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ACCEPTANCE PROCEDURES FOR LARGE BATCHES OF HOSE

by

D. W. Millar

Summary

Several possible procedures for use in testing the quality of large batches of hose are described and their long term operating characteristics have been shown graphically. The merits of the different procedures, and the necessary assumptions for the procedures to be valid are discussed.

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Hose is a variable product manufactured to a specification. It is impracticable for the purchaser to test all the hose in a batch provided by the producer so sampling procedures must be used. This note attempts to provide information on some sampling and acceptance procedures which might be of practical use in buying hose.

a. Hose is usually supplied in lengths of 75 ft. or more while test specimens are usually, but not always, 3 ft. long. It is useful to regard the batch under inspection as made up of a large number of 3 ft. lengths and describe a 3 ft. length as satisfactory or defective according as to whether it passes or fails the test.

b. All acceptance procedures, the performance of which can be measured by statistical theory, require a random choice of the specimens tested. In practice this would mean that every one of the 3 ft. lengths in the batch under examination stands an equal chance of being included in the sample actually tested. To carry this out properly requires each 3 ft. length to be labelled and the choice of items in the sample to be made by picking the labels out of a hat, or more expeditiously, by the use of tables of random numbers. This procedure is perhaps a counsel of perfection, but it is extremely important if reliable conclusions are to be drawn from a sample, that the specimens tested be taken from different 75 ft. lengths and that every 3 ft. length stands an approximately equal chance of being included in the sample.

c. Once a test criterion has been set a given test length of 3 ft. can provide either information on whether it passes the test or not, or in most cases the actual measurement achieved. The latter case is more flexible and so more informative but usually less convenient. Examples of both types of procedure are given.

d. The long term performance of the procedures is shown by their operating curves, Fig.1. The probability of accepting a batch is plotted on the vertical scale against the proportion of items defective in the batch submitted. This latter quantity is, of course, unknown but the surves show the long run proportions of batches which would be accepted, using a given procedure, for different proportions defective in the batch. The curves do <u>not</u> show the probability that an accepted batch is P per cent defective; this depends on the probability of a batch P per cent defective being submitted for inspection, and also on the probability of such a batch being accepted if submitted.

Let P be the proportion defective in the batch and F (P) be the probability of acceptance. Five possible procedures are listed below and illustrated in Fig.1.

1. Test 3 and require all to pass

$$F(P) = (1 - P)^3$$

2. Test 1 - if it passes accept
   - if it fails test two more and accept if both pass

$$F(P) = 1 - P + P(1 - P)^2 = (1 - P) \{ 1 + P (1 - P \}$$

3. Test 3 - if all pass accept
   - if 2 or 3 fail reject
   - if 1 fails test 1 more and accept if this one passes

$$F(P) = (1 - P)^3 + 3P (1 - P)^3 = (1 - P)^3 \{ 1 + 3P \}$$

4. Test 3 - if all pass accept
   - if 2 or 3 fail reject
   - if 1 fails test 2 more and accept if both of these pass

$$F(P) = (1 - P)^3 + 3P (1 - P)^4 = (1 - P)^3 \{ 1 + 3P (1 - P) \}$$

5. Test 3 - if all pass accept
   - if 2 or 3 fail reject
   - if 1 fails test 3 more and require all 3 to pass

$$F(P) = (1 - P)^3 + 3P\,(1 - P)^5 = (1 - P)^3 \left\{ 1 + 3P\,(1 - P)^2 \right\}$$

f. Procedure 1 is the most rigorous. If the proportion defective is 10 per cent, the producer's risk, that is the probability of rejection, is 0.275 or just over one-in-four; for proportions defective of less than 10 per cent the producer's risk is lower; at 5 per cent defective it is 0.15 or about 1 in 7; at 20 per cent defective the chances of acceptance or rejection are almost equal, so that any batch with more than 20 per cent defective is more likely to be rejected than accepted. Procedure 2 is the most lenient to the producer; for proportions defective of 20 per cent or less the maximum producer's risk is only 0.075 or approximately one-in-thirteen. In this procedure the probability of acceptance is always greater than that given by testing a single item, (the operating curve of which procedure is shown by the straight dotted line). Curves 3, 4 and 5 show variations on curve 1 designed to decrease the producer's risk for low proportions defective and to decrease the consumers risk for high proportions defective. Although more than three items <u>may</u> be tested in each of the procedures 3, 4 and 5, the average number tested in the long run will depend upon the proportion defective, but will not be much greater than 3. For example. in procedure 5, the average number tested is given by $ave(n) = 6-3(1-P)^3$. For P equal to 10 per cent ave(n) = 3.81. The operating curves of all these procedures depend only on the assumption of random sampling.

g. If instead of classifying results as defective or non-defective, the measurement of the quality being tested is taken, then it is possible to combine the measurements in a sample of size n = 5 to give the operating curve (6) shown by the dotted curve in Fig.1. This performance curve is achieved by using the fact that in a variable product the same proportion defective can arise either because the population average is high and the variability low or because the population average is low and the variability high. It is assumed not only that the sampling is random, but also that the distribution of the quality being measured is a Normal or Gaussian one, an assumption which can be checked. When the test criterion in terms of the proportion defective has been set, batches are accepted or rejected by calculating the sample mean, adding to it or subtracting from it a constant multiple of a measure of variability, and seeing if the resulting sum is less or greater than the test criterion. The usual measure of variability is the sample standard deviation but in certain circumstances the sample range, that is the difference between the greatest and least values in the sample, can be used. It can be seen from curve (6) that the maximum producer's risk is just over 0.05 for proportions defective of 5 per cent or less, while the maximum consumer's risk is 0.275 for proportions defective of 30 per cent or more. This operating curve or one similar is likely to be more satisfactory than any of the others shown in Fig.1. The computation involved can be reduced to a very simple rule of thumb procedure.

h. The effects on the acceptance procedure if sampling is not carried out at random depend on the method used and on the properties of the hose. If adjacent specimens from one long length are taken, the measurements are likely to be more highly correlated than if they are taken far apart, and results from the sample are therefore more likely to be biased in one direction or the other, than if the sample specimens were separated to a reasonable extent.

<u>Conclusions</u>

It has been shown than different acceptance procedures can provide almost any desired long term operating characteristic curve once a test criterion has been agreed. The performance of the acceptance procedures depends strongly on the rendomness of the sampling carried out.

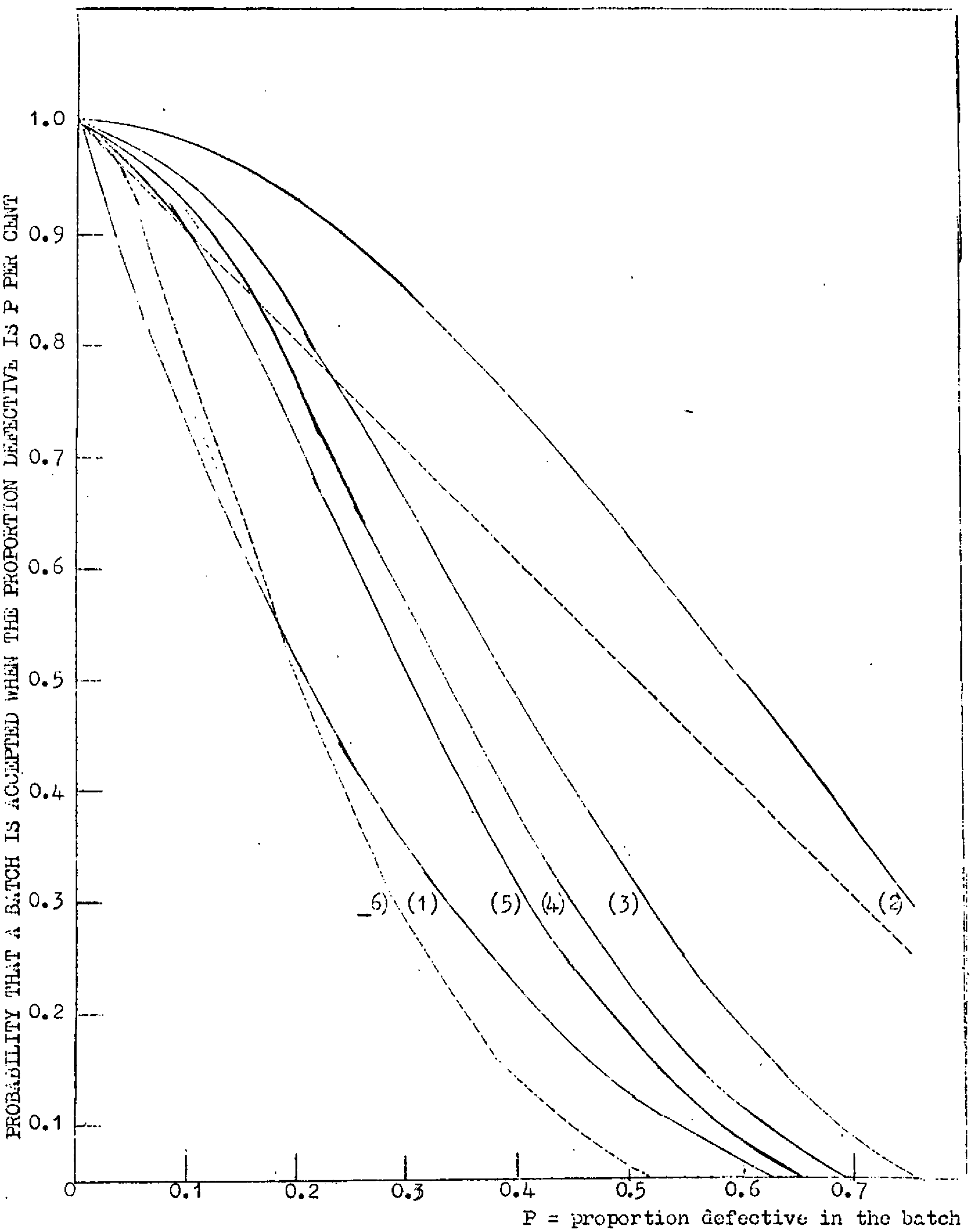

FIG. I. ACCEPTANCE PROCEDURES FOR HOSE.