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DEPARTMENT OF SCIENTIFIC AND INDUSTRIAL RESEARCH AND FIRE OFFICES' COMMITTEE
JOINT FIRE RESEARCH ORGANIZATION

ON THE CONVECTION FROM DISKS AT LOW GRASHOF NUMBERS

by

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Summary

Data originally obtained in the calibration of a 1 in. diameter disk radiometer have been used to evaluate the natural convection heat transfer coefficient for a vertical disk. It is suggested that the correlation between the Nusselt number and the Grashof number for large rectangular vertical plates can be applied to disks if it is modified to allow for the variation in height across the disk and for the limiting value of the Nusselt number at zero Grashof number. It is found that the predicted results are generally in agreement with those found experimentally.

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Introduction

There do not appear to be any published data for the heat transfer coefficient for convection from a small vertical disk so that the few results derived from the calibration of 1 in. diameter disk for use as a radiometer [1] may be of interest to others in this field. These results do not cover a wide range of Grashof numbers since the increase of kinematic viscosity with temperature reduces the effect of increasing the disk temperature on the Grashof number.

The disk consisted of copper with asbestos paper on each face; chromel and constantan wires being attached at opposite ends of a diameter to act as a thermocouple. The instrument was used to measure radiation intensities in the range 0·05 - 2·0 cals cm^{-2} sec^{-1} incident on one side of the disk in terms of the output of the thermocouple. Three radiation sources of different temperature were used in the calibration which is affected by considerable changes in the spectrum of the radiation. Over the useful range of the instrument, the disk temperature varied from a few degrees to 500°C above ambient but because of the increase in kinematic viscosity of air with temperature the actual variation in Grashof number, from 30°C to 500°C, was only about $3 - 10 \times 10^4$.

Calculation of heat transfer coefficient

The temperature was calculated from the thermo-electric output of the chromel constantan thermocouple and calibration tables [2]. A separate direct calibration was made for temperatures up to 100°C and this agreed with the one tabulated. From this calculated value of temperature and the value of incident radiation the heat transfer coefficient was calculated by means of the following heat balance equation.

$$I \epsilon_s = (T - T_A)(2H + C) + 2\sigma \epsilon_T (T^4 - T_A^4)$$

where I is the measured incident intensity of radiation.

- T is the calculated absolute disk temperature (assumed uniform)
- T_A is the absolute ambient temperature
- H is the required convection heat transfer coefficient
- ϵ_s is the emmisivity of the disk to incident radiation
- ϵ_T is the emmisivity of the disk at temperature T
- σ is the Stefan-Boltzman radiation constant
- and C is a constant to allow for conduction loss into thermocouple wires.

Values of I and T were obtained for three radiating sources at different temperatures. These, together with values I and T for a blackened disk enabled the emmisivities ϵ_s and ϵ_T to be calculated, the later being slightly dependent on T. The value of 'C' was calculated approximately from conduction theory and shown to amount to a correction in H of 5% or less. No correction was found to be necessary for the small difference in temperature across the disk.

Mean values of fluid properties

From the measured values of H the Nusselt number can be calculated and these may then be correlated with the Grashof number. In order to obtain appropriate values of the physical properties of air to insert in the dimensionless parameters describing heat transfer the mean properties as defined by Nusselt (3) were used. These are

$$\bar{\lambda} = \frac{1}{T - T_A} \int_{T_A}^{T} \lambda \, dT \qquad (2)$$

where λ may be K the thermal conductivity, ρ the density μ the viscosity or $\frac{1}{T}$

Results

Using the mean values for ρ, μ and $\frac{1}{T}$ as defined in equation (2) the Grashof number

$$N_{Gr} = \frac{g \cdot D^3 (T - T_A) \bar{\rho}^2}{\bar{\mu}^2} \left(\overline{\frac{1}{T}} \right) \qquad (3)$$

was calculated as a function of temperature, 'g' being the gravitational constant and D the disk diameter. Since the kinematic viscosity (μ/ρ) of air increases with temperature faster than the temperature itself the calculated Grashof number passes through a mximum. Thus at two different temperatures there may be only one value of the Grashof number. From the calculated values of H and $\bar{K}$, the Nusselt Number

$$N_{Nu} = \frac{HD}{\bar{K}} \qquad (4)$$

was calculated for various points on the calculated curve of disk temperature against radiation. It was found that at two temperatures giving the same Grashof number the two calculated Nusselt numbers were not the same, so that, when the Nusselt number is plotted against the product of the Grashof number and the Prandtl number N_{Pr} $(= \mu c_p / K$ where c_p is the specific heat at constant pressure, a curve giving two values of Nusselt number for one Grashof Number is obtained. This is shown in Figure (1). The curve turns back on itself when the temperature reaches 300°C, higher values of the Nusselt number being obtained at the higher temperatures, for the same Grashof number. It is not possible on the basis of the limited data available to say how far the ambiguity for temperatures in excess of, say, 300°C is experimental variation or can be removed by a different method of calculating the Grashof number, where there is large temperature difference across the boundary layer.

Dimensionless Correlation for a disk

Also in Figure (1) is shown the correlation (4) between the Nusselt, Grashof and Prandtl numbers for free convection from vertical rectangular plates, the height being taken as the characteristic dimension. This is given by

$$N_{Nu} = 0{\cdot}55 \, (N_{Gr} \cdot N_{Pr})^{\frac{1}{4}} \qquad (5)$$

This equation is seen to underestimate the Nusselt number for the disk by as much as 50%. It is suggested however that this expression can be used as a basis for deriving the appropriate expression for a disk, two modifications being necessary.

The first is found by considering the disk to be a series of elementary rectangles. Thus if 'x' is the height, and '$\delta\omega$' the width of an elementary strip, we have from Fig. (2)

$$x = D\sin\theta \tag{6}$$

$$\delta\omega = \frac{D}{2}\sin\theta\, d\theta. \tag{7}$$

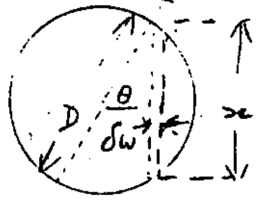

Figure 2. Calculation of effective height of disk

We assume that each elementary strip satisfies a relation of the kind in equation (5). Substituting from expressions (3) and (4) in (5) using x as the characteristic dimension gives

$$H_x = a/x^{1/4} \tag{8}$$

where 'a' is a constant

The total heat transfer is

$$\int H_x\,\delta\omega = \frac{\pi D^2}{4}\bar{H} \tag{9}$$

where $\bar{H}$ is the required mean heat transfer coefficient. From this definition we have from equations (6) - (9)

$$\bar{H} = \frac{4a}{\pi D^{1/4}}\int_0^{\pi/2}\sin^{7/4}\theta\, d\theta$$

$$= \frac{1.05\, a}{D^{1/4}}$$

This suggests that the factor 0·55 in equation (5) should be replaced by 1·05 x 0·55 i.e. 0·58.

The second and more important correction is to allow for the fact that the heat transfer for a finite body is not zero when free convection is absent: the heat transfer is thus determined by conventional heat conduction analysis, the result being obtained by analogy with Weber's problem of the electrified disk (5). The mean heat flux to one half of an infinite medium at temperature T_A from a thin disk of diameter D at uniform temperature T is

$$Q = \frac{8}{\pi}\,\frac{K(T - T_A)}{D}$$

where K denotes the conductivity of the medium.

Substituting the above expression in (4) gives

$$N_{Nu} = \frac{QD}{K(T - T_A)} = \frac{8}{\pi} = 2{\cdot}5_3$$

This may be compared with the value 2 for a sphere and zero for an infinitely wide plate. Frossling (6) and Ranz and Marshall (7) have shown that for a sphere this limiting value can be added to the term involving the Grashof number and by analogy the following equation is suggested for vertical disks.

$$N_{Nu} = 2{\cdot}5 + 0{\cdot}58\,(N_G \cdot N_{Pr})^{1/4}$$

This is shown in Figure (1). In view of the fact that the data were not obtained in the first place in an experiment designed to evaluate free convection the agreement may be regarded as satisfactory in the range of these experiments up to temperatures of 300°C.

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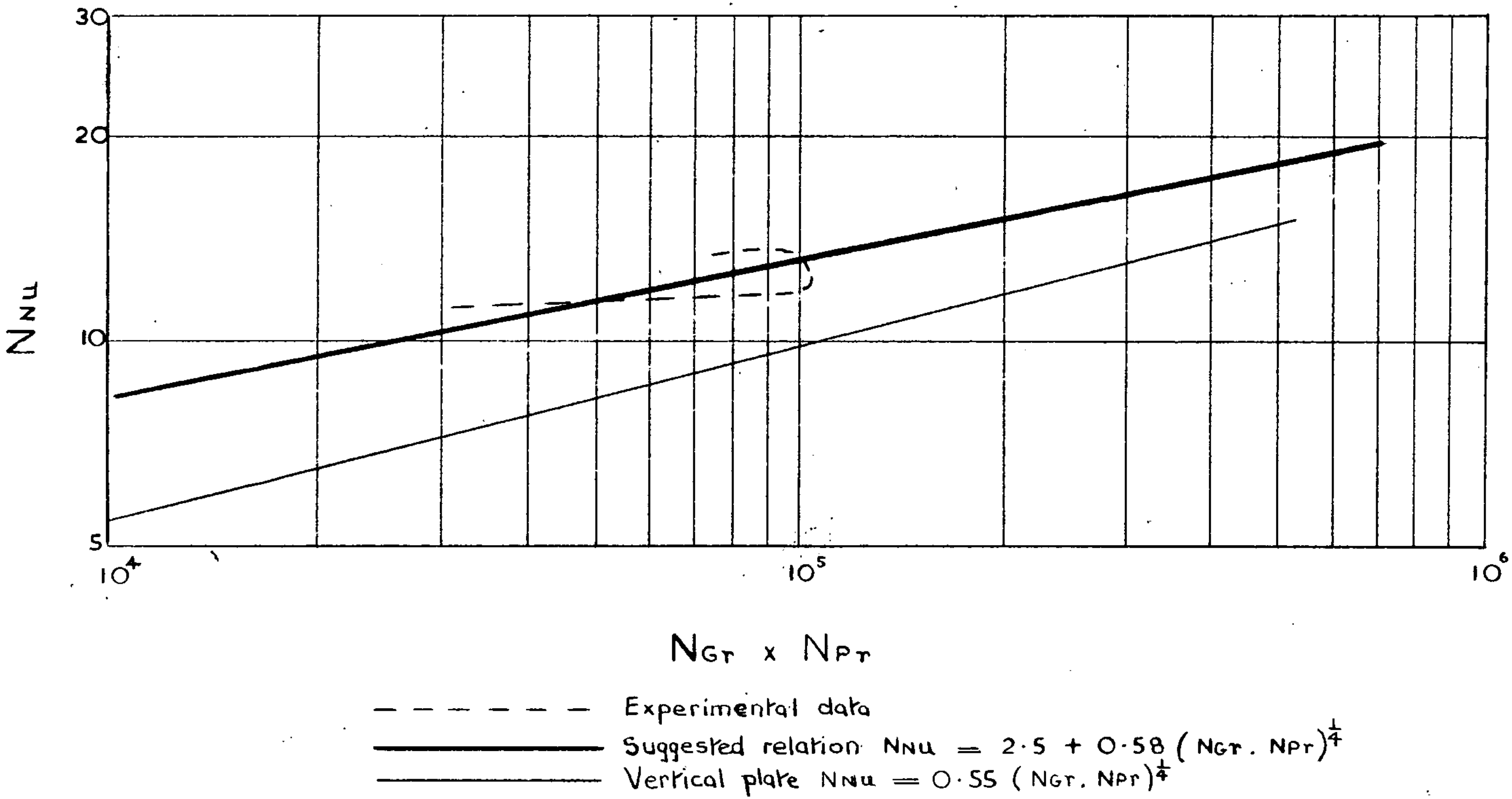

FIG. 1. THE NUSSELT NUMBER AS A FUNCTION OF THE GRASHOF AND PRANDTL NUMBERS FOR A VERTICAL DISK

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