F.R. Note No. 332/1957
Research Programme
Objective B

DEPARTMENT OF SCIENTIFIC AND INDUSTRIAL RESEARCH AND FIRE OFFICES' COMMITTEE
JOINT FIRE RESEARCH ORGANIZATION

NOTES ON THE MOVEMENT OF GASES INDUCED BY FLAMES AND HEAT

by

P. H. Thomas

Summary

A number of results are obtained for some problems associated with the flow of gases in or near fires. Some of these are very approximate, being based on idealised theoretical models, but in absence of more sophisticated methods of calculation, they are discussed as first approximations.

October, 1957.

Fire Research Station,
Boreham Wood,
Herts.

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1. Notation

a, A	=	area of windows or jets of fluid
G	=	width
c	=	specific heat
M	=	mass
m	=	mass flow/unit time
x, y, z	=	cartesian co-ordinates (z vertical)
θ	=	temperature above ambient
T	=	absolute temperature
h	=	heat transfer coefficient
H, L	=	height
K	=	conductivity
p	=	pressure
u, v, w	=	components of velocity (w vertical)
f	=	acceleration
g	=	acceleration due to gravity
d	=	depth of layer
t	=	time
q	=	flux of heat
F	=	force
k	=	thermal diffusivity
ν	=	kinematic viscosity
N_{Nu}	=	Nusselt number
N_{Gr}	=	Grashof number
N_{Pr}	=	Prandtl number

suffices = a - air
f - fuel
0, 1, 2, 3 - position
J - jet.

2. Introduction

In many fire problems it is necessary to obtain an estimate of the gas flow and often it is not difficult to do this within the limits imposed by simple flow theory. In most of the problems considered the flow is induced by pressure differences arising from the difference in weights of hot and cold gases.

3. The mass flow up a flat vertical plate

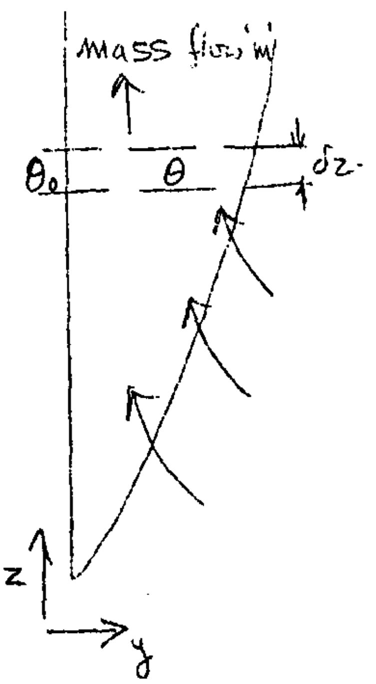
For the laminar flow of gases past a vertical plate, the mean heat transfer coefficient* is obtained from (1)

$$N_{Nu} = 0 \cdot 62 \left[N_{Gr} \cdot N_{Pr} \right]^{1/4} \quad \text{...... (1)}$$

the height and the temperature of the plate being the characteristic length and temperature.

*The constant is that given by Ede (1) - other authors give other figures which may be somewhat different but the difference is hardly likely to be significant in the applications considered here.

It is often of interest to know the mass flow up the heated plate. This can be obtained in a similar way to the heat transfer coefficient, i.e. from the same differential equations. The conventional procedure to obtain the heat transfer coefficient for the plate at uniform temperatures is to assume that a similarity solution exists. As a consequence the increasing heat content of the fluid is manifested as an increase in the mass flow with height - not in the temperature. From the heat balance in the strip between Z and $z + \delta Z$ (see Fig. 1), we have, equating the increase of heat content of the gases to the heat transferred from the hot plate:



$$\frac{d}{dz}\left(mc\,\theta\right)\delta z = h\,\theta_0\,\delta z \qquad \ldots\ldots (2)$$

Since the form of the temperature distribution in "y" is the same for all heights

$$\theta \propto \theta_0 \qquad \ldots\ldots (3)$$

and it follows from equations (1), (2) and (3) that

$$m \propto \frac{K}{c}\left(N_{Gr}\right)^{1/4} \qquad \ldots\ldots (4)$$

Fig. 1. Flow past vertical plate.

The value of the constant of proportionality in equation (4) must be obtained from the distribution of the velocity. For air, the analysis given by Eckert [2] leads to

$$m = 1{\cdot}26\,\frac{K}{c}\left(N_{Gr}\right)^{1/4} \qquad \ldots\ldots (5)$$

(Eckert gives a coefficient of 0·51 for air instead of 0·56 from equation (1). The coefficient of 1·26 may therefore be as high as 1·40).

4. Chimneys and related problems

If the velocity head inside the chimney can be neglected, and there is only one constriction (the throat in domestic chimneys), the whole of the pressure drop may be regarded as acting across it alone; friction losses may be included if necessary, in the usual manner. This treatment, however, is likely to be inadequate for high temperature chimneys because the pressure drop caused by accelerating the expanding gases may not be negligible. Moreover, if combustion is occurring in the chimney, the changes in momentum are associated with a pressure drop though this may not be very important. Some of these points are illustrated below.

4.1. Unrestricted chimney

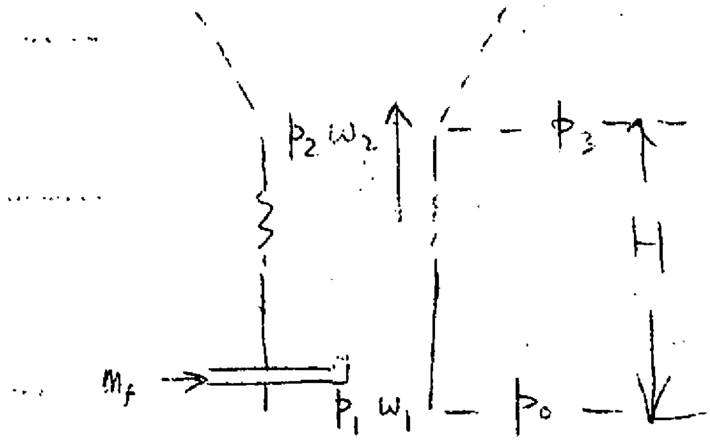


Fig. 2. Flow through unrestricted tube - with combustion.

The pressure p_2 inside the chimney at its top is taken as equal to the external pressure at the same height provided that the flow is as in Fig. 3.

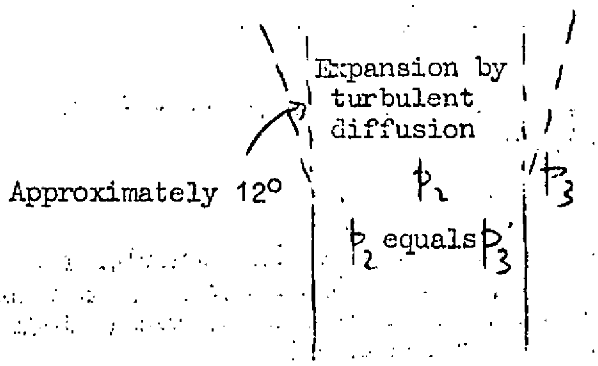


Fig. 3. Conditions at exit of tube.

Hence

$$p_2 = p_3 \qquad \ldots\ldots (6)$$

There is no loss of energy at the inlet hence the pressure drop is equal to the gain in velocity head.

$$p_0 - p_1 = \rho_0 \frac{\omega_1^2}{2g} \qquad \ldots\ldots (7)$$

The static equilibrium outside the chimney gives

$$p_3 = p_0 - \rho_0 H \qquad \ldots\ldots (8)$$

and the momentum equation inside the chimney is

$$p_1 - p_2 = \frac{(m_a + m_f)\,\omega_2}{A g} - \frac{m_a \omega_1}{A g} + \int_0^H \rho\, \delta z \qquad \ldots\ldots (9)$$

The mass flows are given by

$$m_a = A\,\omega_1 \rho_0 \qquad \ldots\ldots(10)$$

$$m_a + m_f = \rho_2 \omega_2 A \qquad \ldots\ldots(11)$$

The state equation gives, assuming an ideal gas

$$\rho_2 = \rho_0 \, T_0/T_2 \qquad \ldots\ldots(12)$$

Also for an ideal gas

$$\rho_0 H - \int_0^H \rho \, dz = \rho_0 \int_0^H \frac{\theta}{T} \, dz \qquad \ldots\ldots(13)$$

For a uniform temperature T_2 this integral becomes $\rho_0 \frac{\theta_2}{T_2} H$

From the above equations (6) to (13) we obtain

$$\omega_1 = \sqrt{\frac{2gH\theta_2}{T_2\left[2\left(\frac{m_a}{m_f}+1\right)^2 \frac{T_2}{T_0} - 1\right]}} \qquad \ldots\ldots(14)$$

w_1 appears implicitly in m_a on the r. h. s of this equation and unless $m_f/m_a \ll 1$, it must be calculated from the roots of a quadratic equation. Usually, however, m_f/m_a can be neglected. The velocity calculated without allowing for the momentum change is

$$\omega_1' = \sqrt{\frac{2gH\theta_2}{T_2}} \qquad \ldots\ldots(15)$$

The correction factor in the square brackets of equation (14) is at least $2T_2/T_0 - 1$ and this is nearly 8 for a chimney at 1000°C thus reducing the estimate of velocity by nearly 3. It should be noted that equation (14) suggests that there is a maximum possible value to w_1. For $m_f/m_a \ll 1$, this occurs when T_2/T_0 is $1 + \frac{1}{\sqrt{2}}$ i.e. for T_0 at 290°K at 510°K. Fig. (4) shows the variation in $\omega_1/\sqrt{2gH}$ with θ_2 for m_f/m_a zero and T_0 290°K).

4.2. Restricted chimney

The opposite extreme of the above problem is one in which the gases can be regarded as stationary in the enclosure or chimney.

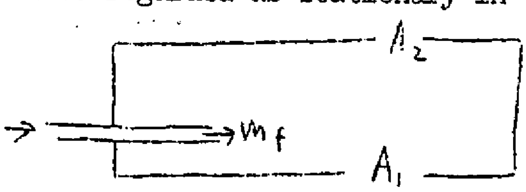


Fig. 5. Restricted chimney.

A_1 and A_2 denote the inlet and exit areas respectively.

We assume that the available draught head is equal to the sum of the pressure drops across the two orifices.

Thus for an ideal gas

$$\rho_0 \frac{H\theta}{T_2} = \rho_0 \frac{w_1^2}{2g} + \rho_2 \frac{w_2^2}{2g} \qquad \ldots\ldots(16)$$

From continuity of mass flow we obtain, as before

$$m_a = A_1 w_1 \rho_0 \qquad \ldots\ldots(17)$$

$$m_a + m_f = A_2 w_2 \rho_2 \qquad \ldots\ldots(18)$$

These together with equation (12) give

$$w_1 = \sqrt{\frac{2g\,H\theta}{T_2 \left\{ 1 + \left(\frac{A_1}{A_2}\right)^2 \left(\frac{m_f}{m_a} + 1\right)^2 \frac{T_2}{T_0} \right.}} \qquad \ldots\ldots(19)$$

Again it has been assumed that the enclosure temperature is uniform but if it is not $H\theta/T_2$ is readily replaced by the integral $\int_0^H \theta/T\,dz$. The same remarks refer to the significance of m_f/m_a as before. The maximum value of w_1 occurs at a value of T_2/T_0 dependent on A_1/A_2

4.2.1. Injection of air etc.

We shall consider here the injection of large quantities of gas. Equations (16) - (19) apply but $m_f/m_a >> 1$.

In these circumstances it is appropriate to rewrite equation (19) as

$$m_f = A_1 \rho_0 \sqrt{\frac{2g\,H\theta}{T_2 \left[\left(\frac{m_a}{m_f}\right)^2 + \left(\frac{A_1}{A_2}\right)^2 \left(1 + \frac{m_a}{m_f}\right)^2 \frac{T_2}{T_0} \right.}} \qquad \ldots\ldots(20)$$

and this gives the value of m_f necessary to reduce m_a - the inlet flow - to zero. When this happens the exit flow through A_2 must account for the complete pressure drop due to buoyancy (w_1 zero in equation (16)). In this simplified treatment the kinetic energy of the flow m_f has been neglected.

4.3. Restricted chimney (enclosures) and vertical plate

We neglect m_H/m_a and we consider the enclosure of Fig. 5 with a part of one vertical surface heated at a temperature above ambient of θ_o (see Fig. 6). Instead of a driving force of T_2 within the enclosure, it is this vertical plate which acts as the driving force in this example.

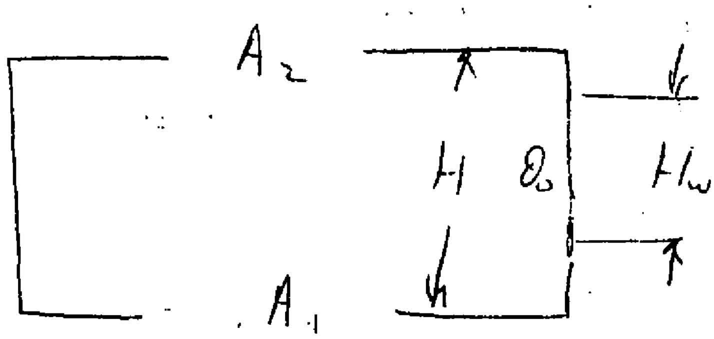


Fig. 6. Enclosure with heated wall.

This can represent a room heated by radiation from outside (in this θ_o increases with time). The difference between horizontal and vertical windows is of secondary significance if their linear dimension is small compared with the vertical dimensions of the enclosure.

The following is suggested as a means of obtaining a first approximation to the solution. The heated wall produces a mass flow of heated gas which, but for the opening A_2 would produce a layer of increasing depth of hot gas.

The opening A_2 provides an outlet for the gases. If $A_1 \gg A_2$ equilibrium is reached when the depth of the hot zone is such as to produce a flow equal to that produced by the hot walls. The equilibrium state could then be found from equation (6) - assuming the heated wall to be in free space - and finding the depth of hot zone that is necessary for this mass to flow through A_2. For this, it is necessary to know the mean temperature of the heated gases. For a vertical plate this is proportional to the temperature of the wall "θ" and using Eckert's simplified treatment, this temperature is $0{\cdot}45\ \theta_o$, so that if $A_1 \gg A_2$

$$m = A_2 \sqrt{\frac{2g\ 0.45\ \theta_o d}{T_o}}$$

where d is the depth of the layer and m is the flow from the vertical plate. From this and equation (5) one can find "d".

This cannot be correct if "d" is more than a small fraction of "H" for then some of the heated gas will be recirculated. If "d" tends to be large for the particular arrangement it is possible that the layer may fill the enclosure and the following solution is for this case.

It is assumed that the problem can be considered in terms of a mean temperature θ_1 for the gases in the enclosure (except those near the plate). The mass flow through the enclosure is then from equation (19).

$$m' = \rho_o A \sqrt{\frac{2gH\theta_1}{(T_o+\theta_1)\left[1+\left(\frac{A_1}{A_2}\right)^2\left(\frac{T_o+\theta_1}{T_o}\right)\right]}} \quad \ldots\ldots (21)$$

and the heat flow is

$$q = m' c\, \theta_1$$

The heat flow neglecting any other loss of heat must at equilibrium be given by

$$q = h H_w b\, (\theta_o - \theta_1)$$

where "b" is the breadth of the heated strip i.e. from these two equations for "q"

$$m' c\, \theta_o = h H_w b (\theta_o - \theta_1) \quad \ldots\ldots (22)$$

Hence, from equations (1), (21) and (22) we have

$$m' = 0.64 \, \frac{kb}{c} \left(N_{Pr} N_{Gr}\right)^{1/4} \left(\frac{\theta_o - \theta_1}{\theta_1}\right) \quad (23)$$

where H_w is the characteristic height in N_{Gr}. From equations (21) and (23) m' & θ_1, can be obtained and the system solved. This treatment can readily be modified to allow for conduction and radiation loss if necessary.

5. Gas velocities above flames

Above the flame the flow is that for a turbulent jet. Normally, the problem of jets is discussed in terms of a definite heat flux (or heat flux per unit length) or a definite momentum flux (or momentum flux per unit length) but not both heat and momentum, although an attempt has been made to obtain a solution with two arbitrary constants (3). Batchelor also gives several references to problems of this kind (4).

These theories may be used to examine the effects of a fire at points beyond say, 5 diameters above it. The jets may be taken as expanding linearly from an origin. The following equations are the basic ones in this analysis and are discussed more fully in the above references.

The momentum equation is equating the mass acceleration of a moving particle to the buoyancy force for the steady state.

$$\rho_j \left(w\frac{dw}{dz} + v\frac{dw}{dy} + u\frac{dw}{dx} \right) = g(\rho_o - \rho_j) = g\rho_j\, \theta / T_o \quad \ldots\ldots (24)$$

The constancy of heat flow across each section for a radial jet gives

$$q \propto w\theta z^2 \quad \ldots\ldots (25)$$

and for a line source

$$q = \omega \theta z \qquad \ldots\ldots (26)$$

Now, it is assumed implicity in equations (25) and (26) that there is a similarity solution, temperature (and velocity) distributions being the same at all values of Z. Equations (24) and (25) can be satisfied by

$$u \propto v \propto \omega \propto \left(\frac{q}{z}\right)^{1/3} \quad \text{for a point source}$$

$$\theta \propto \left(\frac{q^2}{z^5}\right)^{1/3} \quad \text{" " "}$$

equations (24) and (26) give

$$u \propto v \propto \omega \propto q^{1/3} \quad \text{for a line source}$$

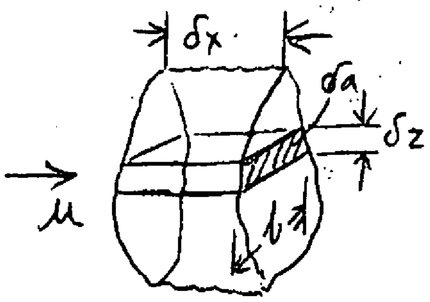
$$\theta \propto \left(\frac{q^2}{z}\right)^{1/3} \quad \text{" " "}$$

6. Effect of buoyancy on horizontal jets

Less well known is the method of calculating the effect of buoyancy on a hot horizontal jet; a detailed derivation is given by Thring and Horne[5].

The approximate analysis below is sufficient to demonstrate the mechanism of the problem, though density differences are neglected except in the buoyancy force.

Let θ (x, y) and u(x, y) be the temperature and velocity at any point (x, y). The force acting on the gas in the elementary section of thickness δx in Fig. 7 is



$$\delta F = g \rho_0 \int_A \frac{\theta}{T_j}\, \delta a \, . \, \delta x \qquad \ldots\ldots (27)$$

Fig. 7.

The mass of gas in the section δx is

$$\delta M = \rho_0 T_0 \int_A \frac{\delta a}{T_j}\, \delta x \qquad \ldots\ldots (28)$$

The mean radial acceleration is

$$f = \frac{1}{R} \frac{\int_A u^2 \frac{\delta a}{T_j}}{\int \frac{\delta a}{T_j}} \qquad \ldots\ldots (29)$$

where R is the radius of curvature of the jet centre line.

Now $$\delta F = \delta M . f.$$

Substituting for each of these three terms from equations (27), (28) and (29) we have, approximately,

$$\frac{1}{R} = \frac{d^2H}{dx^2} = K g \frac{\Theta_m}{T_o u_m^2} \qquad \ldots\ldots (30)$$

where K is a constant depending on the shape on the temperature and velocity profiles. H & x the coordinates of the centre line of the jet and Θ_m and U_m the mean values of θ and u across the jet. They are functions of 'x' - depending on what kind of jet it is. For a wide jet near the orifice they are in effect constants. This gives

$$K \doteq 1$$

and equation (30) integrates to give

$$H = \frac{g\theta}{2T_o u^2} x^2 \qquad \ldots\ldots (31)$$

where x is the distance along the jet from the source.

For a point source the continuity of momentum gives u^2x^2 constant and the continuity of heat flow $u\theta x^2$ constant

i.e. $$u_m \propto \theta_m \propto \frac{1}{x}$$

and equation (30) then gives

$$H \propto x^3$$

For a line source u^2x & $u\theta x$ are constant so that

$$u_m \propto \theta_m \propto x^{1/2}$$

and equation (30) gives

$$H \propto x^{5/2}$$

References

(1) Ede, A. J. "Natural Convection on Vertical Surfaces". Department of Scientific and Industrial Research Mechanical Research Laboratory. Heat Division No. 106.

(2) Eckert, E. R. G. "Introduction to the Transfer of Heat and Mass". McGraw-Hill, 1950.

References (contd.)

(3) Priestley, C. H. B. and Ball, F. K. "Continuous Convection from an Isolated Source of Heat". Quart. J. R. Met. Soc. 1955, 81 144.

(4) Batchelor, G. K. "Heat Convection and Buoyancy in Fluids". ibid 1954, 80 339.

(5) Thring, M. W. and Horne, G. R. J. Inst. Fuel. 1956, 29 437.

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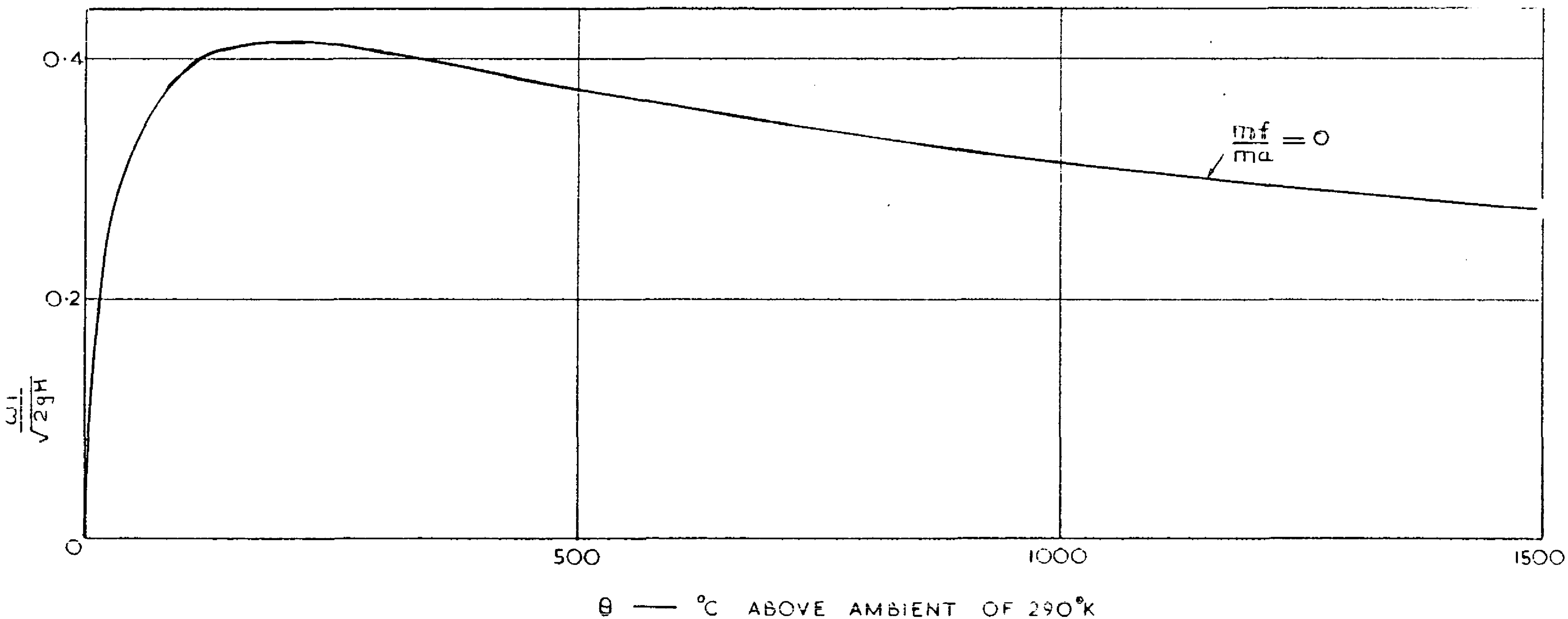

FIG. 4. CALCULATED VARIATION OF $\frac{w_1}{\sqrt{2gH}}$ WITH TEMPERATURE