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THE VECTORIAL PROPERTY OF CONFIGURATION FACTORS

by

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Summary

Expressions are derived determining the orientation of a receiver for the maximum value of the configuration factor with respect to any specified radiator. It is then shown that, provided the radiator always lies entirely on one side of the plane of the receiving element, displacement of the receiver from this orientation by an angle ψ will reduce the configuration factor by $\cos\psi$. Thus, within ~~these~~ limits, a configuration factor has the property of a vector.

Introduction

The configuration factor of a radiator with respect to an elementary receiving area is the dimensionless factor by which the intensity of radiation at the receiver is reduced compared with the intensity at the radiator. It depends purely on the relative geometry of the two surfaces and has a vectorial property within the limits of the orientation of the receiving element. This fact is of importance when the effect on the configuration factor of rotating the receiving element is considered. If the receiving element is turned through an angle ψ from the position for maximum configuration factor then the new value of the configuration factor will be reduced by a factor $\cos\psi$. This is true provided that the plane of the receiving area does not intersect the radiating surface.

2. The configuration factor cosine law

The vectorial property of configuration factors is most easily demonstrated by recourse to the fundamental expression for the intensity of radiation at a receiving element due to a radiating element.

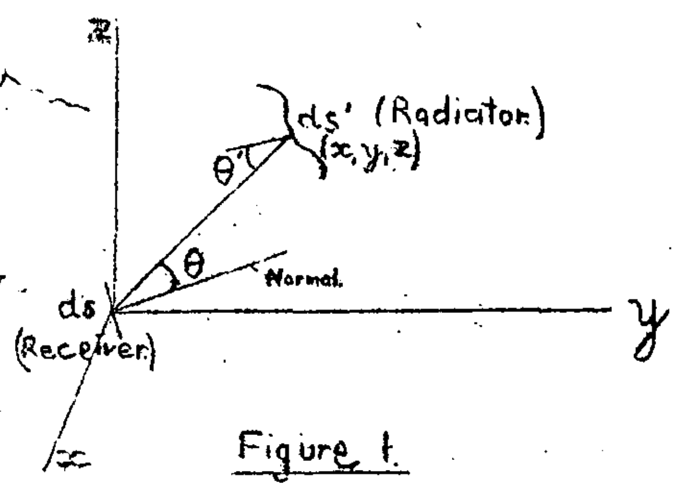

Figure 1.

In figure 1. let ds' be any elemental area of the radiator and, since selection of co-ordinates is arbitrary, let the elemental receiving area be located at the origin, the direction cosines of the normal being $\cos\alpha$, $\cos\beta$ and $\cos\gamma$

The radiation from ds' reaching ds (1) $= \frac{E}{\pi}\, ds' \cos\theta' \,\frac{ds\cos\theta}{d^2}$

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$\therefore$ the intensity at ds due to ds' $= \frac{E}{\pi} ds' \cos\theta' \frac{\cos\theta}{d^2}$ ....(1)

Integrating the expression, the intensity at ds due to an extended radiator

$$= \frac{E}{\pi}\int_0^{S'} \frac{ds' \cos\theta' \cos\theta}{d^2} \quad ....(2)$$

It is important to note here that the above expression only holds provided the radiating surface s' does not cut the plane of the receiver. The mathematical expression treats the radiation from elements behind the receiving plane as negative, which has no physical meaning.

To determine the effect of the orientation of the receiver, the dependence of the various factors in expression 2 must be considered. The only factor dependent on orientation is cos θ . Now the cosine of the angle between any two lines may be expressed in terms of the direction cosines of the lines

Thus $\cos\theta = \frac{x\cos\alpha}{d} + \frac{y\cos\beta}{d} + \frac{z\cos\gamma}{d}$

Substituting this value in expression (2) and remembering that $\alpha, \beta + \gamma$ are independent of s' the expression for the intensity at ds is

$$I = \frac{E\cos\alpha}{\pi}\int_0^{S'} ds' \cos\theta' \frac{x}{d^3}$$

$$+ \frac{E\cos\beta}{\pi}\int_0^{S'} ds' \cos\theta' \frac{y}{d^3}$$

$$+ \frac{E\cos\gamma}{\pi}\int_0^{S'} ds' \cos\theta \frac{z}{d^3} \quad ....(3)$$

Only variations of α, β and γ are being considered and the expressions within integral signs may be represented by the constants. A, B + C

$$\therefore I = E.A.\cos\alpha + E.B\cos\beta + E.C\cos\gamma \text{ ----------- } (4)$$

Writing $\cos a = \dfrac{A}{\sqrt{A^2+B^2+C^2}}$

$$\cos b = \frac{B}{\sqrt{A^2+B^2+C^2}}$$

$$\cos c = \frac{C}{\sqrt{A^2+B^2+C^2}}$$

and $\sqrt{A^2+B^2+C^2} = D$

the expression becomes

$$I = DE(\cos\alpha . \cos a + \cos\beta . \cos b + \cos\gamma \cos c) \qquad (5)$$

The maximum of this expression occurs with an orientation defined by

$\alpha_0 = a,$ $\beta_0 = b$ and $\gamma_0 = c$ and is $I = DE$

Thus $D = \phi max.$, the maximum value of the configuration factor, when orientation of the receiving element is the variable. From the definitions of a, b and c the values of α_0, β_0 & γ_0 are given by the following expressions:-

$$\cos\alpha_0 = \cos a = \frac{\int_0^{s'} ds' \cos\theta' \frac{x}{d^3}}{\pi \phi_{max.}}$$

$$\cos\beta_0 = \cos b = \frac{\int_0^{s'} ds' \cos\theta' \frac{y}{d^3}}{\pi \phi_{max.}} \qquad (6)$$

$$\cos\gamma_0 = \cos c = \frac{\int_0^{s'} ds' \cos\theta' \frac{z}{d^3}}{\pi \phi_{max.}}$$

If the receiver be now rotated by an angle ψ from this position then expression 5 becomes

$$I = \phi_{max} E \cos\psi \qquad (7)$$

where ψ is the angle between the normal to the receiving element and the normal for maximum intensity. Thus ϕ behaves as a vector

The limitation applying to expression 2 also applies to expression 7 viz: the radiating surface must be entirely on one side of the plane of the receiving element.

Acknowledgement

Attention to the vectorial property of configuration factors was drawn by Mr. C. F. Fischl.

References

1. F.P.E. Note No. 38 "Heat Transfer by Radiation" J. H. McGuire para.1.6.