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DEPARTMENT OF SCIENTIFIC AND INDUSTRIAL RESEARCH AND FIRE OFFICES' COMMITTEE
JOINT FIRE RESEARCH ORGANIZATION

THE CALCULATION OF HEAT TRANSFER BY RADIATION

by

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SYNOPSIS

Expressions are derived by means of which the radiation intensity at a point due to various shapes of heated surface (e.g. disc, sphere, rectangle) may be computed. The expressions are concerned only with the relative geometry of the surface and are known as configuration factors. The various properties of configuration factors are listed. Total radiation flux over a finite area of receiver is also considered.

Expressions are derived for the radiative heat transfer between surfaces when multiple reflections can occur.

CONTENTS

1. Introduction

The theoretical background and the laws applicable to thermal radiation have been very satisfactorily detailed in a large number of text books. Prévost's Theory of Exchanges, Lambert's Cosine Law and proofs of Kirchhoff's Laws and Stefan's Law will be found in most text books on Heat. From the above the radiation intensity inside an enclosure or near to a surface may be computed. Problems encountered in practice, however, necessitate the calculation of radiation intensities at a specified distance from a radiator.

It is the purpose of this paper to list expressions whereby the reduction in intensity due to the geometrical relationship of radiator and receiver may be taken into account.

The geometrical relationship factor, known as the configuration factor will first be described and later it will be shown how the expressions for radiant intensity are modified if the radiator and the receiver are reflecting bodies.

2. Configuration factor

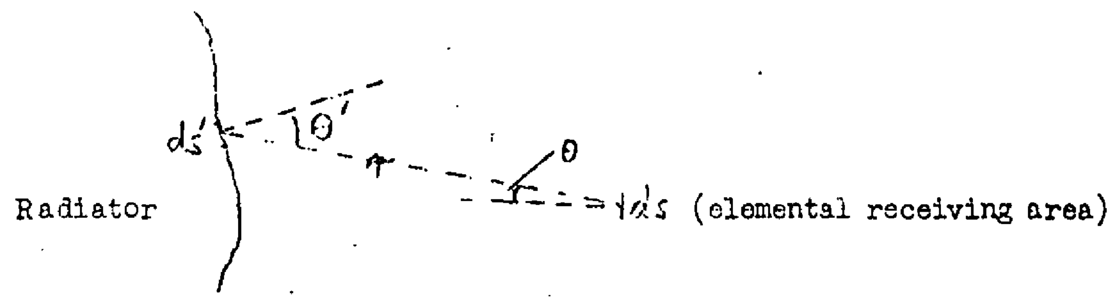


Figure 1

If S' is a radiating surface and S a receiver (figure 1) it will be seen that the radiation falling upon ds per second from the elementary area of radiator ds'

= area of radiating element x i x cos θ' x solid angle subtended by receiver at radiator.

$$dF = ds' \, i . \cos\theta' . d\omega' \qquad \text{..........(2a)}$$

where i is the radiation per second emitted by unit area of surface, normal to the surface. From Stefan's Law it can be shown that

$$i = \frac{\varepsilon \sigma T^4}{\pi}$$

where ε is the emisivity factor or "blackness" of the radiating surface compared with that of a black body and T is the absolute temperature of the surface in °K.

The factor cos θ' appears in expression (2a) because the level of radiation emitted from a surface is dependent on the direction, falling to zero parallel to the surface. The Law is known as Lambert's Cosine Law. Substituting for i and $d\omega'$ (2a) becomes

$$dF = ds' \times \frac{\varepsilon \sigma T^4}{\pi} \times \cos\theta' \times \frac{ds \cos\theta}{r^2} \qquad \text{..........(2b)}$$

The intensity dI at ds due to ds' will be $\frac{dF}{ds}$, i.e.

$$dI = ds' \times \frac{\varepsilon \sigma T^4}{\pi} \times \cos\theta' \times \frac{\cos\theta}{r^2} \qquad \text{..........(2c)}$$

Since $\frac{ds'.\cos\theta'}{r^2} = d\omega$ the solid angle subtended at the receiver by the elemental radiator, then

$$dI = \frac{\varepsilon\sigma T^4}{\pi} \times d\omega.\cos\theta \qquad \text{..........(2d)}$$

Figure 2 illustrates the significance of the symbols $d\omega$ and $\cos\theta$

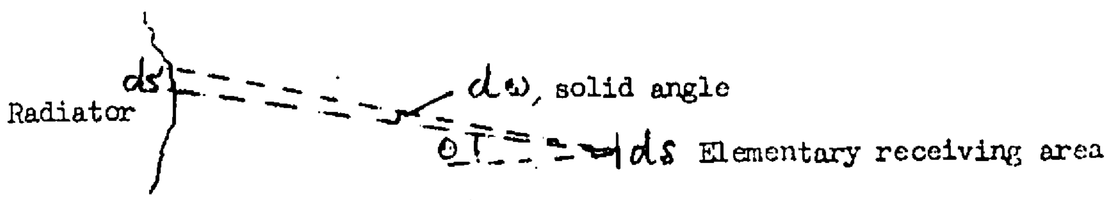


Figure 2

The expression for the intensity at the receiver due to the whole of the radiator will be

$$I = \frac{\varepsilon\sigma T^4}{\pi}\int_0^{s'} d\omega.\cos\theta \qquad \text{..........(2e)}$$

Now Stefan's Law states that the intensity near to a radiator at a temperature T is $\varepsilon\sigma T^4$

where ε

is again the emissivity factor of the radiating surface. The intensity I compared with the intensity $I_0 = \varepsilon\sigma T^4$ near to the radiating surface is therefore

$$I = I_0 \times \frac{1}{\pi}\int_0^{s'} d\omega.\cos\theta \qquad \text{..........(2f)}$$

The factor $\frac{1}{\pi}\int_0^{s'} d\omega.\cos\theta$ which, of course, becomes unity if the receiver is near to the radiator (i.e. gives a solid angular cover of 2π) is dependent purely on the geometrical relationship of radiator and receiving element and is called the configuration factor. It should be noted that it refers to an intensity at a point due to any size or shape of radiator and not to the radiative heat transfer to a receiver of finite size.

The symbol ϕ is given to a configuration factor and it is always defined as

$$\phi = \frac{1}{\pi}\int_0^{s'} d\omega.\cos\theta \qquad \text{..........(2g)}$$

where the limits of integration refer to the radiating area.

Some of the properties of configuration factors will now be described.

3. Additive property of configuration factors

For a fixed radiating area and an elementary receiving area the value of φ may be obtained by dividing up the radiating area, calculating the individual configuration factors and then adding (or subtracting) them.

An example of the usefulness of this property is illustrated by figure 3, where the radiator is a square surface with a disc out from the centre. The configuration factor may be obtained by

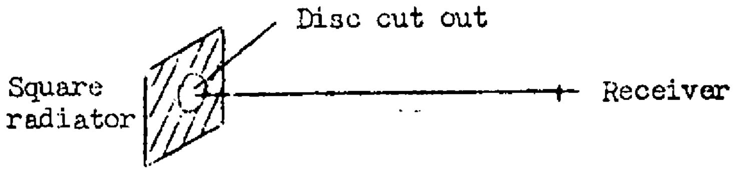


Figure 3

subtracting the value computed for a disc from that for a square, or:-

$$\varphi = \varphi_{Square} - \varphi_{Disc} \qquad \text{..........(3a)}$$

This property has been used in Appendices 4 and 5 where configuration factors for rectangles are considered. Tables are given for the cases where the receiving element lies on the perpendicular from one corner of the rectangle and it is explained that the value of φ for any other case can be obtained by adding (or subtracting) the values of φ for rectangles which conform to the cases to which the tables apply.

4. Equivalent surface

As it is only the limits of the expression $\frac{1}{\pi}\int_0^{S'} d\omega . \cos\theta$ which concern the radiator, it follows that any two radiating surfaces subtending one and the same solid angle will give the same configuration factor. Figure 4 illustrates two surfaces giving the same configuration factor.

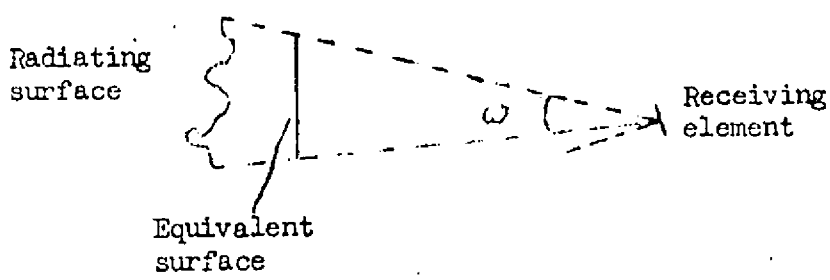


Figure 4

This property of configuration factors is most valuable in evaluating them and has in fact been used in deriving the expression for the sphere given in paragraph 8.

Thus referring to figure 5 the spherical radiating surface has been replaced by the equivalent disc to evaluate:-

$$\phi = \frac{1}{\pi}\int_0^{S'} d\omega . \cos\theta$$

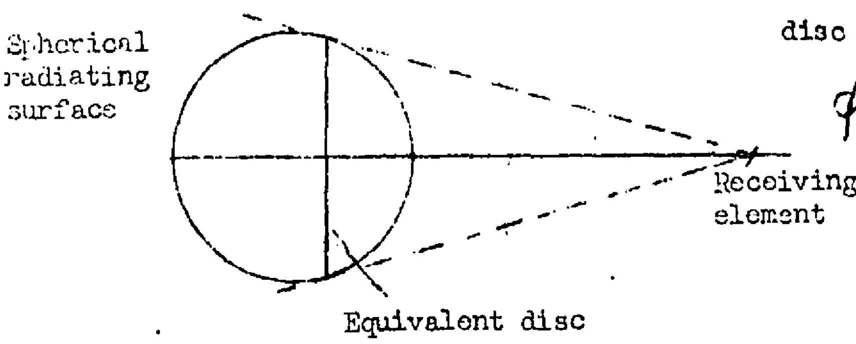


Figure 5

5. Configuration factor Cosine Law

The Configuration Factor Cosine Law concerns the orientation of an elemental reserving area for a specified radiating surface. It states that if a receiving element be orientated such that the configuration factor is a maximum (as regards angular displacement) than any angular displacement of the receiver from this position by an angle θ will reduce the configuration factor by $\cos\theta$ (1)

The application of this law is limited to cases in which the plane of the receiving element does not intersect the radiating surface. This limitation is illustrated by figure 6. Figure 6a illustrates a receiving element orientated such that it receives maximum intensity (as regards angular displacement) from a square radiator.

Figure 6a

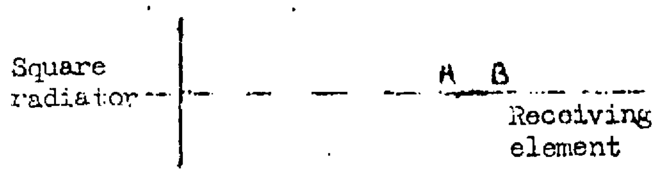


Figure 6b

Figure 6b illustrates the case where the receiver has been turned through 90°. It then receives radiation on both the front and back surfaces and in the mathematical expression for the configuration factor the radiation intensity on the back surface is represented by a negative quantity.

In the particular case cited, the positive and negative quantities are equal and if the limitation were not observed the resulting value of ϕ would be zero.

6. Geometrical interpretation of $\frac{1}{\pi}\int_0^{S'} d\omega \cos\theta$

The calculation of the value of a configuration factor is sometimes simplified by use of the geometrical interpretation of

$$\varphi = \frac{1}{\pi}\int_0^{S'} d\omega . \cos\theta$$

which will now be described.

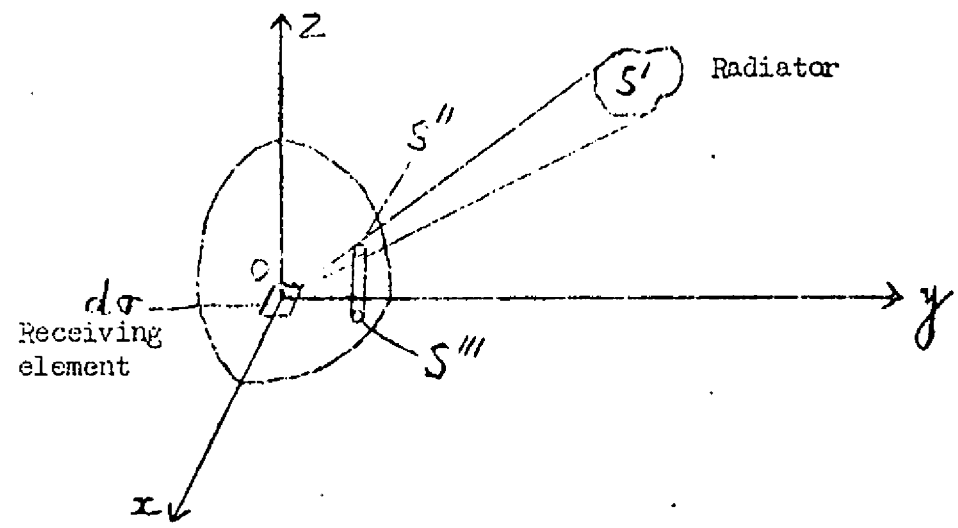


Figure 7

In figure 7 let S' be the radiating surface and let $d\sigma$ be the receiving element located at the origin (and in the xy plane) of a cartesian system of co-ordinates.

Let a sphere, of radius R be drawn around the receiving element. Then if S'' be the projection of S' on the sphere the expression

$$\varphi = \frac{1}{\pi}\int_0^{S'} d\omega . \cos\theta$$

may be written

$$\varphi = \frac{1}{\pi}\int_0^{S''} \cos\theta . \frac{ds''}{R^2} \qquad \text{..........(6a)}$$

But $ds'' \cos\theta$ is the projection of ds'' on the xy plane $= ds'''$

$$\therefore \quad \varphi = \frac{1}{\pi}\int_0^{S'''} \frac{ds'''}{R^2} = \frac{S'''}{\pi R^2} \qquad \text{..........(6b)}$$

The use of the geometrical interpretation is best illustrated by an example.

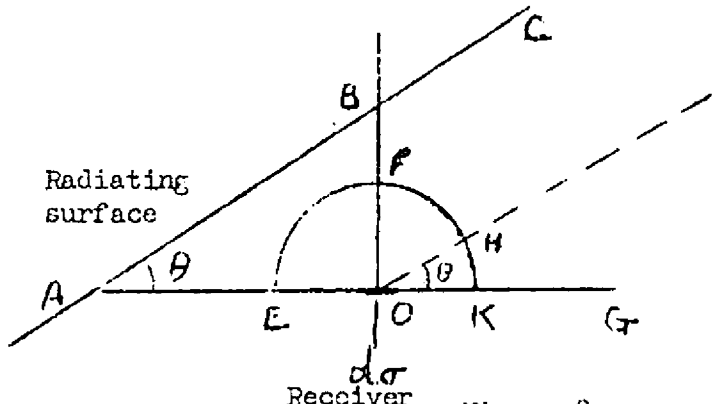


Figure 8

In figure 8 let the infinite plane A B C be the radiating surface and let $d\sigma$ be the receiving element. The hemisphere E F K of radius R is drawn as described above.

Now the projection of the plane A B C on the hemisphere E F K will be the (three dimensional) surface E F H. The further projection of E F H on the plane surface A E O K G will be

$$\frac{\pi R^2}{2}(1+\cos\theta)$$

This fact can be more readily appreciated if the surface is considered to be divided into strips of which the longest dimension is in the direction AG . For simplification the surfaces on either side of the plane O F may also be considered separately.

From expression (6b) the value of φ is immediately given as

$$\varphi = \frac{1}{2}(1+\cos\theta) \qquad \text{..........(6c)}$$

Hottel [2] describes a machine which uses this interpretation to evaluate configuration factors.

7. Optical analogue

Apart from computing configuration factors from the expression

$$\varphi = \frac{1}{\pi}\int_0^{S'} d\omega.\cos\theta$$

or using the geometrical interpretation they may be evaluated by an optical analogue described by Lawson and Hird [3].

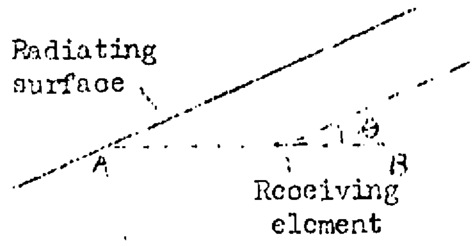
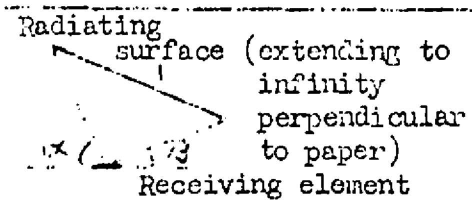
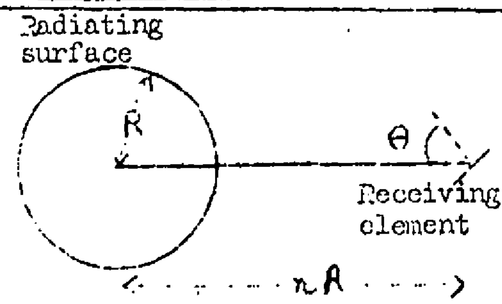
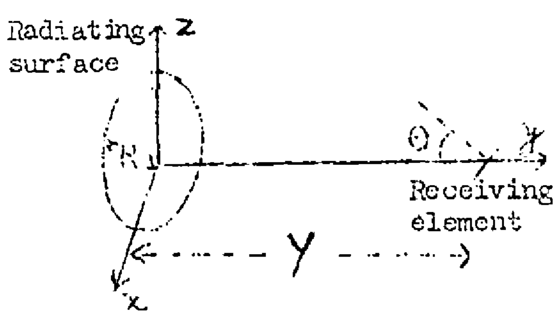
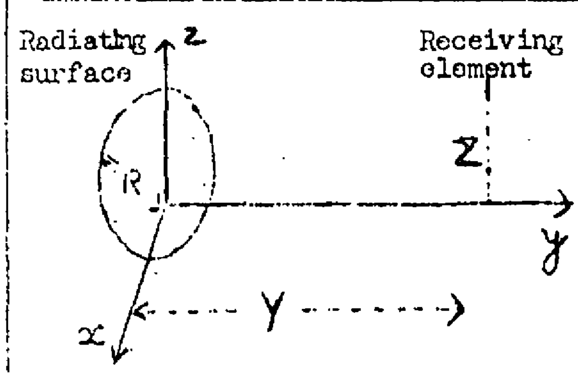
The method is particularly valuable where the surfaces considered are complicated and depends on the fact that the laws of thermal radiation also apply to light since both are electro-magnetic radiations. A model of the radiating surface is made of a material which diffuses light effectively and therefore emits light according to Lambert's cosine Law when illuminated from behind.

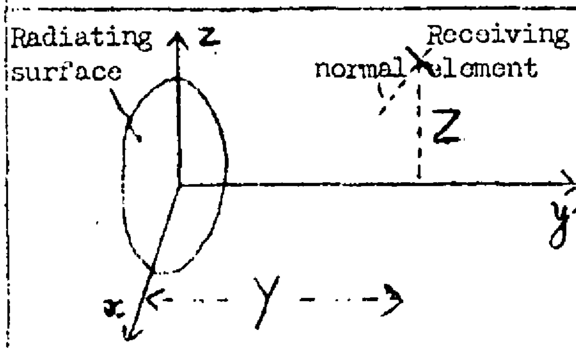
The value of a configuration factor, i.e. the ratio of intensities at a specified point and near to the radiating surface, is measured by means of a photo-cell.

8. Values of various configuration factors

Expressions for certain configuration factors [4] [5] [6] [12] have been derived previously and in the cases of cylindrical [5], rectangular [6] [7] and boiler tube [8] radiators graphs have been plotted giving ranges of values. The following is a list of expressions for the configuration factors of the simple, well known surfaces. The derivations are not given, but are available [9] [10] [11].

It should be noted that, in all the following cases excepting the first, the expressions do not apply if one or more of the limits of the radiating area lies behind the plane of the receiver.

Surface	Configuration factor
 1) Infinite plane Receiving element at an angle θ (see paragraph 5)	$\phi = \frac{1}{2}(1+\cos\theta)$ This expression is derived by the method of paragraph 6 and only the radiating area above the plane AB has been considered. The value of the configuration factor with respect to the rear surface of the receiving element is:- $\phi' = \frac{1}{2}(1-\cos\theta)$ and of course $\phi + \phi' = 1$ since the front and back surfaces, together, receive radiation emitted by an infinite plane.
 2) Infinite strip	$\phi = \frac{1}{2}(\cos\alpha + \cos\beta)$
 3) Sphere	$\phi = \frac{\cos\theta}{n^2}$
 4) Circular disc (in the plane xz). Receiver on the axis.	$\phi = \frac{R^2}{R^2+Y^2}\cos\theta$ A Table for ϕ in terms of the parameter n, where Y = nR, will be found in Appendix 1.
 5) Circular disc (in the plane xz). Receiver off the axis but parallel.	$\phi = \frac{1}{2}\left\{1 - \frac{Y^2+Z^2-R^2}{\sqrt{R^4+2(Y^2-Z^2)R^2+(Y^2+Z^2)^2}}\right\}$ A Table for ϕ in terms of the parameters n and m, where Y = nR and Z = mR, will be found in Appendix 2.

<table>
<tr><th>Surface</th><th>Configuration factor</th></tr>
<tr><td>


Receiving surface has direction cosines cos α , cos β and cos γ with x, y and z axes respectively.

6) <u>Circular disc</u> (in xz plane) General case.
</td><td>

$$\phi = -\frac{\cos\beta}{2}\left\{1 - \frac{Y^2+Z^2-R^2}{\sqrt{R^4+2(Y^2-Z^2)R^2+(Y^2+Z^2)^2}}\right\} + \frac{Y\cos\gamma}{2Z}\left\{1 - \frac{Y^2+Z^2+R^2}{\sqrt{R^4+2(Y^2-Z^2)R^2+(Y^2+Z^2)^2}}\right\}$$

Writing $\phi = -F_1\cos\beta - F_2\cos\gamma$ tables of F_1 and F_2 in terms of the parameters n and m, where $Y = nR$ and $z = mR$, will be found in Appendices 2 and 3.
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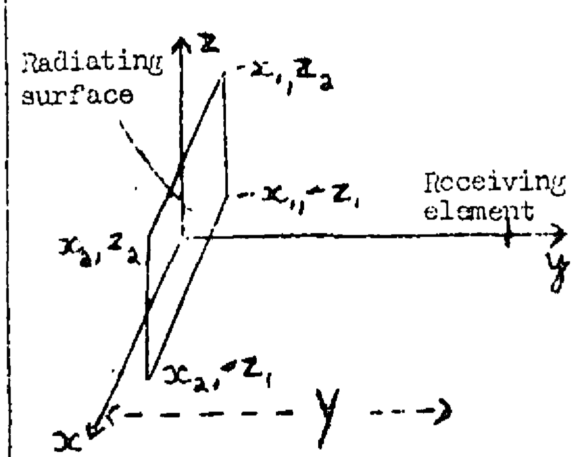

7) <u>Rectangle</u> Receiver parallel to it.
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$$\phi = \frac{1}{2\pi}\left\{\frac{x_2}{\sqrt{x_2^2+Y^2}}\left(\tan^{-1}\frac{z_2}{\sqrt{x_2^2+Y^2}} + \tan^{-1}\frac{z_1}{\sqrt{x_2^2+Y^2}}\right) + \frac{x_1}{\sqrt{x_1^2+Y^2}}\left(\tan^{-1}\frac{z_2}{\sqrt{x_1^2+Y^2}} + \tan^{-1}\frac{z_1}{\sqrt{x_1^2+Y^2}}\right) + \frac{z_2}{\sqrt{z_2^2+Y^2}}\left(\tan^{-1}\frac{x_2}{\sqrt{z_2^2+Y^2}} + \tan^{-1}\frac{x_1}{\sqrt{z_2^2+Y^2}}\right) + \frac{z_1}{\sqrt{z_1^2+Y^2}}\left(\tan^{-1}\frac{x_2}{\sqrt{z_1^2+Y^2}} + \tan^{-1}\frac{x_1}{\sqrt{z_1^2+Y^2}}\right)\right\}$$

Where $x_1 = x_2$ and $z_1 = z_2$ the expression becomes

$$\phi = \frac{2}{\pi}\left\{\frac{x_1}{\sqrt{x_1^2+Y^2}}\tan^{-1}\frac{z_1}{\sqrt{x_1^2+Y^2}} + \frac{z_1}{\sqrt{z_1^2+Y^2}}\tan^{-1}\frac{x_1}{\sqrt{z_1^2+Y^2}}\right\}$$

A Table whereby ϕ may be easily calculated will be found in Appendix 4.
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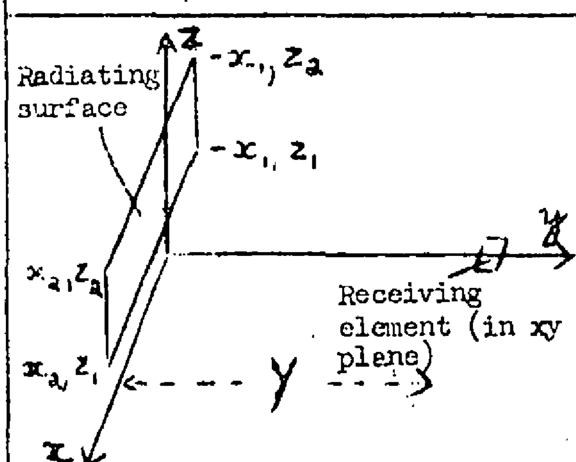

8) <u>Rectangle</u> Receiver perpendicular
</td><td>

$$\phi = \frac{Y}{2\pi\sqrt{z_1^2+Y^2}}\left(\tan^{-1}\frac{x_2}{\sqrt{z_1^2+Y^2}} + \tan^{-1}\frac{x_1}{\sqrt{z_1^2+Y^2}}\right) - \frac{Y}{2\pi\sqrt{z_2^2+Y^2}}\left(\tan^{-1}\frac{x_2}{\sqrt{z_2^2+Y^2}} + \tan^{-1}\frac{x_1}{\sqrt{z_2^2+Y^2}}\right)$$

If $z_1 = 0$ and $x_1 = x_2$ then

$$\phi = \frac{1}{\pi}\tan^{-1}\frac{x_1}{Y} - \frac{Y}{\pi\sqrt{z_2^2+Y^2}}\tan^{-1}\frac{x_1}{\sqrt{z_2^2+Y^2}}$$

A Table whereby ϕ may be easily calculated will be found in Appendix 5.
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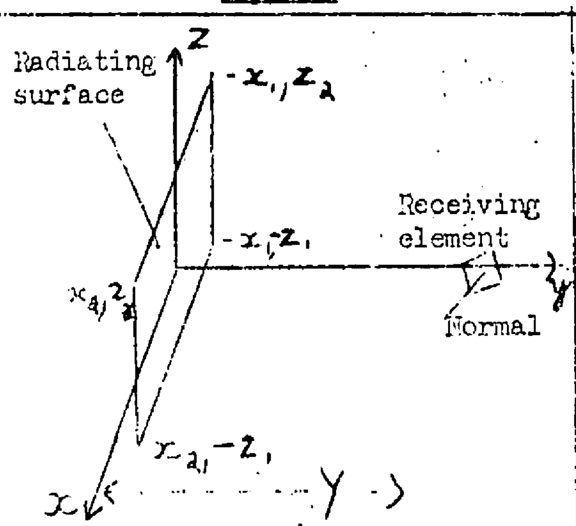
Surface	Configuration factor



Normal to receiving element makes angles α, β and γ with positive directions of x, y and Z axes.

9) Rectangle General case.

$$\phi = \frac{Y\cos\alpha}{2\pi\sqrt{x_1^2+Y^2}}\left(\tan^{-1}\frac{z_2}{\sqrt{x_1^2+Y^2}} + \tan^{-1}\frac{z_1}{\sqrt{x_1^2+Y^2}}\right)$$

$$-\frac{Y\cos\alpha}{2\pi\sqrt{x_2^2+Y^2}}\left(\tan^{-1}\frac{z_2}{\sqrt{x_2^2+Y^2}} + \tan^{-1}\frac{z_1}{\sqrt{x_2^2+Y^2}}\right)$$

$$-\frac{\cos\beta}{2\pi}\left\{\frac{x_2}{\sqrt{x_2^2+Y^2}}\left(\tan^{-1}\frac{z_2}{\sqrt{x_2^2+Y^2}} + \tan^{-1}\frac{z_1}{\sqrt{x_2^2+Y^2}}\right)\right.$$

$$+\frac{x_1}{\sqrt{x_1^2+Y^2}}\left(\tan^{-1}\frac{z_2}{\sqrt{x_1^2+Y^2}} + \tan^{-1}\frac{z_1}{\sqrt{x_1^2+Y^2}}\right)$$

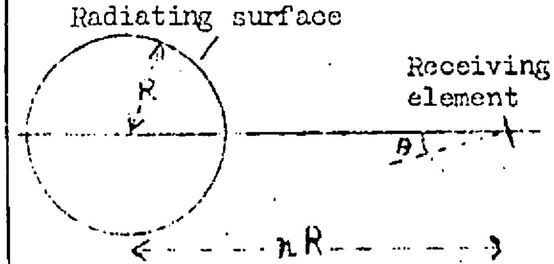
$$+\frac{z_2}{\sqrt{z_2^2+Y^2}}\left(\tan^{-1}\frac{x_2}{\sqrt{z_2^2+Y^2}} + \tan^{-1}\frac{x_1}{\sqrt{z_2^2+Y^2}}\right)$$

$$\left.+\frac{z_1}{\sqrt{z_1^2+Y^2}}\left(\tan^{-1}\frac{x_2}{\sqrt{z_1^2+Y^2}} + \tan^{-1}\frac{x_1}{\sqrt{z_1^2+Y^2}}\right)\right\}$$

$$+\frac{Y\cos\gamma}{2\pi\sqrt{z_1^2+Y^2}}\left(\tan^{-1}\frac{x_2}{\sqrt{z_1^2+Y^2}} + \tan^{-1}\frac{x_1}{\sqrt{z_1^2+Y^2}}\right)$$

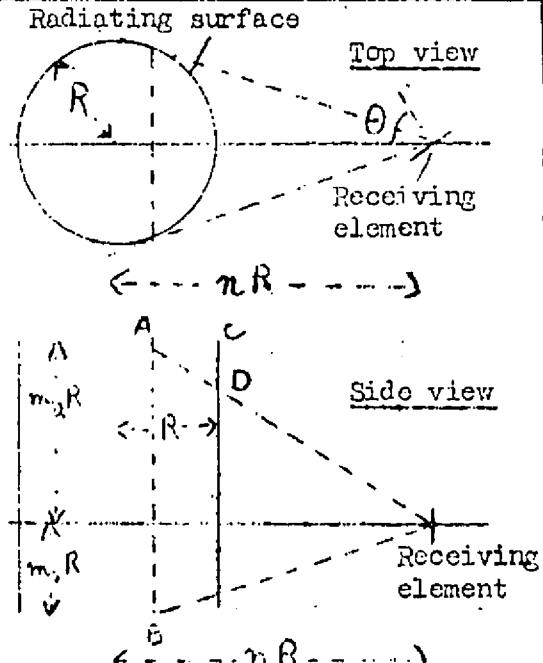
$$-\frac{Y\cos\gamma}{2\pi\sqrt{z_2^2+Y^2}}\left(\tan^{-1}\frac{x_2}{\sqrt{z_2^2+Y^2}} + \tan^{-1}\frac{x_1}{\sqrt{z_2^2+Y^2}}\right)$$

Values of ϕ, for any specified values of x_1, x_2, z_1, z_2 and Y may be obtained by judicious use of Tables 4 and 5 in Appendices 4 and 5.



10) Infinite cylinder

$$\phi = \frac{\cos\theta}{n}$$



11) Finite cylinder (approximate)

$$\phi = \frac{\cos\theta}{\pi}\left\{\frac{1}{n}\tan^{-1}\frac{m_2}{\sqrt{n^2-1}} + \frac{m_2 n}{\sqrt{(n^2-1)^2+m_2^2n^2}}\tan^{-1}\frac{\sqrt{n^2-1}}{\sqrt{(n^2-1)^2-m_2^2n^2}}\right.$$

$$\left.+\frac{1}{n}\tan^{-1}\frac{m_1}{\sqrt{n^2-1}} + \frac{m_1 n}{\sqrt{(n^2-1)^2+m_1^2n^2}}\tan^{-1}\frac{\sqrt{n^2-1}}{\sqrt{(n^2-1)^2+m_1^2n^2}}\right\}$$

N.B. The above expression has been calculated by the method of paragraph 4. The plane surface AB, which was chosen to represent the cylinder, is however not an exact equivalent surface. Radiation from the "cheese" shaped portions CD is neglected.

The approximation is very good if $n > m$.

For $m \rightarrow \infty$ the expression reduces to

$$\phi = \frac{\cos\theta}{n}$$

Tables for ϕ will be found in Appendix 6.

9. Example: Radiation intensities near to a cylindrical flue pipe

The use of the configuration factor tables referred to in paragraph 8 is illustrated by the solution of the following example.

A flue pipe which behaves as a black body is maintained at a temperature of 700°K and has a diameter of 20 cm and an exposed length of 100 cm. Find the intensity of radiation (a) close to its surface (b) 20 cm from the centre (c) 50 cm from the centre.

(a) The intensity of radiation I_0 close to the surface will be given by

$$I_0 = \sigma T^4$$

where σ is Stefan's constant = 1.37×10^{-12} cal/cm^2/°C^4/sec

Thus $I_0 = 1.37 \times 10^{-12} \times (700)^4 = 0.329$ cal/cm^2/sec

(b) The intensity of radiation 20 cm from the flue pipe will be given by

$$I = \phi \sigma T^4 = \phi I_0$$

where the value for ϕ is given by Table 6. This table refers to a receiver on a level with one end of a pipe and thus the required value will be twice the value ϕ' obtained from the table for half the pipe.

The appropriate values of the constants will be

$$m = \frac{\text{half length of pipe}}{\text{radius of pipe}} = \frac{50}{10} = 5$$

and $$n = \frac{\text{distance from axis of pipe}}{\text{radius of pipe}} = \frac{20}{10} = 2$$

Whence $\phi' = 0.247$

and $\phi = 2\phi' = 0.494$

$$\therefore \quad I = \phi I_0 = 0.494 \times 0.329$$
$$= 0.163 \text{ cal/cm}^2\text{/sec}$$

(c) Adopting the same procedure as above, m is again 5 and

$$n = \frac{\text{distance from axis of pipe}}{\text{radius of pipe}} = \frac{50}{10} = 5$$

Whence $\phi' = 0.083$

and $\phi = 2\phi' = 0.166$

$$\therefore \quad I = \phi \sigma T^4 = \phi I_0 = 0.166 \times 0.329$$
$$= 0.055 \text{ cal/cm}^2\text{/sec.}$$

10. Integrated configuration factor

When the area of a receiving surface is considerable it is often required to find the total radiation flux across it and not the intensity at any particular point.

Now the intensity at an element of the receiver will be

$$I = \varepsilon' . \phi . \sigma T^4 \qquad \text{.......... (10a)}$$

where ε' is the emissivity factor of the radiator (assumed to be uniform) and T its absolute temperature.

The flux across the element will be

$$dF = \varepsilon' . \phi . \sigma T^4 ds \qquad \text{.......... (10b)}$$

where ds is its area.

Thus the total flux will be given by the expression

$$F = \varepsilon' \sigma T^4 \int_0^S \phi . ds \qquad \text{.......... (10c)}$$

taken over the whole of the receiver.

The expression $\int_0^S \phi ds$ is called the integrated configuration factor and is given the symbol $\overline{\phi}$

Thus $$F = \varepsilon' \sigma T^4 \overline{\phi} \qquad \text{.......... (10d)}$$

$\overline{\phi}$ includes the effect of the area of the receiver and the mean intensity over the receiver is given by the expression:-

$$I \text{ mean} = \varepsilon' . \sigma T^4 \frac{\overline{\phi}}{S} \qquad \text{.......... (10e)}$$

where S is the area of the receiver.

$\overline{\phi}$ has the very important property that if the roles of receiver and radiator are interchanged the new $\overline{\phi}$ has the same value. In other words, if the emissivity and temperature of the new radiator be again ε' and T the radiation flux falling on the new receiver will again be

$$F = \varepsilon' . \sigma T^4 \overline{\phi} \qquad \text{.......... (10f)}$$

It should be noted that this does not mean that the radiation intensity (i.e. flux per unit area) at the new receiver has the same value. Since the area is different it is now given by:-

$$I'_{mean} = \varepsilon' \sigma T^4 \frac{\overline{\phi}}{S'} \qquad \text{.......... (10g)}$$

where S' is the area of the new receiver.

It is not practicable, here, to give a comprehensive list of expressions for integrated configuration factors for various combinations of simple surfaces, as it would occupy too much space. Instead a simple example, that for the radiative transfer between two parallel discs on the same axis, will be evaluated. It is illustrated in Figure 9.

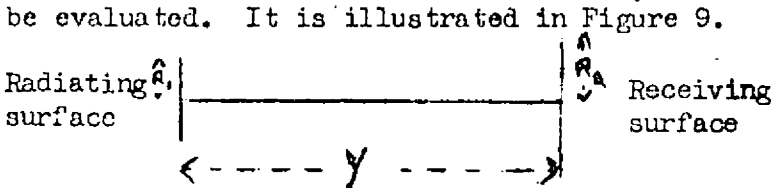


Figure 9a

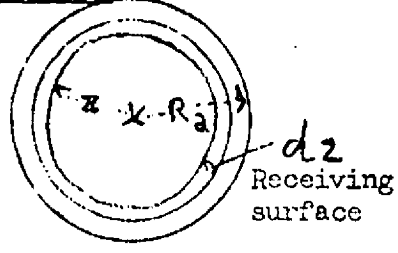


Figure 9b

Consider any one surface to be the radiator, then from example 5, paragraph 8, the value of the configuration factor around an elemental ring of the receiving surface, see figure 9b, will be

$$\phi = \frac{1}{2}\left(1 - \frac{y^2 + z^2 - R_1^2}{\sqrt{R_1^4 + 2(y^2 - z^2)R_1^2 + (y^2+z^2)^2}}\right) \qquad \text{.......... (10h)}$$

The area of this elemental ring is $2\pi z\,dz$. Therefore the integrated configuration factor will be given by

$$\bar{\phi} = \int \phi . ds$$

$$= \int_0^{R_2} 2\pi z . dz \frac{1}{2}\left\{1 - \frac{y^2 + z^2 - R_1^2}{\sqrt{R_1^4 + 2(y^2 - z^2)R_1^2 + (y^2+z^2)^2}}\right\}$$

$$= \frac{\pi}{2}\left\{R_1^2 + R_2^2 + y^2 - \sqrt{(R_1^2 - R_2^2)^2 + 2y^2(R_1^2 + R_2^2) + y^4}\right\}$$

$$\text{.......... (10i)}$$

The symmetry in R_1 and R_2, indicates that the roles of receiver and radiator may be interchanged.

If $R_1 = R_2$ and $R_1 \gg Y$, that is receiver and radiator are large, of the same size, and close together, then $\bar{\phi} = \pi R^2$, the area of receiver or radiator. Thus the mean intensity, which in this case will be the same as the intensity at any point since the latter will be uniform, will simply be $\varepsilon \sigma T^4$.

11. Multiple reflections between infinite parallel plane surfaces

When dealing with problems involving finite receiving areas, the effect of multiple reflections becomes important if (1) the distance separating receiver and radiator is commensurate with the dimensions of the receiver and radiator: and if (2) the emissivity factors of both receiver and radiator differ appreciably from unity. Thus the radiation intensity at the receiver will be due not only to the primary radiation from the radiator, but will include radiation from the receiver reflected once, three times, five times etc. by the radiator and radiation from the radiator which has been reflected by both receiver and radiator.

The intensity at any point is thus the sum of an infinite series. It may be evaluated much more easily, however, as is now shown for the case of two infinite parallel plane surfaces, illustrated in figure 10.

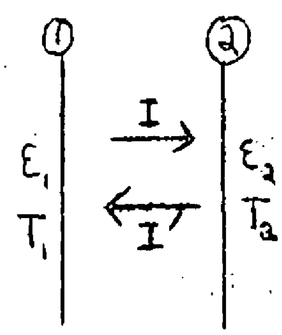


Figure 10

The emissivities of the surfaces are taken as ε_1 and ε_2 and their absolute temperatures as T_1 and T_2. I and I' stand for the radiation intensities in the directions indicated. As the surfaces are infinite, configuration factors are everywhere unity and the intensity of primary radiation from (1) will therefore be $\varepsilon_1 \sigma T_1^4$. By adding the fraction of the intensity I', incident on surface (1), which is reflected, an expression is immediately obtained relating I and I'.

Now Kirchhoff's Law states that the absorptive power of a non-transparent surface equals its emissivity factor. Thus, since the fraction of radiation reflected is that not absorbed, its value for the surface (1) is $(1 - \varepsilon_1)$.

Thus $$I = \varepsilon_1 \sigma T_1^4 + (1-\varepsilon_1) I' \qquad \text{(11a)}$$

by a similar argument

$$I' = \varepsilon_2 \sigma T_2^4 + (1-\varepsilon_2) I \qquad \text{(11b)}$$

Thus $$I = \frac{\varepsilon_1 \sigma T_1^4 + \varepsilon_2 \sigma T_2^4 - \varepsilon_1 \varepsilon_2 \sigma T_2^4}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2} \qquad \text{(11c)}$$

and $$I' = \frac{\varepsilon_2 \sigma T_2^4 + \varepsilon_1 \sigma T_1^4 - \varepsilon_1 \varepsilon_2 \sigma T_1^4}{\varepsilon_2 + \varepsilon_1 - \varepsilon_1 \varepsilon_2} \qquad \text{(11d)}$$

The net heat transfer per unit area per unit time

$$= I - I' = \frac{\varepsilon_1 \varepsilon_2 (\sigma T_1^4 - \sigma T_2^4)}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2} \qquad \text{(11e)}$$

It is of interest to note that if $T_1 = T_2$ then

$$I = I' = \sigma T^4 \qquad \text{(11f)}$$

This result follows from the fact that the assembly has become an enclosure at constant temperature and by the Second Law of Thermodynamics the radiation intensity can be a function only of the absolute temperature.

12. Multiple reflections between two finite surfaces

Where two finite surfaces are considered the effect of multiple reflections can only be easily taken into account if the following approximations can be assumed.

1) That the radiation intensities are uniform over the two surfaces considered. This will be true if the separation between the surfaces is considerably greater than their dimensions.

2) That reflections are diffuse and that the scattered radiation has a uniform angular distribution.

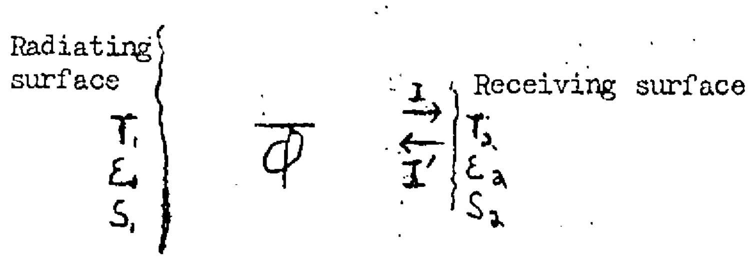


Figure 11

A general case is illustrated in figure 11 in which the integrated configuration factor between the two surfaces is Φ, the areas of the two surfaces are S_1 and S_2, their emissivity factors are ε_1 and ε_2 and they are at temperatures T_1 and T_2. The intensities I and I' are taken to be near the receiving surface. Then by the argument of paragraph 10

$$I = \frac{\frac{\Phi}{S_2}\left\{\varepsilon_1 \sigma T_1^4 + \frac{\Phi}{S_1}\varepsilon_2(1-\varepsilon_1)\sigma T_2^4\right\}}{1 - \frac{\Phi^2}{S_1 S_2}\left(1 - \varepsilon_1 - \varepsilon_2 + \varepsilon_1 \varepsilon_2\right)} \qquad \text{(12a)}$$

$$I' = \frac{\varepsilon_2 \sigma T_2^4 + \frac{\overline{\Phi}}{S_2} \varepsilon_1 (1-\varepsilon_2) \sigma T_1^4}{1 - \frac{\overline{\Phi}^2}{S_1 S_2}(1-\varepsilon_1-\varepsilon_2+\varepsilon_1\varepsilon_2)} \quad \text{.......... (12b)}$$

The rate of gain of heat per unit area, by the receiver, is therefore

$$I - I' = \frac{\frac{\overline{\Phi}}{S_2} \varepsilon_1 \varepsilon_2 \sigma T_1^4 - \varepsilon_2 \sigma T_2^4 \left(1 - \frac{\overline{\Phi}^2}{S_1 S_2} + \frac{\overline{\Phi}^2}{S_1 S_2}\varepsilon_1\right)}{1 - \frac{\overline{\Phi}^2}{S_1 S_2}\left(1-\varepsilon_1-\varepsilon_2+\varepsilon_1\varepsilon_2\right)} \quad \text{(12c)}$$

This expression reduces to expression (11e) if S_1 and S_2 are considered infinite and therefore $\frac{\overline{\Phi}}{S_1} = \frac{\overline{\Phi}}{S_2} = 1$.

If the dimensions of either the radiator or the receiver are small compared with the separation between the two, then the terms including $\frac{\overline{\Phi}^2}{S_1 S_2}$ may be ignored. The expression becomes:-

$$I - I' = \frac{\overline{\Phi}}{S_2} \varepsilon_1 \varepsilon_2 \sigma T_1^4 - \varepsilon_2 \sigma T_2^4 \quad \text{.......... (12d)}$$

Precisely the same result is achieved if $\varepsilon_1 = \varepsilon_2 \to 1$ and it merely implies that reflections have been neglected. The first term in expression (12d) represents the intensity of primary radiation, from the radiator, absorbed by the receiver and the second term represents the intensity of radiation emitted by the receiver.

The treatment may be extended to include the effect, on the two surfaces, of radiation and reflections from the enclosure containing them, but the resulting expressions would probably prove too cumbersome to be of practical use.

13. Acknowledgements

Tables 4 and 5 have been computed by Mr. M. J. Gregsten of the Joint Fire Research Organization and Table 6 has been computed by Mr. D. I. Lawson, M.Sc., M.I.E.E., F.Inst.P., and has been taken from the report "The Heating of Panels by Flue Pipes". (5)

Dr. J. W. Bray, B.Sc., (Eng.) of Queen Mary College, London, corrected and completed the derivation of the expression for the configuration factor for a rectangle (receiver perpendicular to it) and Mr. C. F. Fischl of the Joint Fire Research Organization drew the author's attention to the vectorial property of configuration factors.

Acknowledgement is due to the Director of the Joint Fire Research Organization for permission to publish this paper.

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APPENDIX 1

Circular disc: Receiver on the axis

The expression

$$\phi = \frac{R^2}{R^2 + Y^2}\cos\theta \quad \text{(see paragraph 8 case 4)}$$

$$= \frac{1}{1+n^2}\cos\theta \qquad \text{where } Y = nR$$

Table 1 gives values of $\frac{1}{1+n^2}$ for various values of n.

Table 1 $\left(\frac{1}{1+n^2}\right)$

n	$\frac{1}{1+n^2}$
0.5	0.8
0.6	0.735
0.7	0.671
0.8	0.61
0.9	0.552
1.0	0.5
1.1	0.452
1.25	0.390
1.5	0.308
1.75	0.246
2.0	0.2
2.5	0.138
3.0	0.1

APPENDIX 2

Circular disc receiver parallel but off axis

The expression

$$\phi = \frac{1}{2}\left\{1 - \frac{Y^2 + Z^2 - R^2}{\sqrt{R^4 + 2(Y^2 - Z^2)R^2 + (Y^2+Z^2)^2}}\right\} \quad \text{(see paragraph 8 case 5)}$$

$$= \frac{1}{2}\left\{1 - \frac{n^2 + m^2 - 1}{\sqrt{1 + 2(n^2 - m^2) + (n^2 + m^2)^2}}\right\} \qquad \text{where } Y = nR \text{ and } Z = mR$$

Table 2 gives values of this expression for various values of n and m.

Table 2 $\frac{1}{2}\left\{1 - \frac{n^2+m^2-1}{\sqrt{1+2(n^2-m^2)+(n^2+m^2)^2}}\right\}$

n \ m	0	0·5	1·0	1·5	2·0
0.5	0.800	0.724	0.379	0.084	0.022
0.6	0.735	0.655	0.356	0.099	0.028
0.7	0.671	0.591	0.335	0.110	0.037
0.8	0.610	0.534	0.314	0.118	0.042
0.9	0.552	0.484	0.295	0.124	0.048
1.0	0.5	0.438	0.276	0.126	0.053
1.1	0.452	0.398	0.259	0.127	0.057
1.25	0.390	0.345	0.235	0.126	0.062
1.5	0.308	0.276	0.200	0.120	0.066
1.75	0.246	0.224	0.171	0.112	0.067
2.0	0.2	0.185	0.147	0.102	0.066
2.5	0.138	0.130	0.110	0.084	0.060
3.0	0.1	0.096	0.084	0.069	0.053

APPENDIX 3

Circular disc: General case

The expression:-

$$\phi = -\frac{\cos\beta}{2}\left\{1 - \frac{Y^2+Z^2-R^2}{\sqrt{R^4+2(Y^2-Z^2)R^2+(Y^2+Z^2)^2}}\right\}$$

$$+\frac{Y\cos\gamma}{2Z}\left\{1 - \frac{Y^2+Z^2+R^2}{\sqrt{R^4+2(Y^2-Z^2)R^2+(Y^2+Z^2)^2}}\right\}$$ (see paragraph 8 case 6)

$$= -\frac{\cos\beta}{2}\left\{1 - \frac{n^2+m^2-1}{\sqrt{1+2(n^2-m^2)+(n^2+m^2)^2}}\right\}$$

$$+\frac{n\cos\gamma}{2m}\left\{1 - \frac{n^2+m^2+1}{\sqrt{1+2(n^2-m^2)+(n^2+m^2)^2}}\right\}$$

where $Y = nR$
and $Z = mR$

$$= -F_1\cos\beta - F_2\cos\gamma$$

where F_1 is given by Table 2 (Appendix 2)
and F_2 is given by Table 3.

It will be found that cos β and cos γ are almost always negative since β and γ are the angles between the normal to the receiver and the positive directions of the Y and Z axes respectively.

Table 3 $\quad -\dfrac{n}{2m}\left\{1-\dfrac{n^2+m^2+1}{\sqrt{1+2(n^2-m^2)+(n^2+m^2)^2}}\right\}$

n \ m	0.5	1.0	1.5	2.0
0.5	0.171	0.296	0.157	0.068
0.6	0.166	0.265	0.160	0.076
0.7	0.155	0.238	0.157	0.080
0.8	0.142	0.213	0.152	0.084
0.9	0.130	0.191	0.145	0.085
1.0	0.116	0.171	0.137	0.086
1.1	0.104	0.153	0.129	0.085
1.25	0.088	0.130	0.116	0.082
1.5	0.066	0.100	0.097	0.075
1.75	0.049	0.075	0.079	0.067
2.0	0.038	0.061	0.065	0.058
2.5	0.023	0.039	0.045	0.043
3.0	0.015	0.026	0.031	0.032

APPENDIX 4

Rectangle: Receiver parallel

The expression for ϕ given in paragraph 8 case 7 is in terms of the variables x_1, x_2, Z_1, Z_2 and Y and cannot be easily tabulated. A table with a general application may however be evolved by putting $x_1 = Z_1 = 0$ and by making use of the additive property of configuration factors. Figure 12 illustrates the example for which a table has been prepared.

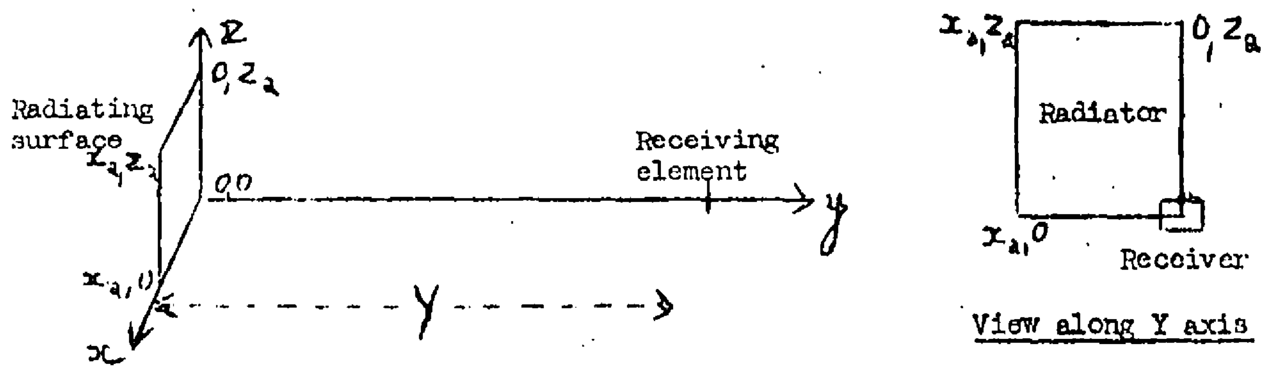

Figure 12

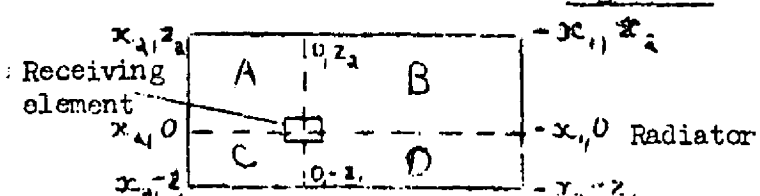

Figure 13

To apply the table to general case illustrated in Figure 13 four values of ϕ are looked up, for areas A, B, C and D respectively, and summed.

If the receiver and radiator are symmetrically placed, i.e. $x_1 = x_2$, $Z_1 = Z_2$, then $\phi = 4\,\phi_A$

The expression for ϕ where $x_1 = Z_1 = 0$ (Figure 12) is

$$\phi = \frac{1}{2\pi}\left\{\frac{x_2}{\sqrt{x_2^2+Y^2}}\tan^{-1}\frac{z_2}{\sqrt{x_2^2+Y^2}} + \frac{z_2}{\sqrt{z_2^2+Y^2}}\tan^{-1}\frac{x_2}{\sqrt{z_2^2+Y^2}}\right\}$$

$$= \frac{1}{2\pi}\left\{\frac{\alpha S}{\sqrt{1+\alpha S}}\tan^{-1}\sqrt{\frac{\alpha/S}{1+\alpha S}} + \sqrt{\frac{\alpha/S}{1+\alpha/S}}\tan^{-1}\sqrt{\frac{\alpha S}{1+\alpha/S}}\right\}$$

where $\delta = \dfrac{z_2}{x_2}$ = ratio of height to width of radiator.

and $\alpha = \dfrac{x_2 z_2}{Y^2}$ = $\dfrac{\text{area of radiator}}{(\text{separation between receiver and radiator})^2}$

Table 4 gives values of this expression for various values of α and δ.

Table 4 $\dfrac{1}{2\pi}\left\{\sqrt{\dfrac{\alpha\delta}{1+\alpha\delta}}\tan^{-1}\sqrt{\dfrac{\alpha/\delta}{1+\alpha\delta}}+\sqrt{\dfrac{\alpha/\delta}{1+\alpha/\delta}}\tan^{-1}\sqrt{\dfrac{\alpha\delta}{1+\alpha/\delta}}\right\}$

α \ δ	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
2.0	0.178	0.178	0.177	0.175	0.172	0.167	0.161	0.149	0.132	0.1[illegible]
1.0	0.139	0.138	0.137	0.136	0.133	0.129	0.123	0.113	0.099	0.0[illegible]
0.9	0.132	0.132	0.131	0.130	0.127	0.123	0.117	0.108	0.094	0.0[illegible]
0.8	0.125	0.125	0.124	0.122	0.720	0.116	0.111	0.102	0.089	0.0[illegible]
0.7	0.117	0.116	0.116	0.115	0.112	0.109	0.104	0.096	0.083	0.0[illegible]
0.6	0.107	0.107	0.106	0.105	0.103	0.100	0.096	0.088	0.077	0.0[illegible]
0.5	0.097	0.096	0.096	0.095	0.093	0.090	0.086	0.080	0.070	0.0[illegible]
0.4	0.084	0.083	0.083	0.082	0.081	0.079	0.075	0.070	0.062	0.0[illegible]
0.3	0.069	0.068	0.068	0.068	0.067	0.065	0.063	0.059	0.052	0.0[illegible]
0.2	0.051	0.051	0.050	0.050	0.049	0.048	0.047	0.045	0.040	0.0[illegible]
0.1	0.028	0.028	0.028	0.028	0.028	0.028	0.027	0.026	0.024	0.0[illegible]
0.09	0.026	0.026	0.026	0.026	0.025	0.025	0.025	0.024	0.022	0.0[illegible]
0.08	0.023	0.023	0.023	0.023	0.023	0.023	0.022	0.022	0.020	0.0[illegible]
0.07	0.021	0.021	0.021	0.021	0.020	0.020	0.020	0.019	0.018	0.0[illegible]
0.06	0.018	0.018	0.018	0.018	0.018	0.017	0.017	0.017	0.016	0.0[illegible]
0.05	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.014	0.014	0.0[illegible]
0.04	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.011	0.0[illegible]
0.03	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.0[illegible]
0.02	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.0[illegible]
0.01	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.0[illegible]

APPENDIX 5

Rectangle: Receiver perpendicular

The expression for ϕ given in paragraph 8 case 8 is, as in the previous example, in terms of the variables x_1 x_2 Z_1 Z_2 and Y and cannot be easily tabulated. Again, a table with a general application may be made by putting $x_1 = Z_1 = 0$ as illustrated by Figure 14 and by making use of the additive (and subtractive) properties of configuration factors.

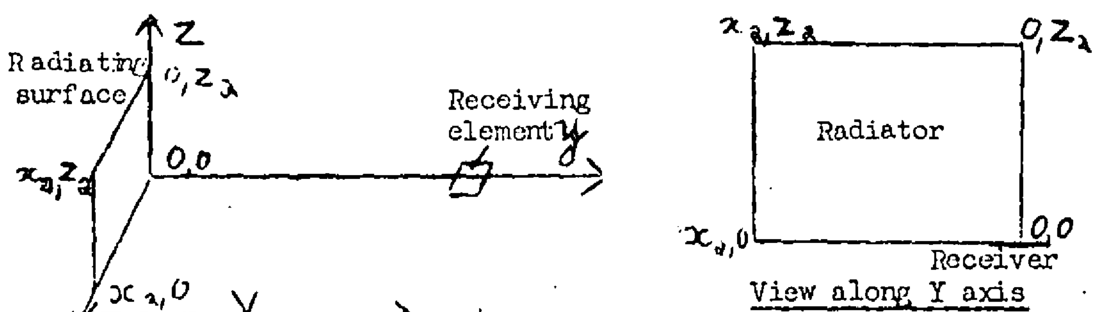

Figure 14

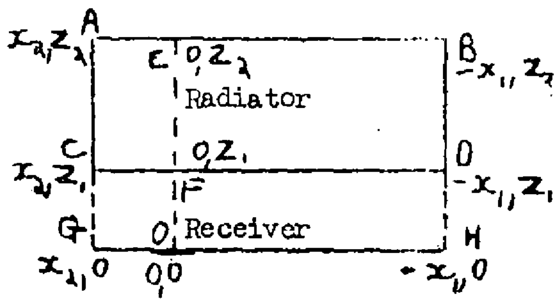

Figure 15

In applying the table to the general case illustrated in Figure 15, four values of are looked up and then

$$\phi_{ABCD} = \phi_{AEDG} + \phi_{EBHO} - \phi_{CFOG} - \phi_{FDHO}$$

The expression for φ where $x_1 = Z_1 = 0$ (Figure 14) is

$$\varphi = \frac{1}{2\pi}\left\{\tan^{-1}\frac{x_2}{Y} - \frac{Y}{\sqrt{Z_2^2+Y^2}}\tan^{-1}\frac{x_2}{\sqrt{Z_2^2+Y^2}}\right\}$$

$$= \frac{1}{2\pi}\left\{\tan^{-1}\sqrt{\frac{\alpha}{S}} - \frac{1}{\sqrt{1+\alpha S}}\tan^{-1}\sqrt{\frac{\alpha/S}{1+\alpha S}}\right\}$$

where $S = \frac{Z_2}{x_2}$ = ratio of height to width of radiator

and $\alpha = \frac{x_2 Z_2}{Y^2} = \frac{\text{area of radiator}}{(\text{separation between receiver and radiator})^2}$

Table 5 gives values of this expression for various values of α and S.

Table 5 $\frac{1}{2\pi}\left\{\tan^{-1}\sqrt{\frac{\alpha}{S}} - \frac{1}{\sqrt{1+\alpha S}}\tan^{-1}\sqrt{\frac{\alpha/S}{1+\alpha S}}\right\}$

α \ S	2	1.8	1.6	1.4	1.2	1.0	0.8	0.6	0.4	0.2
4.0	0.129	0.130	0.130	0.129	0.128	0.124	0.119	0.109	0.093	0.064
2.0	0.095	0.096	0.095	0.094	0.093	0.089	0.084	0.075	0.061	0.038
1.8	0.090	0.090	0.090	0.089	0.087	0.084	0.079	0.070	0.057	0.035
1.6	0.084	0.084	0.084	0.083	0.081	0.078	0.073	0.065	0.052	0.032
1.4	0.078	0.078	0.078	0.077	0.075	0.071	0.066	0.059	0.047	0.029
1.2	0.071	0.071	0.070	0.069	0.067	0.064	0.059	0.052	0.042	0.025
1.0	0.063	0.062	0.062	0.061	0.059	0.056	0.052	0.045	0.036	0.021
0.8	0.053	0.053	0.052	0.051	0.049	0.047	0.043	0.037	0.029	0.017
0.6	0.043	0.041	0.041	0.040	0.038	0.036	0.033	0.028	0.022	0.013
0.4	0.029	0.028	0.028	0.027	0.026	0.024	0.022	0.019	0.014	0.009
0.2	0.014	0.013	0.013	0.012	0.012	0.011	0.010	0.008	0.007	0.004
0.18	0.012	0.012	0.011	0.011	0.010	0.009	0.009	0.008	0.006	0.004
0.16	0.011	0.011	0.010	0.009	0.008	0.008	0.007	0.007	0.005	0.003
0.14	0.009	0.009	0.008	0.008	0.008	0.007	0.006	0.005	0.004	0.003
0.12	0.008	0.007	0.007	0.007	0.006	0.006	0.005	0.004	0.003	0.002
0.10	0.006	0.006	0.006	0.005	0.005	0.004	0.004	0.003	0.003	0.002
0.08	0.005	0.004	0.004	0.004	0.004	0.003	0.002	0.002	0.002	0.002
0.06	0.003	0.003	0.003	0.003	0.003	0.002	0.002	0.002	0.001	0.001
0.04	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001	0.001	0.001
0.02	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0

APPENDIX 6

Cylinder

The expression

$$\varphi = \frac{\cos\theta}{\pi}\left\{\frac{1}{n}\tan^{-1}\frac{m_2}{\sqrt{n^2-1}} + \frac{m_2 n}{\sqrt{(n^2-1)^2+m_2^2n^2}}\tan^{-1}\frac{\sqrt{n^2-1}}{\sqrt{(n^2-1)^2+m_2^2n^2}}\right.$$
$$\left. + \frac{1}{n}\tan^{-1}\frac{m_1}{\sqrt{n^2-1}} + \frac{m_1 n}{\sqrt{(n^2-1)^2+m_1^2n^2}}\tan^{-1}\frac{\sqrt{n^2-1}}{\sqrt{(n^2-1)^2+m_1^2n^2}}\right\}$$

see paragraph 8 case 11

$= \cos\theta_x(F_1 + F_2)$

where $F = \frac{1}{\pi}\left\{\frac{1}{n}\tan^{-1}\frac{m}{\sqrt{n^2-1}} + \frac{mn}{\sqrt{(n^2-1)^2+m^2n^2}}\tan^{-1}\frac{\sqrt{n^2-1}}{\sqrt{(n^2-1)^2+m^2n^2}}\right\}$

Table 6 gives values of F for various values of m and n

Table 6 $\frac{1}{\pi}\left\{\frac{1}{n}\tan^{-1}\frac{m}{\sqrt{n^2-1}} + \frac{mn}{\sqrt{(n^2-1)^2+m^2n^2}}\tan^{-1}\frac{\sqrt{n^2-1}}{\sqrt{(n^2-1)^2+m^2n^2}}\right\}$

n \ m		10	9	8	7	6	5	4	3	2	1
1	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
2	0.250	0.250	0.250	0.249	0.249	0.248	0.247	0.245	0.238	0.222	0.163
3	0.167	0.165	0.164	0.164	0.163	0.161	0.158	0.149	0.140	0.113	0.172
4	0.125	0.123	0.122	0.121	0.120	0.117	0.111	0.104	0.089	0.068	0.04[illegible]
5	0.100	0.099	0.097	0.095	0.093	0.089	0.083	0.075	0.063	0.045	0.026
6	0.084	0.080	0.079	0.076	0.073	0.069	0.064	0.056	0.046	0.032	0.018
7	0.072	0.069	0.066	0.064	0.060	0.055	0.050	0.043	0.035	0.024	0.013
8	0.063	0.060	0.057	0.054	0.050	0.045	0.040	0.034	0.028	0.019	0.010
9	0.056	0.053	0.050	0.047	0.043	0.037	0.033	0.028	0.022	0.015	0.008
10	0.050	0.048	0.045	0.042	0.037	0.032	0.028	0.023	0.018	0.013	0.006