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THE CORRELATION OF IGNITION TIME WITH THE PHYSICAL PROPERTIES OF MATERIALS

PART I : SPONTANEOUS IGNITION OF CELLULOSIC MATERIALS

by

D. L. Simms

Summary

The time taken to ignite cellulosic materials when subjected to thermal radiation of an intensity greater than 1 cal.cm^{-2}s^{-1}, may be correlated with intensity of irradiation in terms of the thermal balance of the solid.

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1. Introduction

Several attempts have been made to correlate the time taken to ignite materials by radiation with the physical properties of the irradiated materials.[1][2][3] More data are now available and in addition it is now known that the time taken to ignite at low radiant intensities increases considerably with decreasing size of irradiated area.[4] Similarly the effect of low absorptivity[5] is to increase significantly the time taken to ignite particularly at high intensities. The correction factors necessary to allow for both these effects are now known.[4][5] These correction factors partly account for the discrepancies between the earlier analyses[1][2] and the experimental results.

This note presents a correlation for spontaneous ignition in terms of the thermal balance of the solid. The amount of chemical heating in the solid may by calculation be shown to be generally negligible up to the point of ignition[6] in the present experiments and is not considered in this discussion. Gardon[7] regards the effect of diathermancy in the surface layers as insignificant after the first two seconds and as most of the ignition times are longer than this, diathermancy is not considered here.

2. Thermal balance in the solid at time 't' and at depth 'x' from the centre

The temperature rise θ at a depth 'x' in an infinite slab of thickness 2ℓ and absorptivity 'a' losing heat from both faces by Newtonian cooling and exposed on one face to an intensity of irradiation I is obtained from

$t > 0$ $$\frac{\partial^2\theta}{\partial x^2} = \frac{1}{k}\frac{\partial\theta}{\partial t} \qquad \text{..... (1)}$$

At the front face

$x = +\ell,\ t > 0$ $$\frac{aI}{K} - h\theta = -\frac{\partial\theta}{\partial x} \qquad \text{..... (2i)}$$

At the rear face

$x = -\ell,\ t > 0$ $$h\theta = -\frac{\partial\theta}{\partial x} \qquad \text{..... (2ii)}$$

and at

$t = 0$ $$\theta = 0 \qquad \text{..... (3)}$$

where $h = H/K$

and H is the Newtonian cooling constant,

K is the thermal conductivity,

ρ is the density,

c is the specific heat of the material,

k is the thermal diffusivity equal to $K/\rho c$

The solution to this equation is complex and is discussed in the Appendix, but rearranging the terms in equations (1) - (3) shows that the solution must be of the form

$$\theta = \frac{aI}{H} f\left(\frac{x}{\ell}, \frac{kt}{\ell^2}, h\ell\right)$$

Within certain ranges of values of the two dimensionless groups, the Fourier number $\frac{kt}{\ell^2}$ and the Biot number $h\ell$, useful approximations may be made. These ranges are shown diagrammatically in Fig. 1.

2.1 The slab with a linear temperature gradient

For the range of values of $\frac{kt}{\ell^2}$ and $h\ell$ in the area A of Fig. 1, there is effectively a linear temperature gradient through the slab (Appendix, equation (14)). The mean temperature rise θ_m is given by (Appendix, equation (17))

$$\theta_m = \frac{aI}{2H}\left(1 - e^{-\frac{Ht}{\rho c \ell}}\right) \quad \text{.....} \quad (4)$$

$\frac{Ht}{\rho c \ell}$ is a dimensionless group obtained from the Biot and Fourier numbers by eliminating K, the thermal conductivity, which is, of course, absent if there are no thermal gradients.

2.2 The semi-infinite solid

For the range of values of $\frac{kt}{\ell^2}$ and $h\ell$ in the area B in Fig. 1, the front surface behaves as though the slab was a semi-infinite solid (Appendix, equation (18)) and

$$\theta_F = \frac{aI}{H}\left(1 - e^{\beta^2} \operatorname{erfc} \beta\right) \quad \text{.....} \quad (5)$$

where $\beta = h\sqrt{kt}$

β is a function of the Fourier and Biot numbers independent of the thickness which has no meaning in the semi-infinite solid.

3. Results

Data are only available at present for materials in the form of non-uniform slabs (Table 1) and semi-infinite solids (Table 2). The data have been corrected for the effects of area.[4]

4. Correlation of data

Experimental results may be correlated in terms of the groups of properties in equations (4) and (5). It has been conventional[8] to consider the absorbed energy, equal to aIt, rather than the absorbed intensity of radiation, aI, as a variable and so the results are plotted in terms of two dimensionless variables:- the energy modulus $\frac{aIt}{2\ell\rho c\theta_m}$, the form of which represents the ratio of the energy received by the surface to the heat content of the sample at ignition, and the cooling modulus $\frac{Ht}{\rho c \ell}$, the form of which represents the ratio of the energy lost by cooling, to the heat content. In terms of these groups equation (4) for slabs with linear temperature gradients may be rearranged as

$$\frac{aIt}{2\ell\rho c\theta_m} = \frac{Ht/\rho c\ell}{\left(1 - e^{-\frac{Ht}{\rho c \ell}}\right)} \quad \text{.....} \quad (6)$$

This is shown in Fig. 2.

Equation (5) for semi-infinite solids may be rearranged as

$$\frac{aIt}{\rho c\sqrt{kt}\,\theta_F} = \frac{\beta}{\left(1 - e^{\beta^2}\operatorname{erfc}\beta\right)} \qquad \text{.....} \quad (7)$$

This is shown in Fig. 3.

Equation (7) contains the same variables as equation (6) except that $\sqrt{kt}$ replaces the thickness of the slab in the formation of the dimensionless groups and θ_F replaces θ_m.

5. Correlation of results

5.1 The slab with the linear temperature gradient

When the cooling modulus $\frac{Ht}{\rho c l}$ tends to zero, as it normally does for short exposure times, then from equation (6)

$$\frac{aIt}{\rho c 2l\,\theta_m} \longrightarrow 1 \qquad \text{.....} \quad (8)$$

Hence from values of $\frac{aIt}{2\rho c l}$ for small values of $\frac{Ht}{\rho c l}$ a first approximation to θ_m may be obtained and the appropriate value chosen for the Newtonian cooling constant H over the temperature range. The value of θ_m is then adjusted to give the best fit between the experimental points and equation (6). The correlation obtained (Fig. 2) is adequate considering the assumptions made and the experimental variations.

5.2 Semi-infinite solids

When the cooling modulus $\frac{Ht}{\rho c\sqrt{kt}}$ tends to zero, as it normally does for short exposure times, we have from equation (7)

$$\frac{aIt}{\rho c\sqrt{kt}\,\theta_F} \longrightarrow \frac{\sqrt{\pi}}{2} \qquad \text{.....} \quad (9)$$

Hence from values of $\frac{aIt}{\rho c\sqrt{kt}}$ for small values of the cooling modulus $\frac{Ht}{\rho c\sqrt{kt}}$, a first approximation to θ_F may be obtained and the appropriate value for the Newtonian cooling constant, H, over the temperature range chosen. The value of θ_F is then adjusted to give the best fit between the experimental points and equation (7). (Fig. 3.)

6. Discussion and conclusions

In this series of experiments, ignition was not obtained below 1 cal.cm^{-2}s^{-1}, but above this level the ignition time may be obtained for slabs of cellulosic materials from Fig. 2 and for semi-infinite solids from Fig. 3, or from equations (6) or (7). The best fit between the experimental results and these equations is obtained with a value of 525°C for θ_F or θ_m. It is fortuitous that the same value may be used for the mean temperature of a slab and for the surface of a semi-infinite solid. Examination of the energy modulus shows that the group $2l\rho c\,\theta_m$ is the actual heat content of the slab at ignition and this may be linked with the production of a critical amount of flammable gases, but the group $\rho c\sqrt{kt}\,\theta_F$ is only the equivalent dimensional form and is not the actual heat content of the semi-infinite solid. The greater scatter obtained with the results for the semi-infinite solid (Fig. 3) is presumably associated with the incompleteness of the thermal model.

Despite this, a large part of ignition behaviour may be readily understood in terms of thermal heating.

The principal use of this type of analysis is to obtain dimensionless forms of the variables and provided that the role of chemical heating may be neglected similar analyses could be employed for non-cellulosic materials.

Flaming persisted for most of the results reported in Fig. 2, though for some of the ignitions at high intensities, flaming did not persist after the removal of the radiation. It is hoped to consider this aspect of ignition in a future report.

TABLE 1

Description of materials which behave as slabs with a linear temperature gradient

The specific heat of all these materials was taken as 0.34 cal/gm/deg C.[11]
All properties are taken as the values applying at room temperature.

Material	Colour	J.F.R.O. Ref. No.	Density g cm^{-3} ρ	Thermal conductivity (K)[8] cal $cm^{-1}s^{-1}$deg C^{-1} x 10^4	Thickness 2ℓ cm.	Source of radiation	Range of intensities of radiation cal $cm^{-2}s^{-1}$	Symbol in Fig 2	$h\ell$
Cotton	Black	R.169	0.57	1.8	0.045	Carbon arc[9]	1.7 - 13.5	●	0.12
						1 ft^2 radiant panel[10]	1.2 - 2.3	■	
Cotton	White	R.54	0.57	1.8	0.045	"	1.0 - 1.8	○	0.12
Filter paper	Black	R.110	0.53	1.7	0.02	Carbon arc	1.2 - 12.9	▲	0.06
Denim	Khaki	R.26	0.52	1.6	0.06	1 ft^2 radiant panel	1.3 - 1.9	▼	0.17
Drill	Blue	R.24	0.46	1.5	0.065	"	1.3 - 2.0	△	0.22
Drill	Black	R.7	0.39	1.4	0.056	"	1.5 - 1.9	□	0.20
Gaberdine	Olive	R.30	0.59	1.9	0.029	"	1.1 - 2.0	▽	0.08

TABLE 2

Description of materials which behave as semi-infinite solids

The specific heat of all these materials was taken as 0.34 cal/gm/deg^{C}. (11)
All properties are taken as the values applying at room temperature.

Material	Colour	Density $g\ cm^{-3}$ ρ	Thermal conductivity (12) $cal\ cm^{-1}s^{-1}deg\ C^{-1} \times 10^4$	Source of radiation	Range of intensities of radiation $cal\ cm^{-2}s^{-1}$	Symbol in Fig 3
Oak	Natural	0.7	3.9	Carbon arc	2.4 - 14.0	○
	Black (candle)			" "	1.9 - 2.9	◆
	" "			" "	1.5 - 2.5	●
Fibreboard	Natural	0.25	1.7	" "	2.4 - 12.0	△
	"			Radiant panel	1.2 - 2.1	•
	Black (carbon)			Carbon arc	1.7 - 7.1	▲
	Black (candle)			" "	2.0 - 3.6	+
Cedar	Natural	0.4	2.4	" "	1.9 - 12.0	□
	"			Radiant panel	1.6 - 2.1	■
	Black (candle)			Carbon arc	1.4 - 2.9	◇
	" "			Radiant panel	1.7 - 2.4	×
Mahogany	Natural	0.6	3.4	Carbon arc	1.9 - 12.0	▽
	Black (candle)			" "	1.9 - 12.0	▼

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14. ibid, Appendix IV, Table 2.

15. ibid, p.55, equation (2) section 24.

APPENDIX

The temperature rise of an irradiated slab

1. General solution

The solution to equations (1) (2) and (3) is given by[13]

$$\theta = \frac{aIhx}{2H(1+hl)} - \frac{aIhl}{H}\sum_{1}^{\infty}\frac{e^{-k\gamma_n^2 t}\sin\gamma_n x}{[hl+l^2(\gamma_n^2+h^2)]\sin\gamma_n l} \quad \text{.....} \quad (10)$$

$$+ \frac{aI}{2H} - \frac{aIhl}{H}\sum_{1}^{\infty}\frac{e^{-k\alpha_n^2 t}\cos\alpha_n x}{[hl+l^2(\alpha_n^2+h^2)]\cos\alpha_n l}$$

where α_n, $(n = 1, 2, 3 \text{ etc.},)$ are the positive roots of

$$\alpha l \tan\alpha l = hl$$

and γ_n, $(n = 1, 2, 3, \text{etc.},)$ are the positive roots of

$$\gamma l \cot\gamma l = -hl.$$

The temperature θ_F of the front surface, $x = l$, is thus

$$\theta_F = \frac{aIhl}{2H(1+hl)} - \frac{aIhl}{H}\sum_{1}^{\infty}\frac{e^{-k\gamma_n^2 t}}{[hl+l^2(\gamma_n^2+h^2)]}$$

$$+ \frac{aI}{2H} - \frac{aIhl}{H}\sum_{1}^{\infty}\frac{e^{-k\alpha_n^2 t}}{[hl+l^2(\alpha_n^2+h^2)]} \quad \text{.....} \quad (11)$$

and the temperature θ_B of the back surface, $x = -l$, is

$$\theta_B = -\frac{aIhl}{2H(1+hl)} + \frac{aIhl}{H}\sum_{1}^{\infty}\frac{e^{-k\gamma_n^2 t}}{[hl+l^2(\gamma_n^2+h^2)]}$$

$$+ \frac{aI}{2H} - \frac{aIhl}{H}\sum_{1}^{\infty}\frac{e^{-k\alpha_n^2 t}}{[hl+l^2(\alpha_n^2+h^2)]} \quad \text{.....} \quad (12)$$

The temperature difference between front and back surfaces

$$\theta_F - \theta_B = \frac{aIhl}{H(1+hl)} - \frac{2aIhl}{H}\sum_{1}^{\infty}\frac{e^{-k\gamma_n^2 t}}{[hl+l^2(\gamma_n^2+h^2)]} \quad \text{.....} \quad (13)$$

This does not contain a term in αl, which has a small first value.[14] This enables the approximations of Section 2.1 to be made.

2. Special cases

The two independent dimensionless parameters for the surface temperatures of the slab are hl, the Biot number and $\frac{kt}{l^2}$ the Fourier number, of which $\frac{\theta H}{aI}$ is a function. Useful simplifications to the above equations may be made for certain values of these parameters (Fig. 1).

2.1 The slab with a linear temperature gradient

This is defined as a slab in which the steady state temperature gradient has in effect been attained.

The condition for this from equation (13) is

$$\frac{1}{1+hl} >> 2\sum_{1}^{\infty}\frac{e^{-k\gamma_n^2 t}}{[hl+l^2(\gamma_n^2+h^2)]} \quad \text{.....} (14)$$

The relation between hl and $\frac{kt}{l^2}$ for the left hand side of equation (14) to be ten times the right hand side has been computed and is the lower boundary of the region A in Fig. 1.

With the above condition equation (13) reduces to

$$\theta_F - \theta_B \doteqdot \frac{aIhl}{H(1+hl)} \quad \text{.....} (15)$$

The temperature gradient through the slab is thus effectively the steady state value.

The mean temperature θ_m of the slab may be obtained directly by integrating equation (10) so that

$$\theta_m = \frac{aI}{2H} - \frac{aIhl}{H}\sum_{1}^{\infty}\frac{e^{-k\alpha_n^2 t}}{[hl+l^2(\alpha_n^2+h^2)]}\frac{hl}{(\alpha_n l)^2} \quad \text{.....} (16)$$

with $\alpha_1^2 \doteqdot h/l$

and

Terms in this series other than the first may be neglected if hl is small[14] (<0.25). Equation (16) then reduces to

$$\theta_m = \frac{aI}{2H}\left(1 - e^{-\frac{Ht}{\rho c l}}\right) \quad \text{.....} (17)$$

Computations have been made to find the relation between hl and $\frac{kt}{l^2}$ so that $\frac{\theta_F - \theta_m}{\theta_m}$ has a given value, e.g. 0.05, 0.25 and 0.5. These give respectively the lower boundaries of the regions C, D and E in Fig. 1

2.2 Semi-infinite solid

This is normally defined as a material of such thickness that there is no appreciable rise in temperature of the rear surface. In the context of this report, it is sufficient for the front surface temperature rise of the slab to be the same as for a semi-infinite solid and the limits of hl and $\frac{kt}{l^2}$ for which this is true to within 10 per cent, are shown in area B of Fig. 1.

Equation (2ii) may then be replaced by

$$\theta = 0 \quad , \quad x = 0$$

and the solution to equation (1) may be shown to be[15]

$$\theta_F = \frac{aI}{H}\left(1 - e^{\beta^2}\operatorname{erfc}\beta\right) \qquad \text{..... (18)}$$

where $\beta = h\sqrt{kt}$

In particular, for $hl < 1.0$, $\frac{kt}{l^2}$ must be less than 0.2 for the approximation to be valid.

KEY TO FIG. 1

Key	Region	Equations for front face temperature θ_F	Remarks	
A	Uniform slab	θ_F and θ_m from equation (4)	Temperature gradient within 10 per cent of steady state value.	Maximum difference between front and mean temperatures less than 5 per cent of mean temperature.
B	Semi-infinite solid	θ_F from equation (5)	Front surface temperature within 10 per cent of that for semi-infinite solid.	—
C	Linear gradient in slab	θ_m from equation (4) and θ_F from equation (15)	As for A.	As for A but 25 per cent.
D	ditto	ditto	As for A.	As for A but 50 per cent.
E	ditto	ditto	As for A.	Difference between front and mean temperatures greater than 50 per cent of mean.
Overlap of region B and E		θ_F either from equation (5) or from (15) with θ_m from equation (4)	The front face temperature can be calculated from E or B approximations.	

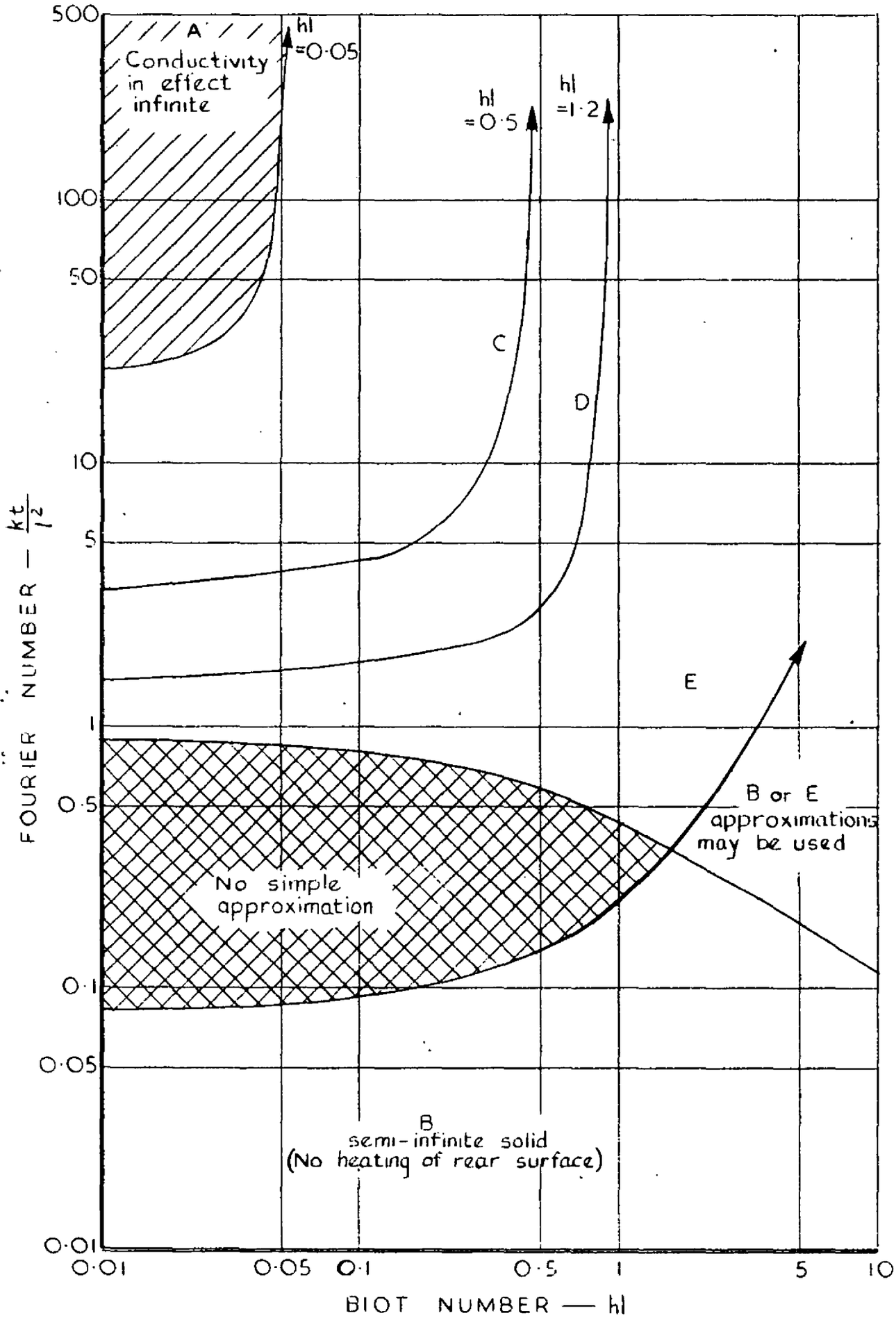
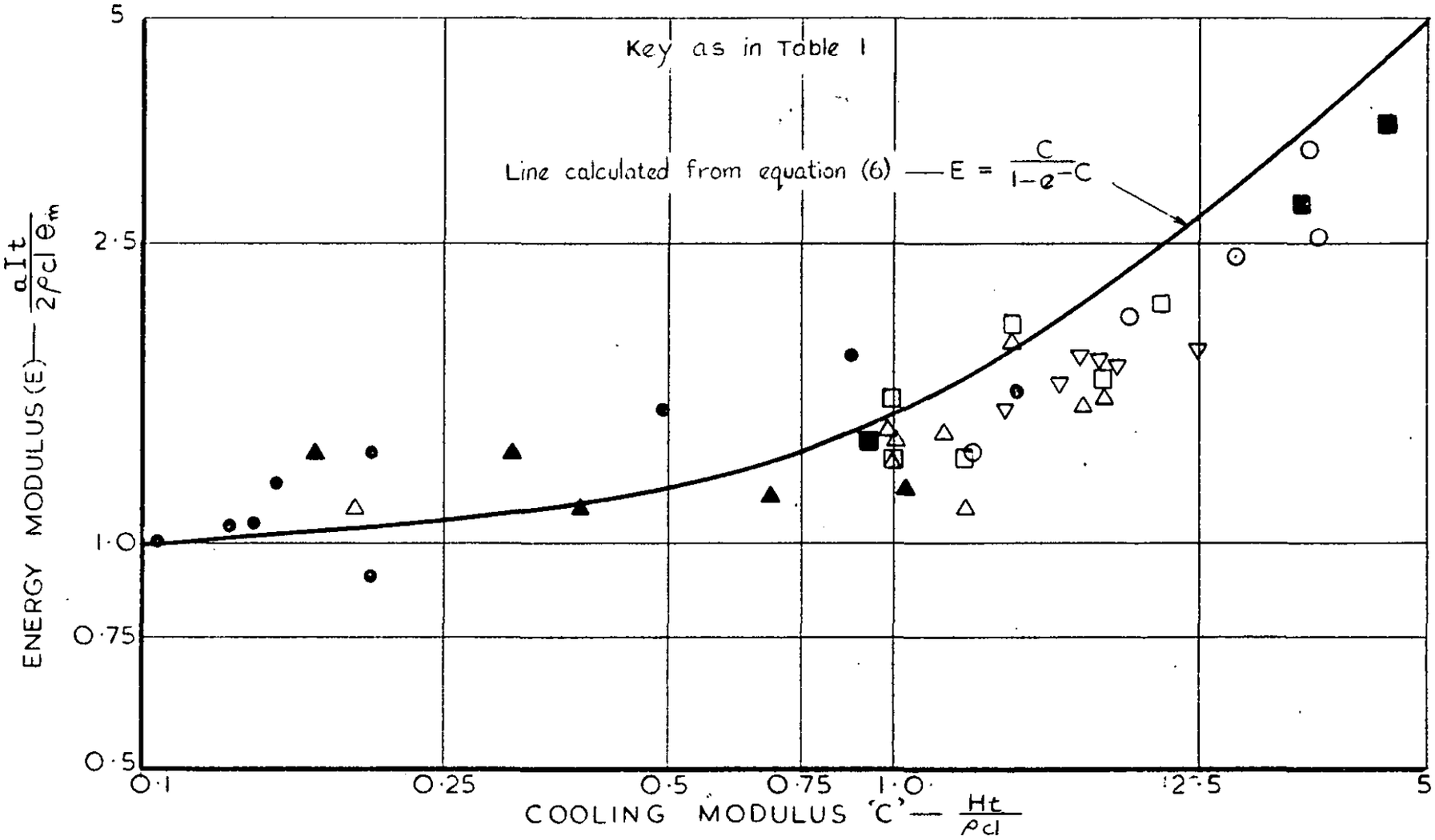
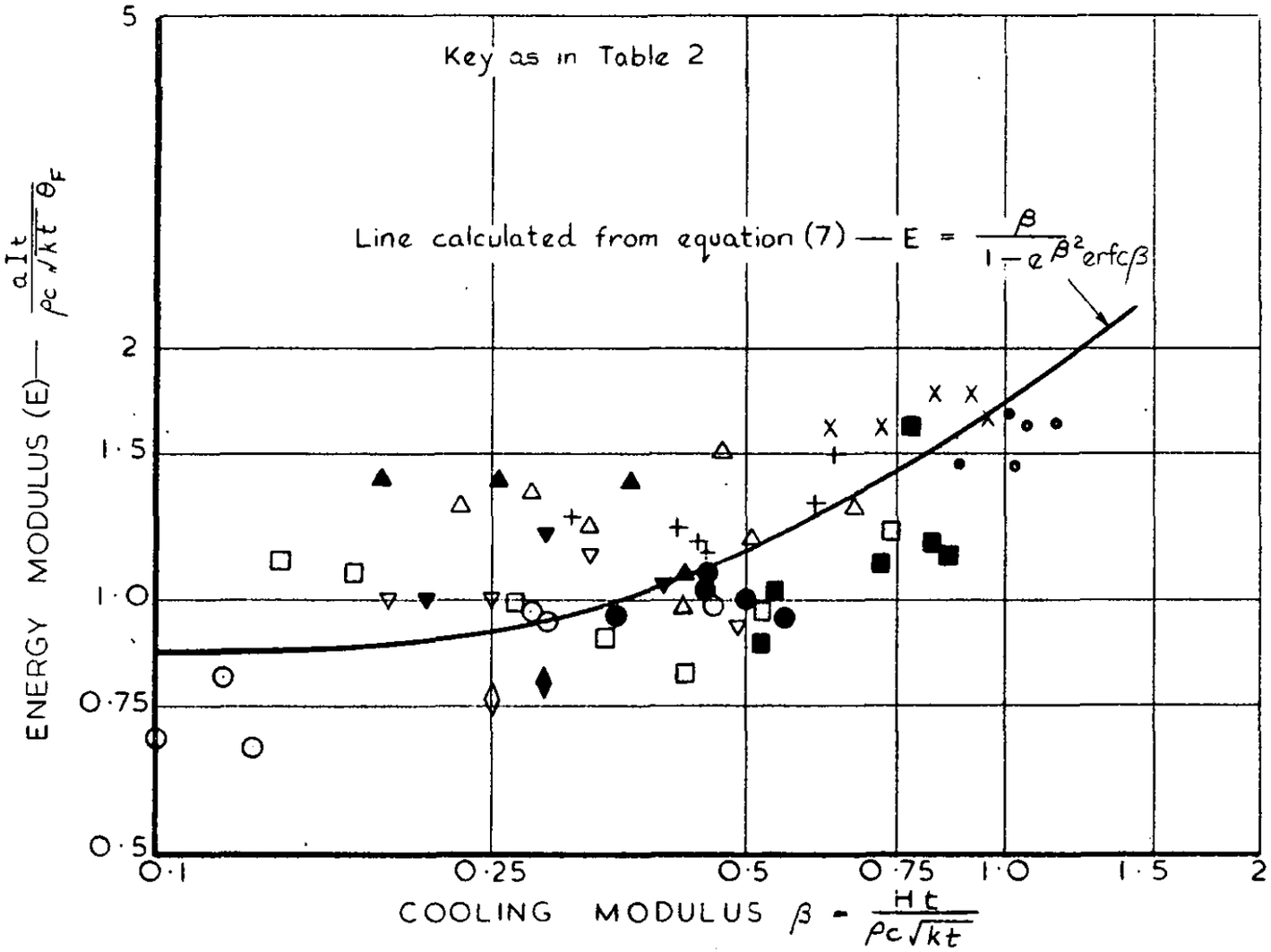


FIG.1. ROLE OF DIMENSIONLESS GROUPS IN DETERMINING SURFACE TEMPERATURE RISE



Range of Intensities —— 1 — 14 cal. $cm^{-2} s^{-1}$
Range of Thicknesses —— 0·02 — 0·065 cm
Range of Times —— 1 — 20 s
Correlating Temperature — 525 °C

FIG. 2. CORRELATION OF IGNITION TIMES FOR SLAB WITH LINEAR TEMPERATURE GRADIENT



Range of Radiation Intensities — 1·2 — 14 cal $cm^{-2}s^{-1}$
Range of Densities — 0·25 — 0·7 g / cm^3
Range of Ignition Times — 2 — 30 s
Correlating Temperature θ_0 — 525 °C

FIG. 3. CORRELATION OF IGNITION TIMES FOR SEMI-INFINITE SOLIDS