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ON THE MINIMUM SPEED OF FLAME PROPAGATION IN  
FABRICS

by

F. H. Thomas and D. I. Lawson.

Summary

When propagation of flame along a fabric on the semi-circular apparatus ceases, the velocity just prior to extinction is observed to be an approximately constant fraction of the vertical flame speed. This and the measured variation in the distance of spread for different thicknesses of paper are discussed in the light of a theoretical analysis of flame spread in terms of heat transfer, in which the conduction along the material determines the minimum velocity. Insufficient heat transfer data are known from which to effect a quantitative comparison but the experimental results are qualitatively consistent with theory.

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ON THE MINIMUM SPEED OF FLAME PROPAGATION IN FABRICS

by

F. H. Thomas and D. I. Dawson.

INTRODUCTION

In observing the propagation of flame along fabrics at various angles, by means of the semi-circular test (1) it appears that near extinction the velocity of propagation does not decrease gradually to zero but that propagation ceases abruptly at a point and the flame goes out without progressing any further. The velocity of propagation at the point of extinction is an approximately constant fraction, about one-seventieth, of the vertical flame speed, measured on a torsion balance (2). Some results are given in Fig. 1, where for various materials the minimum velocity is shown as a function of vertical flame speed. (Line A)

This paper discusses the interpretation of this limiting condition for flame propagation in terms of the heat conduction along the fabric.

An equation has been derived elsewhere (3) based on the heat conduction equation for a moving heat source (4) and assuming steady burning of a fabric thin enough for transverse thermal gradients to be neglected this equation is:-

$$K \frac{d^2 \theta}{dx^2} = -v \rho c \frac{d\theta}{dx} - 2Q \phi(x, z) + 2H \theta \dots \dots \dots (1)$$

and  $z = x - vt$

where  $\theta$  is the temperature

$\Delta$  is the thickness

$c$  is the specific heat

$\rho$  is the density

$K$  is the thermal conductivity

$k = \frac{K}{\rho c}$

$t$  is the time

$V$  is the velocity of flame propagation

$Q$  is the maximum heat flux to the fabric

$H$  is the surface cooling coefficient.

$h = H/K$

$\phi$  is a function ( $< 1$ ) giving the distribution of heat flux along the fabric.

$x$  is a stationary co-ordinate along the fabric

and  $z$  is a co-ordinate moving along the fabric at the speed of the flame.

Heat is generated within the fabric by chemical reaction but for a thin fabric this can in effect be combined with the heat transfer from the flames and hot gas in the term  $Q \phi(x, z)$ . The boundary conditions to be employed are

$$\theta \rightarrow 0 \quad \text{as} \quad z \rightarrow \infty \quad (2A)$$

$$\theta = \theta_0 \quad \text{at} \quad z = 0 \quad (2B)$$

$\theta_0$  can be regarded as an "ignition temperature"

In order to enable an analytical solution of the above equation to be obtained simply, we assume

$$\phi(x, z) = \begin{cases} e^{-\alpha z} & z \geq 0 \\ 0 & z < 0 \end{cases} \quad (3)$$

In terms of heat transfer this is equivalent to a uniform heat flux of  $Q$  over a length  $\frac{1}{\alpha}$  which may be defined as an "equivalent flame length".

The qualitative aspects of the following theoretical discussion would not be affected by assuming a different distribution of heat flux provided

- (a) only one independent parameter (e.g. the characteristic length  $\frac{1}{\alpha}$ ) were involved.
- (b) the heat flux were zero at positive and negative infinite values of "z".

ANALYSIS

We employ the Complex Fourier Transform

$$\bar{\psi} = \int_{-\infty}^{\infty} \psi e^{ifz} dz \quad (4a)$$

$$\psi = \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{\psi} e^{-ifz} df \quad (4b)$$

Hence, from equations (1), (3) and (4a)

$$\bar{\theta} = \frac{2Q}{K\Delta} \frac{1}{(\alpha - if)} \frac{1}{(f^2 + \frac{2h}{\Delta} + i\nu f)} \quad (5)$$

From equations (2b), (4b) and (5) we have

$$\theta_0 = \frac{Q}{K\Delta\pi} \int_{-\infty}^{\infty} \frac{df}{(\alpha - if)(f^2 + \frac{2h}{\Delta} + i\nu f)} \quad (6)$$

which reduces to

$$\theta_0 = \frac{2Q}{K\Delta\pi\alpha} \int_0^{\infty} \frac{(y^2(1+B) + A) dy}{(1-y^2)(y^2 + A)^2 + B^2 y^2} \quad (7)$$

where  $A = \frac{2h}{\Delta\alpha}$  =  $\frac{\text{surface loss of heat}}{\text{conduction in direction of flame propagation}}$

and  $B = \frac{\nu}{R\alpha}$  =  $\frac{\text{heat given to material}}{\text{conduction in direction of flame propagation}}$

Equation (7) yields, on integration

$$\frac{B+A-1}{A} \frac{2H\theta_0}{Q} = 1 + \left(\frac{B-2}{B}\right) \frac{1}{\left(1 + \frac{4A}{B^2}\right)^{1/2}}$$

We define  $\theta_0/H\theta_s$  as  $R$ , the ratio of the maximum heat flux to the surface to the maximum heat loss from the surface.

Fig. 2 shows the values of  $D = \frac{B}{A}$ , i. e.  $\frac{\text{heat given to material}}{\text{surface loss of heat}}$  or  $\frac{\nu PC \Delta \alpha}{2H}$  against  $A$  for various values of  $R$ .

At large values of  $A$ , i. e. when the transfer by conduction can be neglected

$$D \rightarrow R - 1$$

$$\nu PC \Delta \alpha = \frac{2(Q - H\theta_s)}{C\theta_0\alpha}$$

i. e.

a result obtained and discussed elsewhere (3).

Here we are concerned with the fact that for any given  $R$  there is a minimum value of  $V$  at which propagation is possible.

EXTINCTION CONDITION

In the semi-circular test,  $\frac{1}{2}L$ , the equivalent flame length decreases around the semi-circle and we should expect extinction to occur at an appropriate critical value of  $A$ ; the value of  $R$  at extinction may be smaller than the value appropriate for vertical flame speed and we introduce the suffixes "v" for vertical flame spread and "e" for extinction.

We have from equation (11)

$$\frac{\rho \Delta V_v c \alpha_v}{2H} = D_v \quad (9)$$

and at extinction

$$\frac{\rho \Delta V_e c \alpha_e}{2H} = D_e \quad (10)$$

Therefore from equations (9) and (10)

$$V_e/V_v = \frac{D_e \alpha_v}{D_v \alpha_e} \quad (11)$$

But at extinction

$$A_e = \frac{2H}{\Delta \alpha_e} \quad (12)$$

Therefore from equations (9), (11) and (12)

$$V_e/V_v = \frac{D_e}{(\rho \Delta V_v c)} \sqrt{2 A_e \Delta H K} \quad (13)$$

Although this equation is written with  $V_v$  as a denominator on both sides,  $\rho \Delta V_v$  is in effect a constant for any one type of material as that for given values of  $\alpha$ ;  $D_0 = kc$

$$V_e/V_v \propto \sqrt{\Delta} \quad (14)$$

It can be shown by dimensional analysis of equation (1) that equation (14) is valid generally for a heat flux satisfying the two conditions mentioned above.

COMPARISON OF EXPERIMENTAL DATA AND THEORY

Equation (13) contains a number of terms whose values are dependent on the particular material. These are  $K$ ,  $c$  and  $\rho$  and the product  $(\rho \Delta V_v)$ . If we assume these properties to be approximately the same for the different materials tested then  $V_e/V_v \propto \sqrt{\Delta}$  and is therefore proportional to  $\sqrt{m}$  where  $m$  is the mass per unit area of the material.

If the flame speed at extinction  $V_e$  is normalised to a constant mass per unit area, say  $10 \text{ mg./cm}^2$

$$V'_e = \sqrt{\frac{10}{m}} V_e$$

where  $V'_e$  is the normalised flame speed at extinction, then a linear relation should hold between the vertical flame speed  $V_v$  and  $V'_e$ . This is shown in Fig. 1 by Line B.

There are insufficient data in Fig. 1 for any one material to test whether the power of the thickness term in equation (14) is  $1/2$ , but it may be seen that the proportionality between  $V'_e$  and  $V_v$  is as good if not better than that between  $V_e$  and  $V_v$ .

In order to estimate  $V'_e$  from equation (13) the value of various physical properties and the value of  $R_e$  would have to be known.

The flames from the fabric when burning feebly near the point of extinction are non-luminous and about 1.5 cm in size. The heat transfer would therefore be mainly convective from the flame to the unburnt material but the heat loss from the surface would include radiation as well. There are insufficient heat transfer data available to calculate the heat transfer from the flame to the surface for this type of flame. Even if conventional convection data are to be used it must allow for the fact that the size of the flame is small in width as well as length. Also the lower the speed of propagation the more important is the heating by chemical decomposition.

It is therefore not at present practical to consider the quantitative aspects of the above theory but there are however data other than in Fig. 1. which support equation (14).

It is of interest to consider the empirical formula derived by Webster (5) for the relation between the semi-circular test and the vertical flame speed.

$$V_v = \frac{0.078 d^{3/2}}{T} \quad (15)$$

where  $d$  is the final distance of spread in cm  
and  $T$  is the time of spread in s.

It has been shown (6) that with certain assumptions equation (17) can be used to derive a value for the velocity at any point on the semi-circle.

A short formal derivation has also appeared (7)

Thus from equation (15)

$$\frac{d}{dt} \left( \frac{d}{dt} \right) = \frac{V_v}{0.078 d^{3/2}}$$

$$V_v = \frac{d(d)}{dt} = \frac{V_v}{0.078 d^{3/2}} \cdot d^{3/2}$$

$$V_v / V_v = \frac{5.05}{d^{3/2}} \quad (16)$$

From equations (14) and (16) we have

$$d \propto \Delta^{-3} \quad (17)$$

Webster (5) gives data for paper of various thicknesses made up from layers of a single sheet and in Fig. 3 the distance of spread  $d$  is plotted on log-log scales against the number of layers which is proportional to the thickness. The slope is  $-1/3$  as expected from equation (17). For a mass per unit area of  $10 \text{ mg./cm}^2$

$$\frac{V_t}{V_v} = \frac{1}{73} \quad (\text{calculated from equation (16)})$$

compared with  $\frac{V_t}{V_v} = \frac{1}{55}$   
for the materials in Fig. 1.

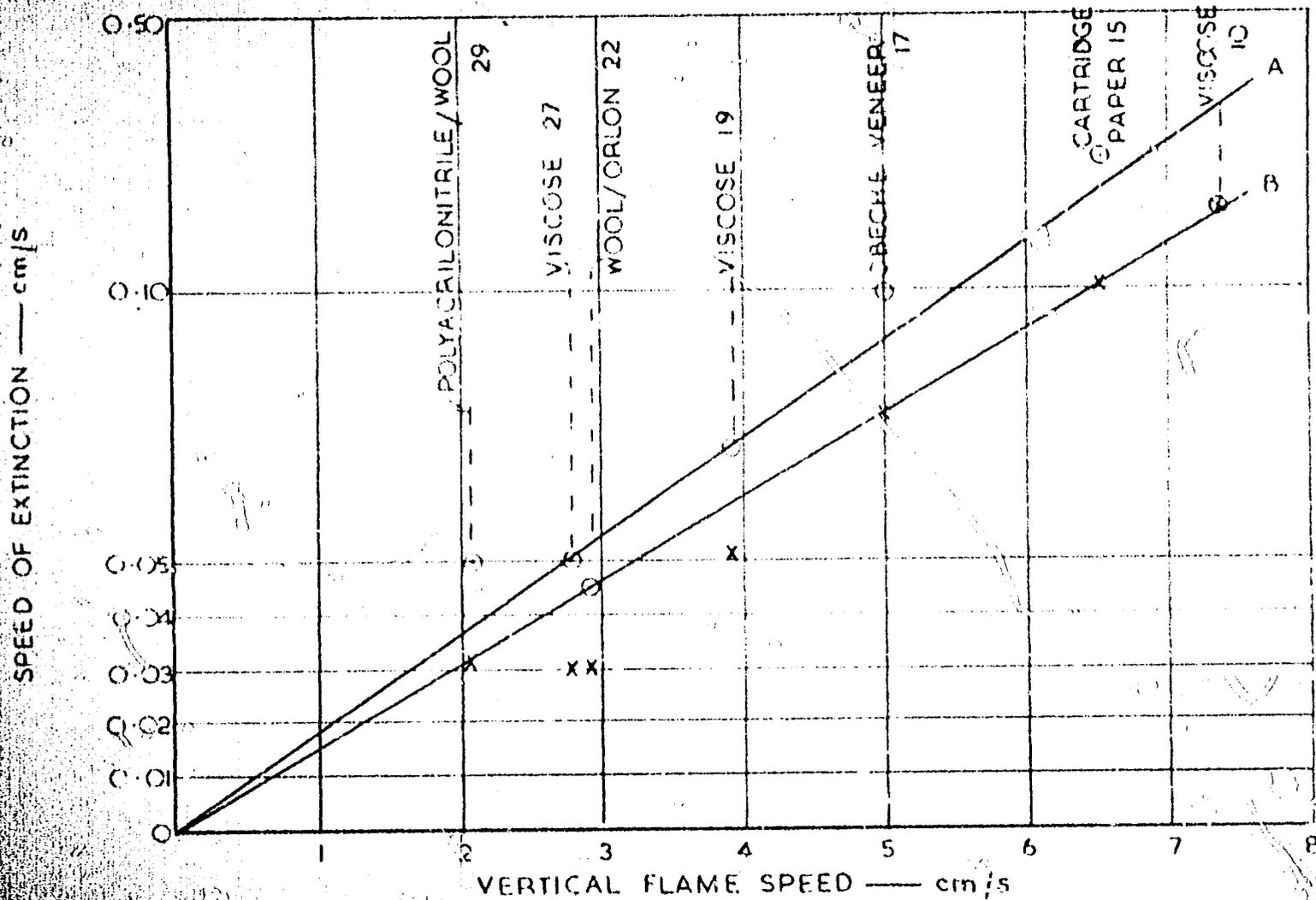
The similarity of these figures is a reflection of the validity of equation (15)

Until it is possible to measure or calculate the heat fluxes in and near the flame when propagation is ceasing, it will not be possible to compare directly experiment and theory. The evidence of the measured flame speeds and the variation in flame speed with thickness of paper is, however, consistent with the qualitative aspects of the theory.

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The figures next to the material refer to the wt/unit area in mg/cm<sup>2</sup>

- O actual extinction speed
- X extinction speed reduced to a weight/unit area of 10 mg/cm<sup>2</sup> by formula  $V_e \propto \sqrt{w}$

(FIG. 1) RELATION BETWEEN EXTINCTION SPEED AND VERTICAL FLAME SPEED

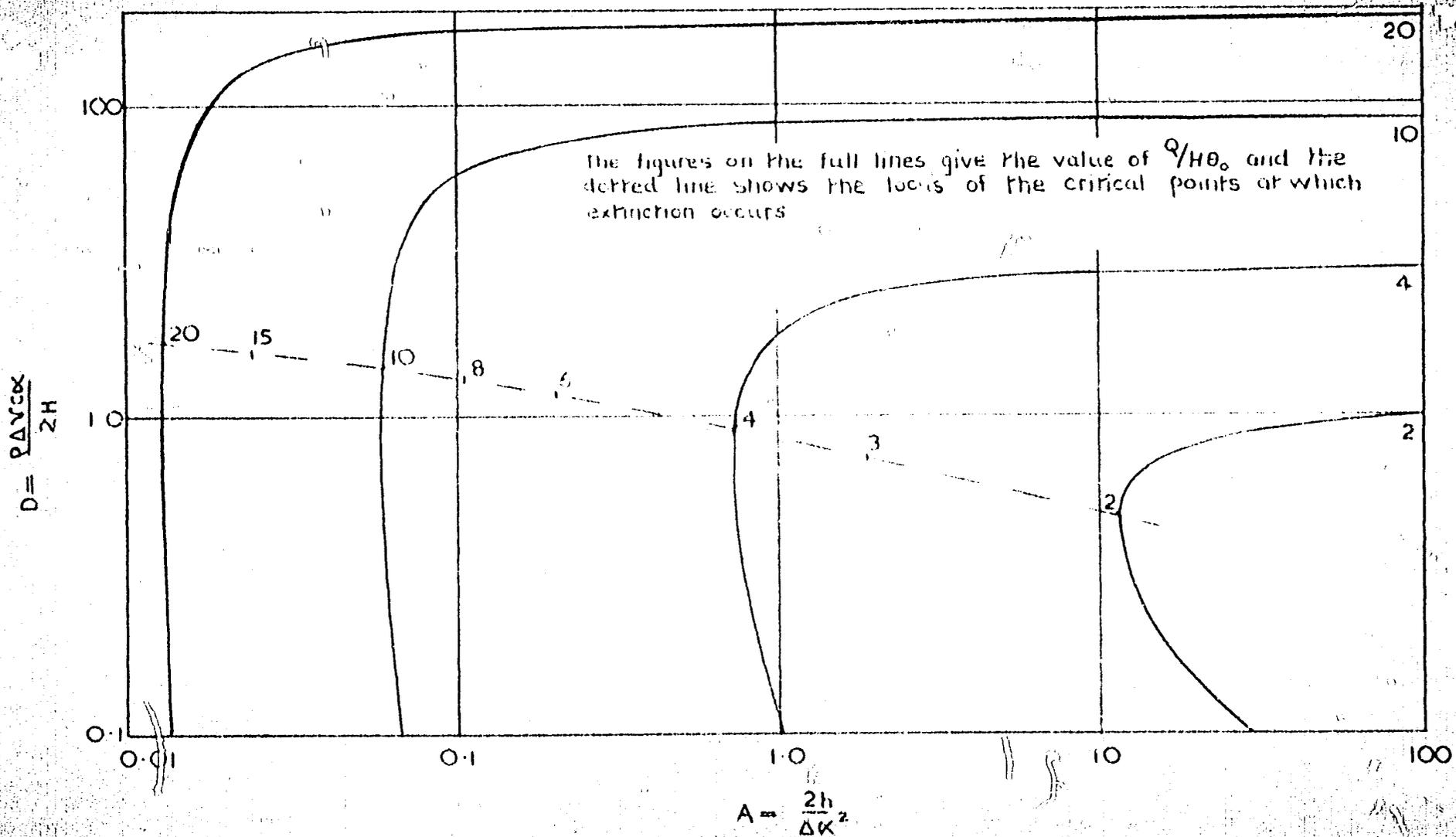


FIG 2. RELATIONS DETERMINING FLAME SPEED

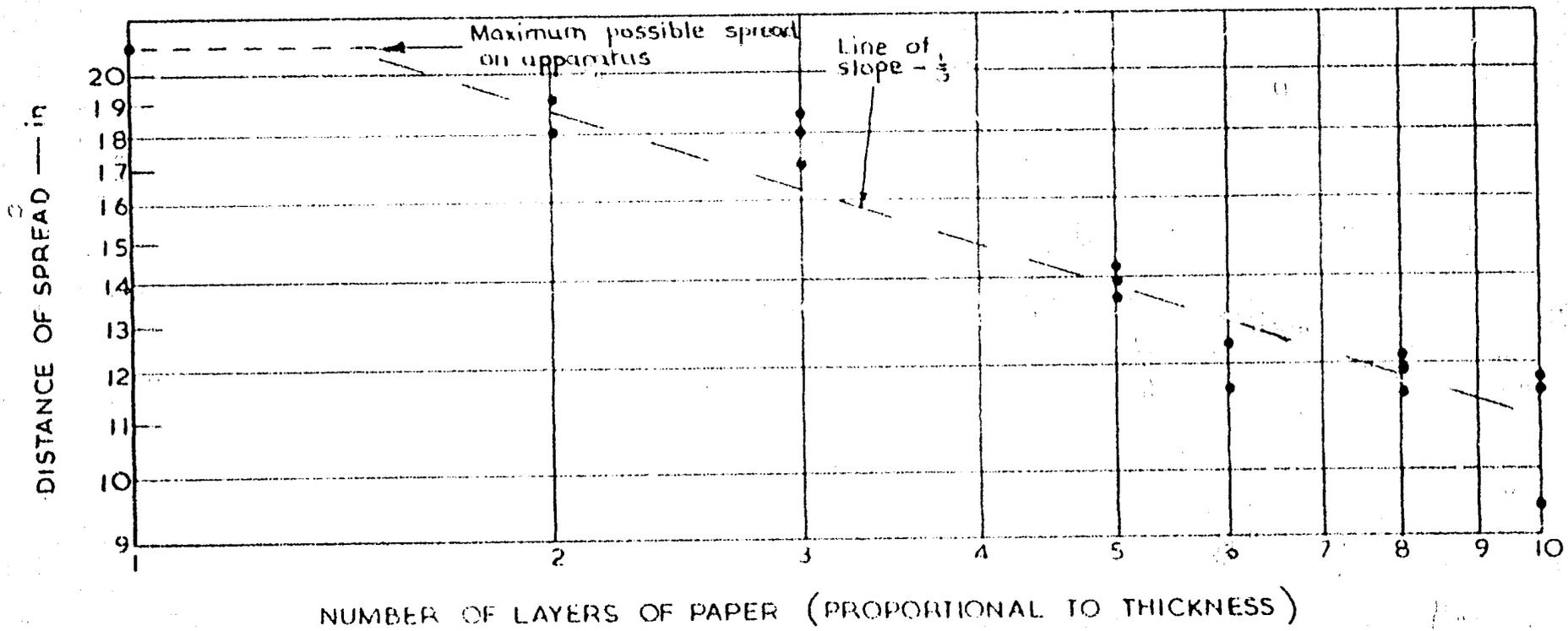


FIG. 3. DISTANCE OF SPREAD ON SEMI-CIRCULAR TEST FOR DIFFERENT THICKNESSES OF PAPER

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