1}{(\pi k\lambda)^{1/2}} - h\, e^{h^2 k\lambda}\,\mathrm{erfc}\, h\sqrt{k\lambda}\right) \quad \ldots\ldots (3)$$

The solution to the integral of the first term in the first bracket in equation (3) is the result for a heated disk with no surface cooling and is equal to Blok's (2) result

$$\theta_{H=0} = \frac{IR}{K}\left\{\frac{2}{\sqrt{\pi}}\left(\frac{kt}{R^2}\right)^{1/2}\left(1 - e^{-R^2/4kt}\right) + \operatorname{erfc}\frac{R}{2\sqrt{kt}}\right.$$

...... (4)

Hence

$$\theta_{H=0} - \theta = \frac{Ih}{\rho c}\int_0^t d\lambda \cdot e^{h^2 k\lambda} \operatorname{erfc} h\sqrt{k\lambda}\left(1 - e^{-\frac{R^2}{4k\lambda}}\right)$$

...... (5)

Let $\lambda = \frac{x^2}{h^2 k}$

Then

$$\theta_{H=0} - \theta = \frac{IR}{K}\cdot\frac{2}{hR}\int_0^{h\sqrt{kt}}\left(1 - e^{-\frac{R^2h^2}{4x^2}}\right) x\, e^{x^2} \operatorname{erfc} x\, dx$$

...... (6)

This has been numerically evaluated and in Fig. (1) are shown the resulting values of θ for various value of hR and kt/R^2 For the steady state the value of θ has been obtained analytically (1) as

$$\theta = \frac{IR}{K}\left[1 + \frac{1}{hR} - \frac{\pi}{2}\left(H_1(hR) - Y_1(hR)\right)\right]$$

...... (7)

Where Y_1 is Weber's Bessel function of the second kind of the first order and H_1 is Struve's function of the first order.

It is possible to find the values of kt/R^2 at which the temperature at the centre of the circle is, say, 10% and 20% less than it would be for a plane source. These values of kt/R^2 are shown in Fig. (2) as a function of h R, while in Fig. (3) h^2kt is shown as a function of hR for 10% & 20% error. For any h, R, and k it is thus possible to find the maximum duration of heating for which a small source is sensibly similar to a large source.

Acknowledgment

The author is indebted to Mr. Philips who performed the bulk of the computations.

References

(1) "Some practical limitations to the use of small irradiated areas on solids for the study of thermal damage". Thomas, P. H. and Simms, D. L. Department of Scientific and Industrial Research and Fire Offices' Committee Joint Fire Research Organization. F.R. Note No. 163/1955.

(2) "General discussion on lubrication". Blok. H. Inst. Mech. Eng. 1937. Vol II. P. 222.

(3) "Conduction of heat in solids". Carslaw, H. S., Jaegar, J. C. O.V.P. 1947. P. 308.

(4) ibid. P. 55.

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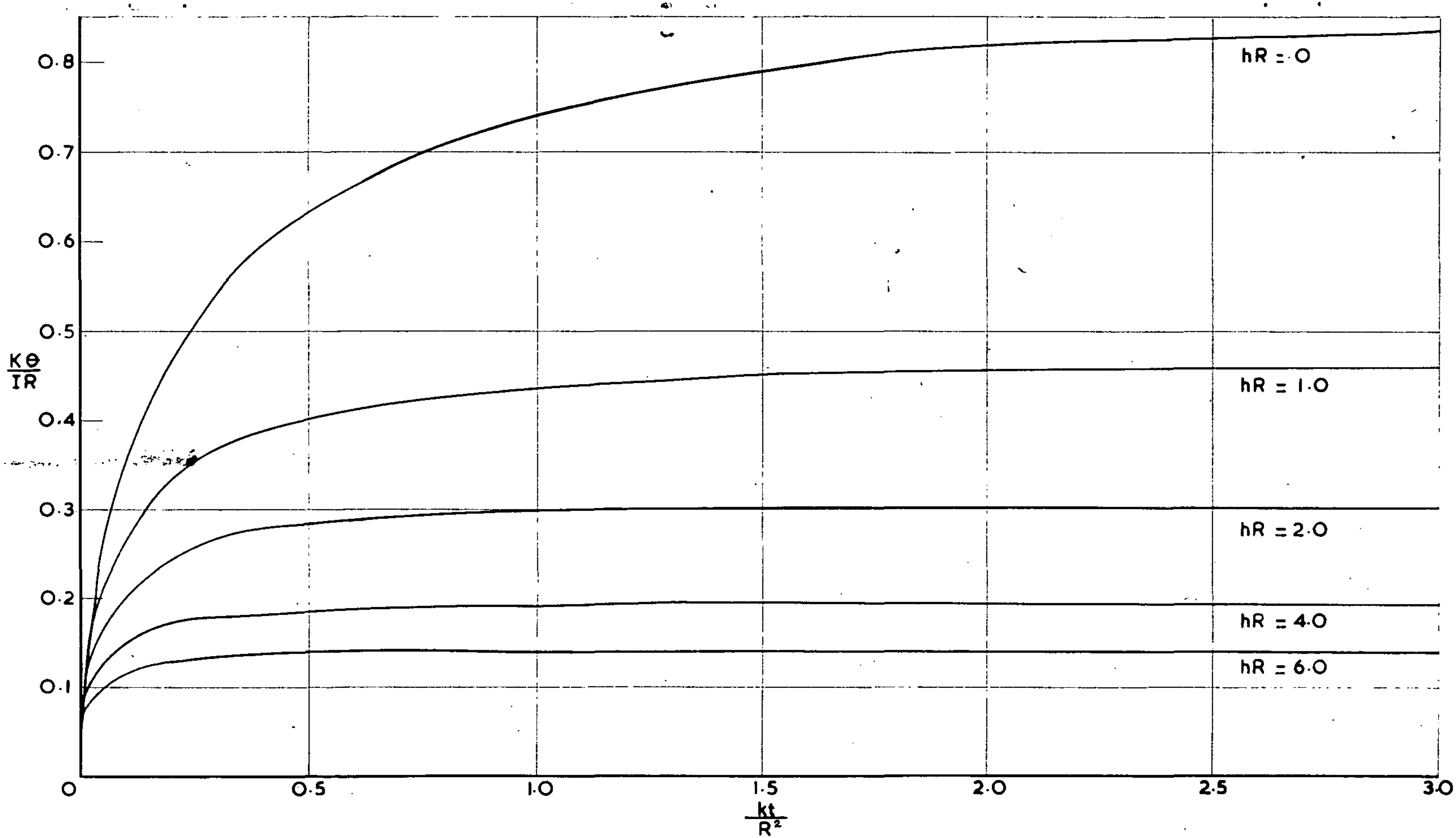

FIG. 1. TRANSIENT TEMPERATURE AT CENTRE OF HEATED DISK

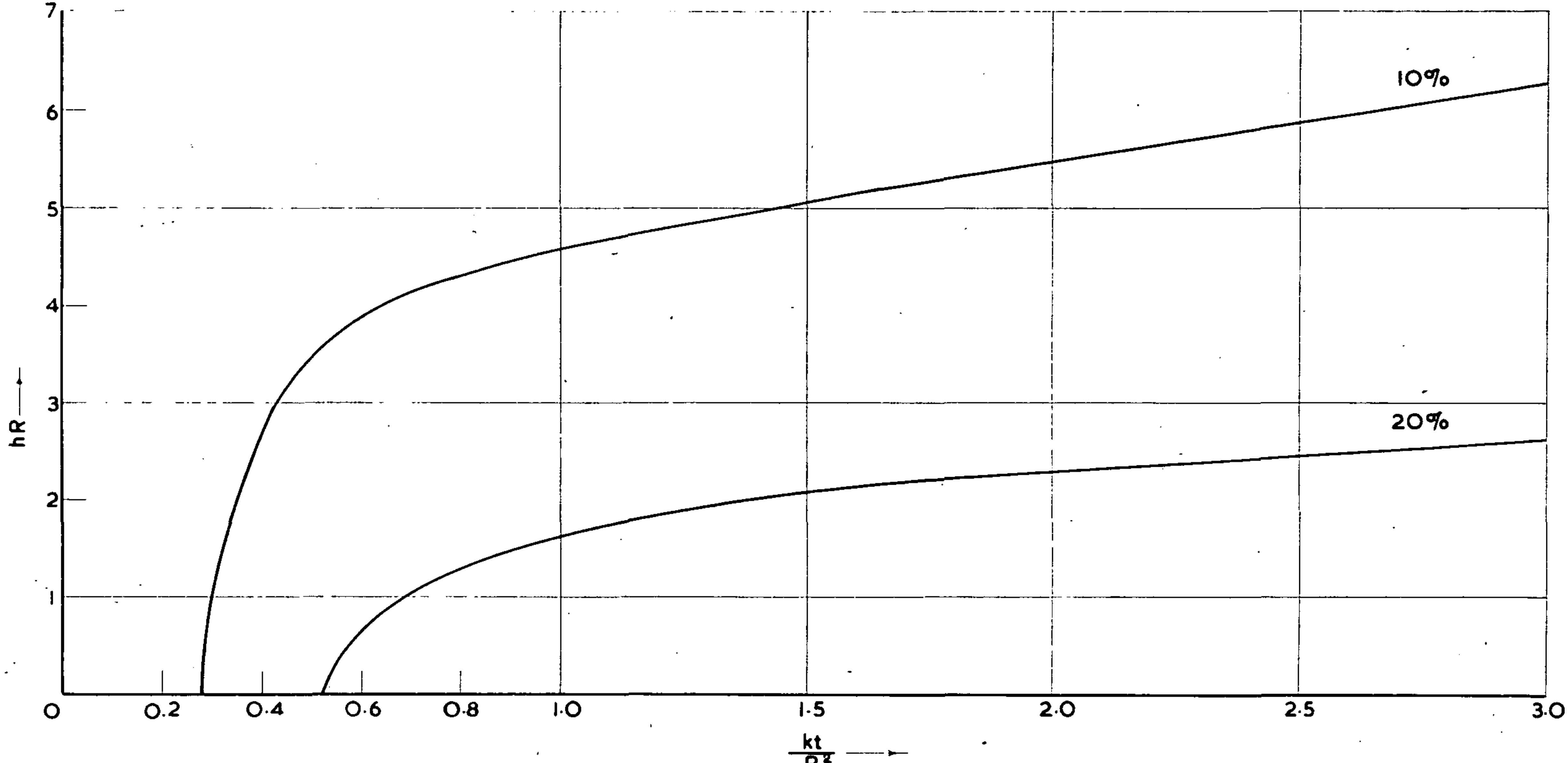

FIG. 2. VALUES OF $\frac{kt}{R^2}$ AT WHICH TEMPERATURE OF DISK CENTRE IS 10% AND 20% BELOW THAT OF HEATED PLANE

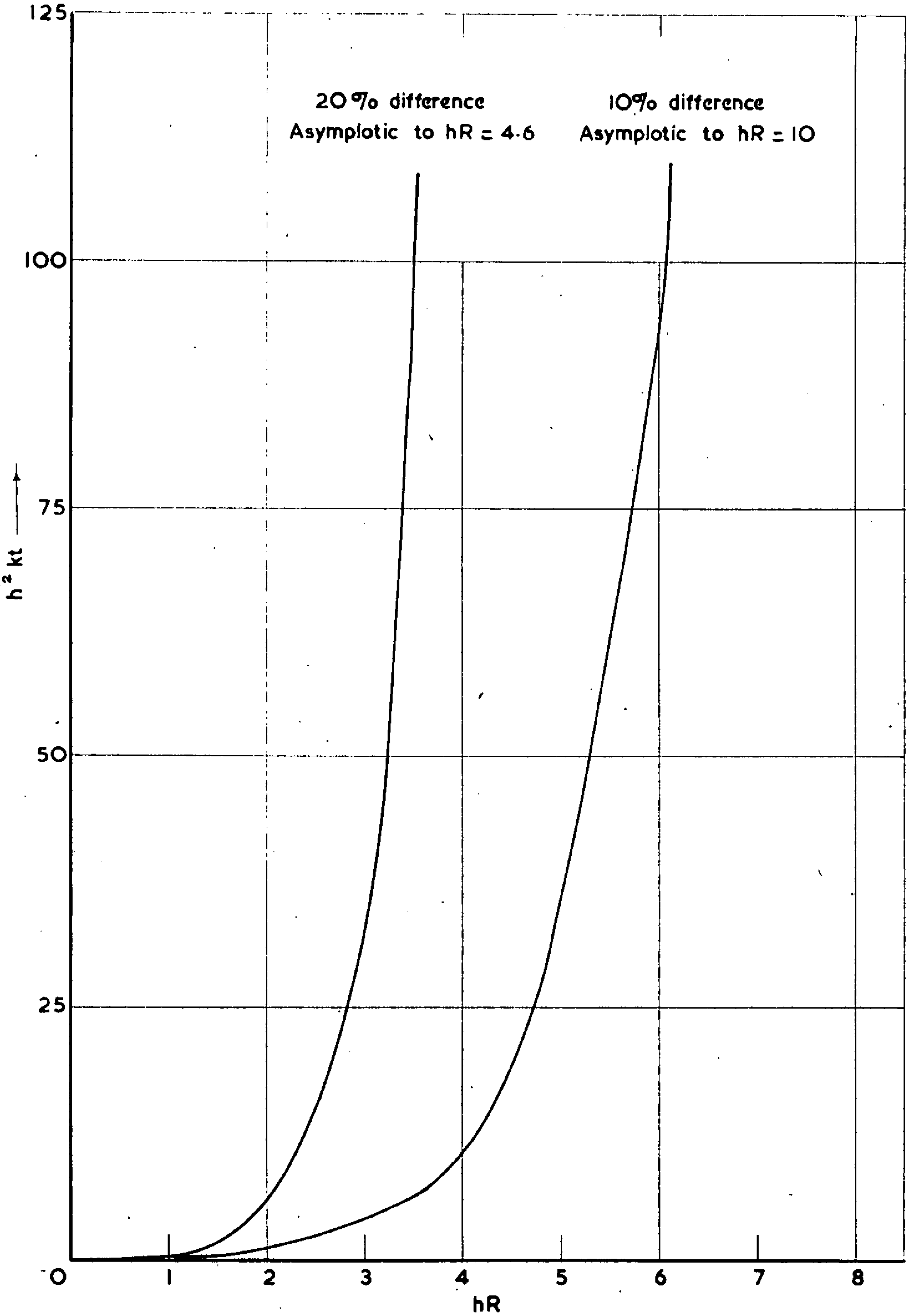

FIG. 3. VALUES OF h^2kt AS FUNCTION OF hR FOR 10 % & 20 % DIFFERENCE BETWEEN DISK AND PLANE.