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NOMOGRAMS FOR SOLVING ONE DIMENSIONAL STEADY STATE CONDUCTION PROBLEMS INVOLVING COOLING TO THE ATMOSPHERE

by

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Summary

The method of using nomograms for solving one dimensional steady state conduction problems, concerning walls and involving cooling to the atmosphere, is described. Examples concerning a composite wall are given.

1) Introduction

Although the equation of steady state heat conduction is simple, problems encountered in practice are usually not easy to solve, as cooling to the atmosphere is involved for which the relevent expression is somewhat complex.

In this note nomograms are described which will solve one dimensional problems, that is problems concerning large walls of uniform thickness, or, if they are composite, of uniform thicknesses of the different materials comprising the wall.

2) The steady state temperatures either side of a prescribed wall

When a wall is heated on one side by a constant temperature fire, the far side will eventually attain a steady temperature and the nomogram constituting Figure 1 relates the temperatures either side of the wall to the thermal resistance per unit area of the wall. The values attributed to certain constants to evolve it will be found in the Appendix.*

The variables involved are:-

(a) the temperature θ_2 (°C or °F) of the heated face, above the ambient (i.e. atmospheric) temperature,

(b) the temperature θ_1 (°C or °F) of the unheated face above the ambient (i.e. atmospheric) temperature and

(c) the thermal resistance R (°C cm^2 sec/cal or °F ft^2 hr/B.Th.U) of the wall per unit area.

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*The unheated surface of the wall is considered to have an emissivity factor of unity, which is almost always a valid approximation not merely of itself but also because cooling is by both radiation and convection.

If any two of these quantities be known then the third can be determined from the nomogram in the following way:- Lay a straight edge across the scales θ_1, θ_2, and R such that it intersects two of the scales at the values given for these quantities in the statement of the problem. Then the value of the third quantity will be that marked on the appropriate scale at the point of intersection with the straight edge.

In some cases the thermal conductivities (K_1, K_2 etc.) and thicknesses (l_1, l_2 etc.) of the various layers of different materials comprising the wall will have been specified in place of the thermal resistance per unit area (R). The value of R will then be given by the expression

$$R = R_1 + R_2 + R_3 + \text{------}$$

$$= \frac{l_1}{K_1} + \frac{l_2}{K_2} + \frac{l_3}{K_3} \text{------}$$

where the subscripts refer to the layers of material, l is thickness in cm or in and K is thermal conductivity in cal/sec/cm/°C or B.Th.U. in./hr/ft^2/°F.

K and l will generally be specified in British units (B.Th.U. in./ft^2/hr/°F and inches) and two additional scales will be found on the left of the nomogram of Figure 1 to give values of R (°C cm^2 sec/cal or °F ft^2 hr/B.Th.U.). For prescribed values of K and l, R may be found by laying a straight edge so that it intersects the K and l scales at the appropriate values. Then the value of R will be that marked on the R scale at the point of intersection with the straight edge.

Where a composite wall is being considered the value of R for each of the component layers must be determined as described and the results added. The right hand portion of the nomogram may then be used as previously described to give, say, a value of θ_2 if θ_1 has been given in the statement of the problem.

3) Example

A $4\frac{1}{2}$ inch brick wall of thermal conductivity 5·5 B.Th.U. in./ft^2/hr/°F is faced with half-an-inch of plaster of thermal conductivity 4·5 B.Th.U. in./ft^2/hr/°F. If a fire maintains the temperature of one side at 1,000°C above ambient temperature, what temperature will be eventually attained at the far side of the wall?

From the left hand side of Figure 1

R_{Brick} = 6,000 °C cm^2 sec/cal.
and $R_{Plaster}$ = 825 °C cm^2 sec/cal.

$\therefore$ Total R = 6,825 °C cm^2 sec/cal.

From the right hand side of Figure 1

θ_1 = 215°C rise above ambient.

4) Intermediate steady state temperatures

It is often necessary to know not merely the steady state temperatures either side of a wall but the temperature at some specified depth within the wall and such problems may be solved by using the nomogram of Figure 2.

The variables involved, as illustrated in Figure 2, are:-

(a) the temperature difference θ_{12} (°C or °F) across the whole wall (known).
(b) the temperature difference θ_a (°C or °F) across the particular thickness considered (unknown).
(c) the thermal resistance per unit area R_a (°C cm^2 sec/cal or °F ft^2 hr/B.Th.U.) of the thickness considered (known, or found by the method described in the second half of paragraph 2).
(d) the total thermal resistance per unit area R (°C cm^2 sec/cal or °F ft^2 hr/B.Th.U.) of the wall (known).

The expression relating the variables is

$$\theta_a = \frac{R_a}{R}\,\theta_{12}$$

or $$n = \frac{R_a}{R} \quad \text{and} \quad \theta_a = n\,\theta_{12}$$

The procedure for using the nomogram, which solves the simple equations above, is as follows. Firstly n is determined by laying a straight edge across the scales R_a, R and n such that it intersects the R_a and R scales at the specified values. Then the value of n will be that marked on the n scale at the intersection of the straight edge and the n scale.

It is possible that the values specified for R_a and R may not be found on the scales. In this case, the two scales (R_a and R) may be considered to be multiplied (or divided) by ten (or in fact any other number).

θ_a may be determined by laying the straight edge across the n, θ_{12} and θ_a scales, intersecting the first two at the appropriate values. The third point of intersection gives the value of θ_a. For certain values of θ and θ_{12} more accurate results may be obtained by considering the θ^a scales to be multiplied by 10.

5) Example

In the example of paragraph 3 find the temperature of the brick-plaster interface, supposing the value of θ_2 to have been measured at the exterior face of the brick.

Then $R_a = R_{Plaster} = 825$ °C cm^2 sec/cal.
and $R = R_{Total} = 6{,}825$ °C cm^2 sec/cal.

From the nomogram

$$n = 0.12$$

Considering the second part of the nomogram,

$$\theta_1 = 215°\text{C rise and } \theta_2 = 1{,}000°\text{C rise}$$

$$\therefore \quad \theta_{12} = 1{,}000° - 215° = 785°\text{C}$$

From the nomogram of Figure 2

$$\theta_a = 93°\text{C}$$

$\therefore$ the temperature of the interface, above ambient is $\theta_a + \theta_1 = 308°$C.

N.B. The value of θ_a was obtained by laying the straight edge against the mark 7,850 for θ_{12} instead of 785°C. The result therefore appeared as 930°C.

6) Rate of flow of heat

The steady state rate of flow of heat through a wall may be immediately obtained from the graph of Figure 3 provided θ_1 is known.

Thus, in the example given, the rate of flow of heat, since θ_1 = 215°C, is 0·116 cal/cm²/sec.

7) Acknowledgements

The work described in this paper forms part of the programme of the Joint Fire Research Organization of the Department of Scientific and Industrial Research and Fire Offices' Committee; the paper is published by permission of the Director of Fire Research.

8) References

1. The efficient use of fuel p 130. Ministry of Fuel and Power. (London, 1944) H.M. Stationery Office.

Appendix

The nomogram of Figure 1 solves the equation

$$\frac{\theta_2 - \theta_1}{R} = \sigma(\theta_1 + T_o)^4 - \sigma T_o^{\,4} + H\,\theta_1^{\,1\cdot25} \quad \ldots\ldots \quad (1)$$

where θ_2 = temperature of heated surface above ambient temperature (°C)
θ_1 = temperature of unheated surface above ambient temperature (°C)
T_o = ambient temperature (degrees absolute)
= 290°K (= 17°C)
R = thermal resistance per unit area (°C cm² sec/cal)
σ = Stefan-Boltzmann constant
= $1\cdot37 \times 10^{-12}$ cal/cm²/sec/°C⁴

and H = convection coefficient (1)
= $4\cdot7 \times 10^{-5}$ cal/cm²/sec/°C$^{1\cdot25}$

Expression 1 relates the heat flowing through the wall (L.H.S.) to the cooling of the unheated face to the atmosphere by radiation and convection. It has been assumed that the emissivity factor of the unheated face is approximately unity. This is true for the radiation of heat by most building materials and is further justified as an approximation by the fact that the right hand side of expression 1 contains an additional term ($H\,\theta_1^{\,1\cdot25}$) which is of the same order of magnitude as the sum of the terms representing radiation.

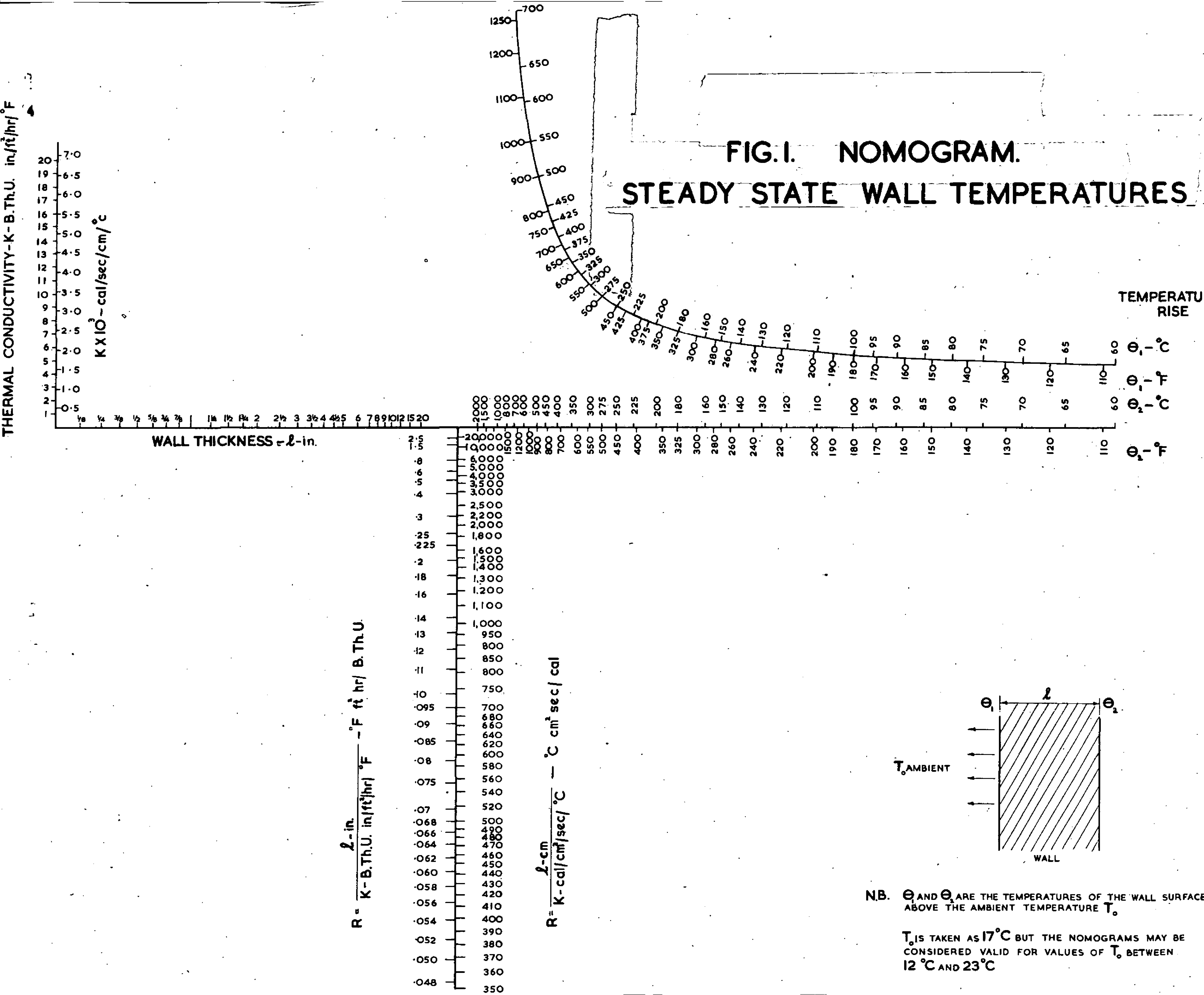

FIG. 1. NOMOGRAM.

STEADY STATE WALL TEMPERATURES

N.B. Θ_1 AND Θ_2 ARE THE TEMPERATURES OF THE WALL SURFACES ABOVE THE AMBIENT TEMPERATURE T_o

T_o IS TAKEN AS 17°C BUT THE NOMOGRAMS MAY BE CONSIDERED VALID FOR VALUES OF T_o BETWEEN 12 °C AND 23°C

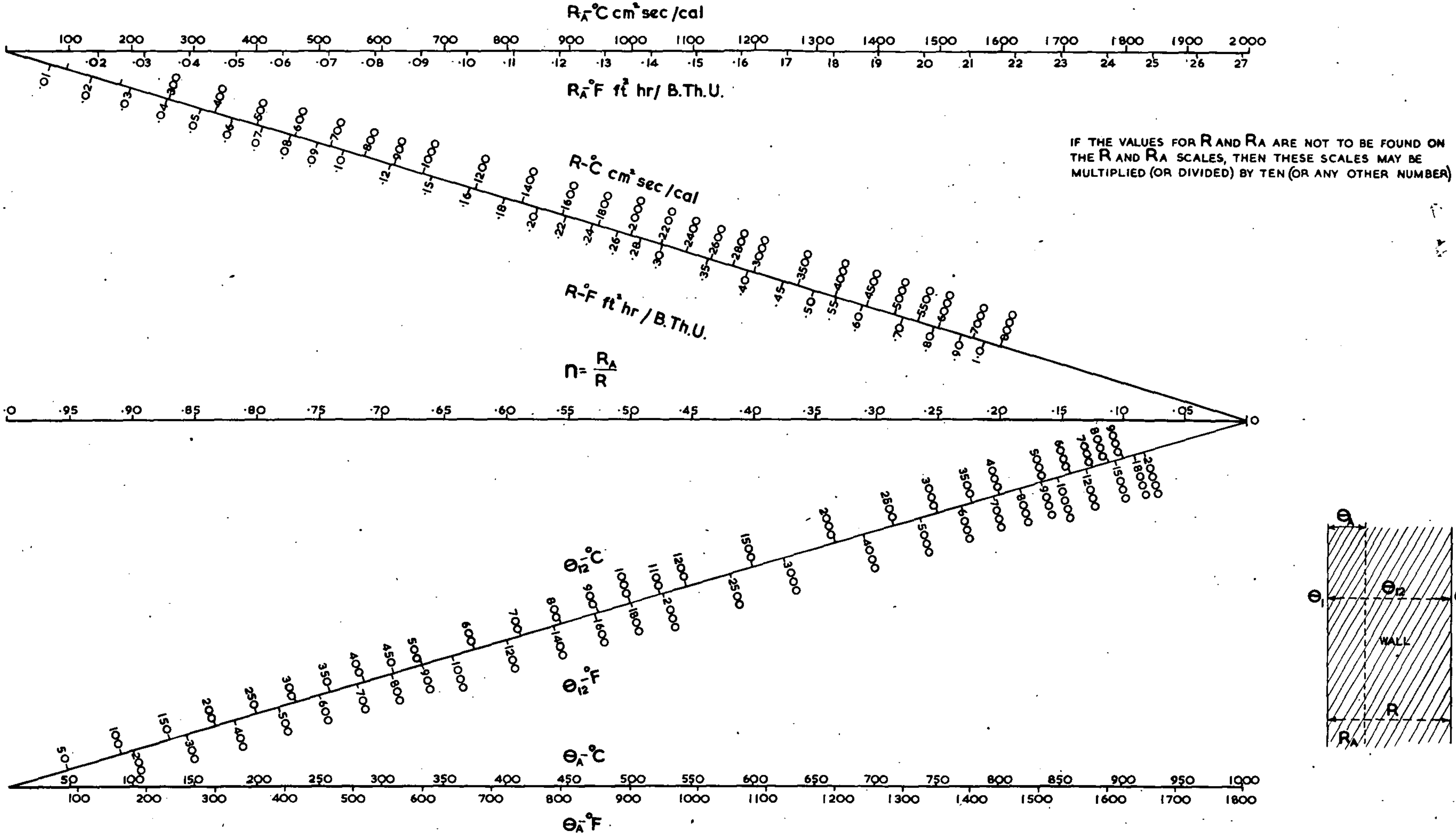

FIG.2. NOMOGRAM FOR INTERMEDIATE TEMPERATURES IN WALLS

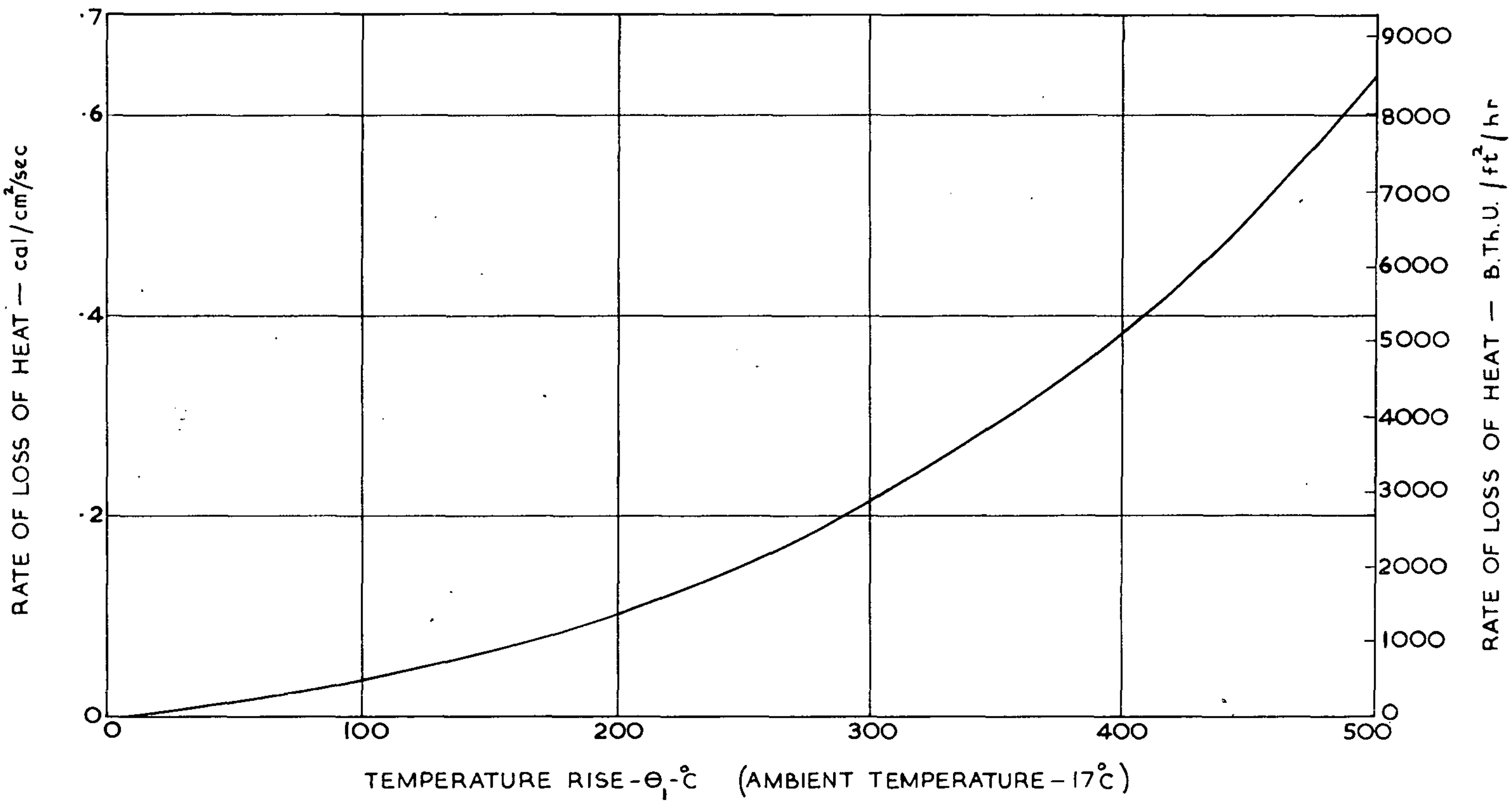

FIG. 3. COOLING TO THE ATMOSPHERE FROM A VERTICAL SURFACE