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SPONTANEOUS IGNITION IN SLABS WITH FACES AT DIFFERING TEMPERATURES

by

P. C. Bowes and P. H. Thomas

Summary

The theory of thermal explosion is extended to ignition in
self-heating slabs with one face maintained at a relatively high
temperature and the other face cooling to the surroundings. The
theory is of use in connection with ignition in dust layers on hot
surfaces and ignition in combustible lagging. Wider applications
are considered.

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1. INTRODUCTION

Critical conditions for spontaneous ignition or thermal explosion in bodies generating heat and exposed to surroundings at constant uniform temperature, have been obtained mathematically by Frank-Kamenetsky [1][2] and Chambré [3] for bodies whose surface temperature remains equal to the ambient temperature, and by Thomas [4] for bodies whose surface temperature is governed by Newtonian cooling.

The present note extends the analysis used by the above authors to the spontaneous ignition of a plane slab, one of whose faces is maintained at a constant high temperature while the other cools to the atmosphere.

The solution of the problem is approximate and has been possible only for a limited range of the variables, but it appears to be adequate for the applications contemplated. These applications are, primarily, the ignition of fairly thick layers of dusts on hot surfaces and the ignition of combustible lagging.

A question of more general importance is the relationship between conditions for ignition in a given material in different environments. In particular, there is the possibility of using data relating to the ignition of a slab of material in contact with a hot surface to predict the conditions for ignition of a body of the material exposed to a uniform ambient temperature, and vice versa. This question is considered in part in this note.

The validity of an approximation used in the analysis has been assessed by reference to experiments on the ignition of wood fibre insulating board.

2. THEORETICAL

2 (i) General

We consider a plane slab one surface of which is maintained at a constant temperature T_p, greater than the atmospheric temperature T_0, while the other surface is exposed to the atmosphere and assumes a temperature T_s, between T_p and T_0, which depends on the heat balance at the surface.

An exothermic reaction takes place in the material of the slab. It is assumed that at any point in the slab the rate of the reaction depends solely on the temperature at that point, and that the rate is related to the temperature by the Arrhenius equation. We then have

$$q = Qf \, e^{-\frac{E}{RT}} \qquad \dots\dots\dots\dots\dots\dots(1)$$

where

q = rate of heat evolution per unit mass (cal/g/sec),
Q = heat of reaction per gram,
f = pre-exponential constant of Arrhenius equation,
E = apparent activation energy (cal/mol),
T = absolute temperature ($^{\circ}$K),
R = gas constant (cal/mol/$^{\circ}$K).

For heat transfer by conduction, the temperature distribution in the slab in the steady state, when the rate of heat generation is equal to the rate of heat loss to the surroundings, will be given by solution of

$$\frac{d^2T}{dx^2} = \frac{Qf\rho}{K} \, e^{-\frac{E}{RT}} \qquad \dots\dots\dots\dots\dots\dots(2)$$

for the boundary conditions

$$x = 0, \qquad T = T_p \qquad \qquad \dots\dots\dots\dots\dots\dots(3i)$$

and

$$x = 2r, \qquad \frac{-K \, d \, T_s}{dx} = H(T_s - T_o)$$
$$= H(T_s - T_p) - H(T_o - T_p) \dots\dots\dots\dots\dots(3ii)$$

where

x = co-ordinate normal to surface of slab (cm),
r = semi-thickness of slab[*] (cm),
ρ = density of slab (g/cm^3),
K = thermal conductivity of slab (c.g.s. units),
H = overall coefficient of heat transfer between slab and surroundings (c.g.s. units).

It is required to find the limiting conditions at which a steady state ceases to exist in the slab and the temperature rises indefinitely to ignition.

Following Frank-Kamenetsky, equation (2) and the boundary conditions are put into dimensionless form as follows:-

$$\frac{d^2\theta}{d \, z^2} = - \delta e^\theta \qquad \qquad \dots\dots\dots\dots\dots\dots(4)$$

$$z = 0, \qquad\qquad \theta = 0 \qquad\qquad \dots\dots\dots\dots\dots\dots(5i)$$

$$z = 2, \qquad -\frac{d \, \theta}{d \, z} = \propto\theta_s - \propto\theta_o \qquad \dots\dots\dots\dots\dots(5ii)$$

where $\quad z = \dfrac{x}{r}$ $\qquad\qquad\qquad\qquad\qquad\qquad$ $\dots\dots\dots\dots\dots(5iii)$

$$\theta = \frac{E}{R \, T_p^2}(T - T_p) \qquad\qquad \dots\dots\dots\dots\dots(5iv)$$

$$\delta = \frac{Q \, f \rho \, E \, r^2 \, e^{-\frac{E}{R \, T_p}}}{K \, R \, T_p^2} \qquad\qquad \dots\dots\dots\dots\dots(5v)$$

$$\propto = \frac{H \, r}{K} \qquad\qquad\qquad\qquad \dots\dots\dots\dots\dots(5vi)$$

The definition of the dimensionless temperature, θ, is an approximation that neglects the second and higher powers of $(T - T_p)/T_p$ in the expansion of the right hand side of the identity

$$\frac{E}{R \, T} \equiv \frac{E}{R \, T_p}\left(1 + \frac{T - T_p}{T_p}\right)^{-1}$$

and it is strictly valid only for $(T - T_p) \ll T_p$.

However, in many applications for which this analysis is intended $(T - T_p)$ is of order $0.5 \, T_p$. The magnitude and effect of the error thereby introduced is considered later. The immediate justification for use of the approximation is that, in the region where it ceases to hold, the rate of heat production is negligible compared with the rate in the region where the approximation is good.

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[*] The semi-thickness is introduced here for the sake of conformity with the equations for bodies in surroundings symmetrical with respect to temperature.

The limiting conditions for ignition will be determined by the maximum possible values of the parameter δ, for given values of α and θ_0.

On physical grounds we may expect the temperature in the slab, in the steady state, to have a maximum; and this will be defined by

$$\frac{d\theta}{dz} = 0 \text{ when } \theta = \theta_m = \text{maximum.}$$

The solution of equation (4), subject to this condition, is

$$\theta = \log_e A - 2 \log_e \cosh\left[\sqrt{\frac{\delta A}{2}} \; (z - C)\right] \qquad \dots\dots\dots(6)$$

where A and C are integration constants, of which $A = e^{\theta_m}$

If $z = z_m$ when $\theta = \theta_m$, we have $C = z_m$.

Inserting the boundary condition (5i) and (6) gives

$$A = \cosh^2 (C\sqrt{\delta A/2}) \qquad \dots\dots\dots(7)$$

Differentiating (6) with respect to z,

$$\frac{d\theta}{dz} = -\sqrt{2\delta A} \; \tanh\left[\sqrt{\frac{\delta A}{2}} \; (z - C)\right] \qquad \dots\dots\dots(8)$$

Combining (8) with (5ii) and (6) we have

$$\frac{P}{\sqrt{2\delta}} e^{- (\frac{P\emptyset}{2\alpha} + \frac{\theta_0}{2})} = \cosh(P - \frac{CP}{2}) \qquad \dots\dots\dots(9)$$

where $P = \sqrt{2\delta A}$ and $\emptyset = \tanh (P - \frac{C P}{2})$

Eliminating A between P and (7) we have

$$\cosh \frac{C P}{2} = \frac{P}{\sqrt{2\delta}} ,$$

whence,

$$\sinh \frac{CP}{2} = (\cosh^2 \frac{CP}{2} - 1)^{\frac{1}{2}} = (\frac{P^2}{2\delta} - 1)^{\frac{1}{2}}$$

Substituting these expressions in (9) we obtain, finally

$$\delta = \frac{P^2}{\sinh^2 P} e^{- (\frac{P\emptyset}{2\alpha} + \frac{\theta_0}{2})} \left[\cosh P - \cosh (\frac{P\emptyset}{2\alpha} + \frac{\theta_0}{2})\right] \dots\dots\dots(10)$$

In order to proceed further it is necessary to introduce restrictions on the range of some of the quantities in equation (10). Thus, we suppose P is sufficiently large, and C sufficiently small, for $\emptyset$ to be unity; this is justified later.

Differentiating equation (10) with respect to P (with $\emptyset = 1$) and equating to zero, we find δ is a maximum when

$$2(1 - P \coth P) \left[\cosh P - \cosh (\frac{P}{2\alpha} + \frac{\theta_0}{2})\right] + \frac{P}{2\alpha} e^{-(\frac{P}{2\alpha} + \frac{\theta_0}{2})}$$
$$+ (1 - \frac{\coth P}{2\alpha}) P \sinh P = 0 \qquad \dots\dots\dots(11)$$

Since we are restricted to values of P large enough for $\emptyset = 1$, we may put coth P = 1 in (11).

For given values of θ_0 and $\propto$, the value of P that satisfies equation (11) can be found by trial, and the corresponding value of δc can be obtained by inserting the θ_0, $\propto$ and P in equation (10). This has been done for values of $\propto$ between 0.5 and ∞, and θ_0 between -8 and -18; these values being chosen as covering the probable range that will be encountered in the applications outlined in the introduction. Calculated values of δc are tabulated in Table 1, and plotted in Fig. 1 with a logarithmic scale for $\propto$. It may be noted that the lowest value of $\emptyset$ in the range covered was 0.99 at $\theta_0 = -10$, $\propto = 0.5$. It thus happens that the restriction $\emptyset = 1$ is not unduly severe from the point of view of practical application. The range may, if necessary, be extended down to about $\theta_0 = -4$ for high $\propto$. This may cover ignition in thin layers of highly reactive materials at low hot-surface temperatures.

Values of A, θ_m and C have also been calculated and are given in Table 1. In the ranges of $\propto$ and θ_0 covered, $\sqrt{A}$, and hence θ_m, appears to be independent of $\propto$ for given θ_0, but decreases as $|\theta_0|$ increases. C, which defines the plane in which $\theta = \theta_m$, varies with both θ_0 and $\propto$.

2 (ii) Value of δ_c when $\propto = 0$

When II = 0 and, hence, $\propto = 0$ we have, from condition (5ii),

$$\frac{d\theta}{dz} = 0 \text{ at } z = 2,$$

i.e. from equation (6)

$$\theta = \theta_m \text{ at } z_m = 2 = C.$$

Equation (7) then becomes

$$\sqrt{A} = \cosh \sqrt{2\delta}/A$$

The maximum value of $\sqrt{2\delta}$ for which this equation has a solution is 0.663. Hence, when $\propto = 0$,

$$\delta_c = 0.22 \text{ and } \theta_m = 1.19$$

δ_c here is exactly one quarter of the value of δ_c for a slab with both faces at equal fixed temperatures; the present slab, when $\propto = 0$, representing one half of the symmetrically heated slab

3. APPLICATION OF THEORY TO IGNITION EXPERIMENTS

3 (i) General

It follows from the definition of δ, equation (5v), that a plot of $\log_e (\delta_c T_p^2/r^2)$ vs. $1/T_p$ will be linear with slope $- E/R$; and further, substitution of equation (1) into the definition of δ provides an expression for the rate of heat evolution at temperature T_p.

Thus,

$$q(T_p) = \frac{K R \delta_c T_p^2}{\rho E r^2} \qquad \dots\dots\dots\dots(12)$$

Mitchell's [5] results on the ignition of fibreboard blocks exposed to uniform ambient temperatures have been shown to obey the above linear relationship [6], and results on the ignition of dust layers on a hot surface (to be described elsewhere) also obey it. T_p is now the minimum hot-surface temperature, or minimum ambient temperature, for ignition and is close to the maximum hot surface, or ambient, temperature for a steady state.

TABLE 1 (PART 1)

IGNITION OF SLABS WITH FACES AT DIFFERING TEMPERATURES. CRITICAL CONSTANTS

θ_0	α	P	δ_c	$\sqrt{A}$	A^*	θ_m^*	C^*	θ_0	α	P	δ_c	$\sqrt{A}$	A^*	θ_m^*	C^*
-8	0.5	–	–	–			–	-12	0.5	3.44	5.74	1.015			0.098
	1.0	3.33	5.18	1.035			0.155		1.0	4.59	10.25	1.015			0.074
	2.5	4.16	8.08	1.036			0.124		2.5	5.73	16.00	1.013			0.059
	5.0	4.52	9.52	1.035			0.114		5.0	6.26	19.00	1.015			0.054
	10.0	4.74	10.55	1.031			0.108		10.0	6.55	20.85	1.014			0.052
	20.0	4.85	11.04	1.032			0.106		20.0	6.71	21.87	1.014			0.051
	∞	4.98	11.65	1.032			0.103		∞	6.88	23.0	1.015			0.049
			Mean	1.034	1.068	0.066					Mean	1.014	1.029	0.028	
-10	0.5	2.96	4.19	1.022			0.139	-13	0.5	3.68	6.62	1.012			0.087
	1.0	3.95	7.46	1.022			0.104		1.0	4.91	11.72	1.016			0.066
	2.5	4.94	11.65	1.024			0.083		2.5	6.14	18.3	1.013			0.053
	5.0	5.38	13.88	1.020			0.076		5.0	6.69	21.9	1.010			0.048
	10.0	5.64	15.28	1.018			0.073		10.0	7.01	24.0	1.012			0.046
	20.0	5.74	16.02	1.022			0.071		20.0	7.19	25.1	1.014			0.045
	∞	5.92	16.80	1.021			0.069		∞	7.36	26.4	1.016			0.044
			Mean	1.021	1.042	0.041						1.013	1.026	0.026	
-11	0.5	3.20	4.93	1.020			0.119	-14	0.5	3.92	7.53	1.010			0.076
	1.0	4.26	8.76	1.018			0.089		1.0	5.24	13.42	1.012			0.056
	2.5	5.33	13.70	1.019			0.071		2.5	6.54	21.05	1.010			0.045
	5.0	5.83	16.35	1.019			0.065		5.0	7.13	25.00	1.009			0.042
	10.0	6.09	17.91	1.018			0.062		10.0	7.48	27.35	1.012			0.040
	20.0	6.24	18.8	1.016			0.061		20.0	7.66	28.75	1.010			0.039
	∞	6.40	19.8	1.018			0.059		∞	7.84	30.08	1.011			0.038
			Mean	1.018	1.037	0.036					Mean	1.011	1.022	0.022	

$*$ Calculated from mean value of $\sqrt{A}$

TABLE 1 (PART II)

IGNITION OF SLABS WITH FACES AT DIFFERING TEMPERATURES.   CRITICAL CONSTANTS

θ_0	α	ρ	δ_c	$\sqrt{A}$	A^*	θ_m^*	c^*	θ_0	α	ρ	δ_c	$\sqrt{A}$	A^*	θ_m^*	c^*
-15	0.5	4.17	8.53	1.010			0.069	-17	0.5	4.66	10.50	1.008			0.055
	1.0	5.56	15.19	1.010			0.052		1.0	6.21	19.00	1.007			0.042
	2.5	6.95	23.56	1.011			0.041		2.5	7.77	29.70	1.009			0.033
	5.0	7.61	28.26	1.011			0.038		5.0	8.47	35.43	1.007			0.030
	10.0	7.94	30.94	1.010			0.036		10.0	8.88	38.84	1.009			0.029
	20.0	8.14	32.50	1.010			0.035		20.0	9.1	40.65	1.009			0.028
	∞	8.34	34.12	1.010			0.034		∞	9.32	42.66	1.009			0.028
			Mean	1.010	1.021	0.020					Mean	1.008	1.017	0.017	
-16	0.5	4.41	9.63	1.007			0.061	-18	0.5	4.91	11.89	1.007			0.047
	1.0	5.89	17.06	1.010			0.045		1.0	6.54	21.15	1.005			0.035
	2.5	7.36	26.60	1.010			0.036		2.5	8.19	33.02	1.009			0.028
	5.0	8.03	31.70	1.009			0.033		5.0	8.92	39.28	1.007			0.026
	10.0	8.41	34.74	1.009			0.032		10.0	9.35	43.15	1.006			0.025
	20.0	8.61	36.50	1.008			0.031		20.0	9.58	45.3	1.007			0.024
	∞	8.82	38.38	1.009			0.030		∞	9.81	47.5	1.006			0.024
			Mean	1.009	1.019	0.018					Mean	1.007	1.014	0.014	

* Calculated from mean value of $\sqrt{A}$.

At the least the theory provides a linear relationship, based on a simple model of the self-heating process, for correlating the results of ignition experiments. The theory will be of more general interest, as indicated in the introduction, if it can be shown that the values of the activation energy E and the rate of heat evolution at a given temperature q (or, more generally, the product Q f in equation (1)) yielded by the theory are truly characteristic of a given material and are independent of the conditions under which ignition experiments are carried out. If this is so, it will be possible to predict ignition temperatures under "hot surface" conditions from "oven" experiments and vice versa. The possibility of this general application of the theory depends on two considerations:- .

(a)   The validity of the simple model on which the theory is based;
        i.e. a rate of reaction depending solely on temperature and
        obeying the Arrhenius equation, and heat transfer by conduction.

(b)   the error introduced, in the theory for slabs with faces at
        differing temperatures, by the approximation in the definition of
        the dimensionless temperature, θ (equation 5iv)).

In what follows we are primarily concerned with (b). The error is estimated by applying the approximate theory to the results of some ignition experiments with wood fibreboard, and then making a comparison with a numerical integration of the exact equation (2) appropriate to the conditions of the experiments.

3 (ii) Experimental

Ignition temperatures were determined for two different thicknesses of wood fireboard, in pieces 6 in. square, in contact with a horizontal metal surface capable of being maintained at any desired tamperature to within 1°C. A fine thermocoupe was stretched across the centre of the upper face of the fibreboard specimens. The fibreboard was oven dried at 105°C for two hours in order to prevent it curling during tests.

If ignition occurred at a given temperature of the hot surface, and no ignition occurred at a temperature 5° lower, then the upper temperature was regarded as the ignition temperature.

3 (iii) Results

The experimental results are given in Table 2 together with the calculated activation energy and rates of heat evolution. Details of the calculations are given in Appendix 1. The rate of heat evolution at 250°C has been calculated for comparison with earlier results.

TABLE 2

Ignition of wood fibreboard on hot surface

Thickness of fibreboard, cm.	1.27	2.54
Ignition temperature, °C.	305	275
Indicated temperature of cool surface, °C.	90	60
Activation energy, cal/mol.	27,800	
Rate of heat evolution, cal/g/min, at 305°C	24.60	
at 275°C	6.64	
at 250°C	1.96	
Value of product Qf (equation 1)	1.17×10^{10}	

3 (iv) Temperature distribution in steady state

Assuming that a steady state temperature distribution is possible in the fibreboard with the hot surface at the ignition temperatures given in Table 2, certain features of this temperature distribution, and some associated quantities, have been calculated and listed in Table 3. The values are obtained quite simply from equations in section 2 (i); details are given in Appendix 2.

Since δ_c varies rapidly with α, at low α, it is necessary to calculate α with fair accuracy. This requires a knowledge of the temperature of the cooling surface, which initially can be calculated only by assuming that the slab of fibreboard is inert and does not generate heat.

Where appropriate, the quantities given in the Table for the reactive fibreboard are compared with corresponding quantities calculated on the assumption that the fibreboard is inert.

The Table is self-explanatory except perhaps for the entry called "Maximum temperature rise due to reaction". This is calculated at the plane in the slab where the temperature exceeds, by the greatest amount, the temperature that would exist in that plane by virtue of the temperature gradient $(T_s - T_p)/2r$ if the fibreboard were inert; the maximum temperature rise is the value of this excess.

TABLE 3

Steady-state temperature distribution in fibreboard

Thickness of slab, cm.	1.27	2.54
Temperature of hot surface, °C.	305	275
Maximum temperature in slab, °C.	305.77	275.65
Position of maximum* cm.	0.04	0.08
Temperature gradient at cool surface, °C/cm.		
Reactive	-197	-98
Inert	-173	-85
Heat flux at cool surface, cal/sec/cm².		
Reactive	23.6×10^{-3}	11.7×10^{-3}
Inert	20.7×10^{-3}	10.2×10^{-3}
Temperature at cool surface, °C.		
Reactive	93	63
Inert	86	60
α. Dimensionless.		
Reactive	1.84	3.28
Inert	1.80	3.07
Maximum temperature rise due to reaction, °C.	29	26
Position of maximum temperature rise*, cm.	0.35	0.64

* measured from hot surface

There are three main points of interest arising from the data in Table 3. First, the maximum temperature is less than 1°C above the temperature of the hot surface and the maximum occurs close to the hot surface. Second, the heat flux at the cool surface due to the reaction, (e.g. $23.6 \times 10^{-3} - 20.7 \times 10^{-3} = 2.9 \times 10^{-3}$ cal/sec/cm² for the slab 1.27 cm. thick), is not large compared with the total flux, and the steady state is clearly associated with a relatively small amount of self heating. Third, the value of α, calculated on the assumption that the fibreboard is inert, is adequate for practical purposes.

It may be noted that as the thickness of the slab is increased, the heat flux at the cool surface due to reaction will become an increasing fraction of the total flux and α, which will also increase, will be subject to increasing error. At the same time, however, it is evident from Fig. 1 that, as α increases, errors in α become less important.

3 (v) Integration of exact equation for temperature distribution

For the conditions

$$T = T_p \text{ at } x = 0,$$
$$T = T_s \text{ at } x = r,$$
$$T = T_m \text{ (maximum)}, \frac{dT}{dx} = 0,$$

the solution of the exact equation (2), obtained by multiplying the equation by dT/dx and integrating twice, may be expressed as follows:-

$$\int_{T_s}^{T_m}\left(\int_{T_s}^{T_m} e^{-E/RT} dT\right)^{-\frac{1}{2}} dT + \int_{T_p}^{T_m}\left(\int_{T_p}^{T_m} e^{-E/RT} dT\right)^{-\frac{1}{2}} dT = \left(\frac{8Qf\rho r^2}{K}\right)^{\frac{1}{2}} \dots (13)$$

Inserting experimental values of T_p, and values of T_m and T_s calculated on the basis of the approximate equation (4), the left-hand side of the above has been integrated numerically and the equation solved for the product Qf. For the two sets of temperature limits available from the tests on the two thicknesses of fibreboard, we find:-

Fibreboard 2.5 cm. thick, Qf $= 0.93 \times 10^{10}$
Fibreboard 1.27 cm. thick, Qf $= 0.91 \times 10^{10}$

T_m, calculated on the basis of the approximate theory, will be subject to very little error; and, because $T_m - T_s$ is large, the integration of $e^{-E/RT}$ between these limits is not sensitive to error in the lower limit T_s. Hence, we may expect the error in Qf to be small.

Rates of heat evolution at given temperatures calculated from equation (1) with the mean value of Qf (above) inserted are given in Table 4, where the figures in brackets are the values obtained previously from equation (12).

TABLE 4

Temperature °C	Rate of heat evolution cal/g/min
305	19.80 (24.60)
275	5.30 (6.64)
250	1.55 (1.96)

It can be shown that the error, ΔT, in ignition temperature calculated from (5v), due to an error Δ (Qf) in Qf, is given by

$$\frac{\Delta T}{T} \approx \frac{T \, \Delta(Qf)}{Qf(2T - E/R} \approx \frac{RT}{E} \cdot \frac{\Delta(Qf)}{Qf}.$$

Since, here, $\Delta(Qf) = 0.25 \times 10^{10}$ the error in calculating the ignition temperature of a block of fibreboard, exposed to a uniform ambient temperature, with data obtained from ignition in contact with a hot surface, and using the approximate theory, will be about 1 per cent °K; i.e. about 4°C at temperatures near 200°C. An error of this order is quite acceptable for practical purposes.

3 (vi) Validity of Model

The rate of heat evolution at 250°C calculated for the fibreboard used in these tests is one-fifth of the rate calculated by Thomas [6] from Mitchell's ignition results, [5] and the activation energy is slightly higher. These differences may be due to differences in the fibreboard and do not necessarily afford an indication of the validity or otherwise of the model on which the theory is based, or of the possibility of relating the two types of ignition experiment.

However, rates of heating calculated from Mitchell's ignition experiments differ considerably from rates calculated from his self-heating tests in which no ignition occurred [6]. The reason for these differences is not clear; but it does not depend on the mathematical analysis, and it suggests that the model may be inadequate. This question is being investigated further experimentally.

4. CONCLUSIONS

1. It has been possible to extend the thermal theory of explosion to the spontaneous ignition of a slab with one face maintained at a relatively high temperature, and the other face cooling to the surroundings, for a limited though useful range of the variables involved.

2. Provided the simple model of the ignition process on which the theory is based is valid, it should be possible to calculate ignition temperatures for the ignition of a given material, in a uniform ambient temperature, with the aid of data obtained from experimental ignition of the material on a hot surface (and vice versa), to within about 1 per cent (°K); i.e. about 4°C at 200°C.

ACKNOWLEDGMENTS

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Part of Table 1 was calculated by Miss P. M. Cheshire.

5. REFERENCES

1. FRANK-KAMENETSKY, D. A. On the mathematical theory of thermal explosions. Acta phys-chim. U.R.S.S. 1942, 16 (5 - 6), 357-61.

2. FRANK-KAMENETSKY, D. A. Diffusion and heat exchange in chemical kinetics. Princeton University Press. 1955.

3. CHAMBRÉ, P. L. On the solution of the Poisson-Boltzmann equation with application to the theory of thermal explosions. J. Chem. Phys., 1952, 20, 1795.

4. THOMAS, P. H. On the thermal conduction equation for self-heating materials with surface cooling. Trans. Farad. Soc. 1958, 54 Part 1, 60.

5. MITCHELL, N. D. New light on self-ignition. National Fire Protection Association, Quarterly, October 1951, 3.

6. THOMAS, P. H. Comments on experiments on the self-heating of fibre insulating board. Department of Scientific and Industrial Research and Fire Offices' Committee. Joint Fire Research Organization, F.R. Note No. 306/1957.

7. MCADAMS, W. H. Heat Transmission. New York, 1942, McGraw-Hill Book Company Inc.

APPENDIX 1

Details of calculation of activation energy, etc.

1. Properties of wood fibreboard

Density ... ...(7)...	0.23 g/cm^3.
Thermal conductivity (7)..	1.2×10^{-4} c.g.s. units.
Emissivity	assumed unity.

2. Calculation of α

The fibreboard is assumed to be inert so that the heat flux at the cool surface is given by the simple conduction equation. With this assumption the cool surface temperature is determined by graphical solution of the following equation for the thermal balance at the cool surface:-

$$\frac{K(t_p - t_s)}{2\,r} = \sigma\left[(t_s + 273)^4 - (t_o + 273)^4\right] + h_c(t_s - t_o) \ldots\ldots\ldots\ldots(17)$$

where

$t_p,\ t_s,\ t_o$ = temperatures of hot surface, cool surface, and surroundings respectively, °C.

σ = Stefan-Boltzmann constant, c.g.s. units.
h_c = convective heat transfer coefficient.

The following expression for h_c, for horizontal plates facing upwards, has been used:-

$$h_c = 0.38\,(t_s - t_o)^{0.25} \times 1.36 \times 10^{-4} \text{ cal/cm}^2/\text{sec}/°C \ (7)$$

In the above expressions the temperature of the surroundings was taken as 25°C.

α is obtained from

$$\alpha = \frac{t_p - t_s}{2(t_s - t_o)}$$

3. Activation energy

We have

$$E = 2.303\,R\,\frac{\log_{10}(\frac{\delta_c\,T_p^2}{r^2})_1 - \log_{10}(\frac{\delta_c\,T_p^2}{r^2})_2}{(1/T_p)_2 - (1/T_p)_1}$$

where the subscripts 1 and 2 refer to the two thicknesses of fibreboard.

It is necessary, first, to assume a value of E in order to calculate θ_o and so obtain δ_c from Fig. 1. The value of E calculated from the above equation is then used to correct θ_o and δ_c. Further approximations may be made if desired, but will usually be unnecessary since δ_c appears in the logarithms and E is therefore relatively insensitive to systematic errors in δ_o.

The rate of heat evolution is obtained from equation (12) and converted to cal/g/min.

4. Dimensionless quantities

   The values of the dimensionless quantities appearing in the analysis and used in these and later calculations are tabulated below for the two thicknesses of fibreboard.

Fibreboard thickness, cm.	2.54	1.27
Dimensionless quantity:-		
α	3.1	1.8
θ_o	-11.5	-11.6
δ_c	15.8	13.2
p	5.75	5.20
θ_m	0.032	0.032
C	0.062	0.070

Temperature distribution in steady state

(a) Maximum temperature.

$$t_{max} = \frac{R\,T_p^2}{E}\,\theta_m + T_p \,, \quad {}^{o}C$$

(b) Position of maximum.

$$x = r\,C, \quad cm$$

(c) Temperature gradient at cool surface.

$$\frac{d\,t_s}{d\,x} = \frac{R\,T_p^2}{r\,E}\,\frac{d\,\theta\,s}{d\,z} \,, \quad {}^{o}C/cm.$$

where $\frac{d\,\theta\,s}{d\,z}$ is calculated from equation (8) for $z = 2$.

For inert slab,

$$\frac{d\,t_s}{d\,x}_{inert} = \frac{t_s - t_p}{2\,r} \quad {}^{o}C/cm.$$

(d) Flux (F_s) at cool surface.

$$F_s = -K\frac{d\,t_s}{d\,x}, \quad cal/sec/cm^2$$

(e) Temperature of cool surface.

Obtained by substituting flux F_s in left-hand side of equation (17)

and solving for t_s graphically.

(f) $\propto$ for reactive slab.

From boundary condition (5ii) we obtain

$$-\frac{R\,T_p^2}{E}\,\frac{d\,\theta s}{d\,z} = \propto(t_s - t_o)$$

whence

$$\propto = -\frac{r}{t_s - t_o}\cdot\frac{d\,t_s}{d\,x}$$

(g) Maximum temperature rise due to reaction.

If the slab were inert the steady state temperature distribution would
be linear with gradient

$$\frac{T_s - T_p}{2\,r}$$

If T_1 is the temperature at x we have

$$\frac{T_1 - T_p}{T_s - T_p} = \frac{x}{2\,r} = \frac{z}{2}$$

Converting to dimensionless form we have

$$\theta_1 = \frac{z}{2}\,\theta_s$$

In the presence of reaction the temperature, θ , at x is given by equation (6). Hence

$$\theta - \theta_1 = -\frac{z}{2}\ \theta_s + \log_e A - 2\log_e \cosh \frac{P}{2}\ (z - C) \quad \dots\dots\dots\dots(18)$$

This difference is a maximum when

$$\frac{\theta_s}{2} + P \tanh \frac{P}{2}\ (z - C) = 0 \quad \dots\dots\dots\dots(19)$$

For given θ_s, P and C we solve (19) for z and then obtain $(\theta - \theta_1)_{max}$ by inserting these values in (18).

Then $(T - T_1)_{max} = \frac{R\,T_p^2}{E}\ (\theta - \theta_1)_{max}.$

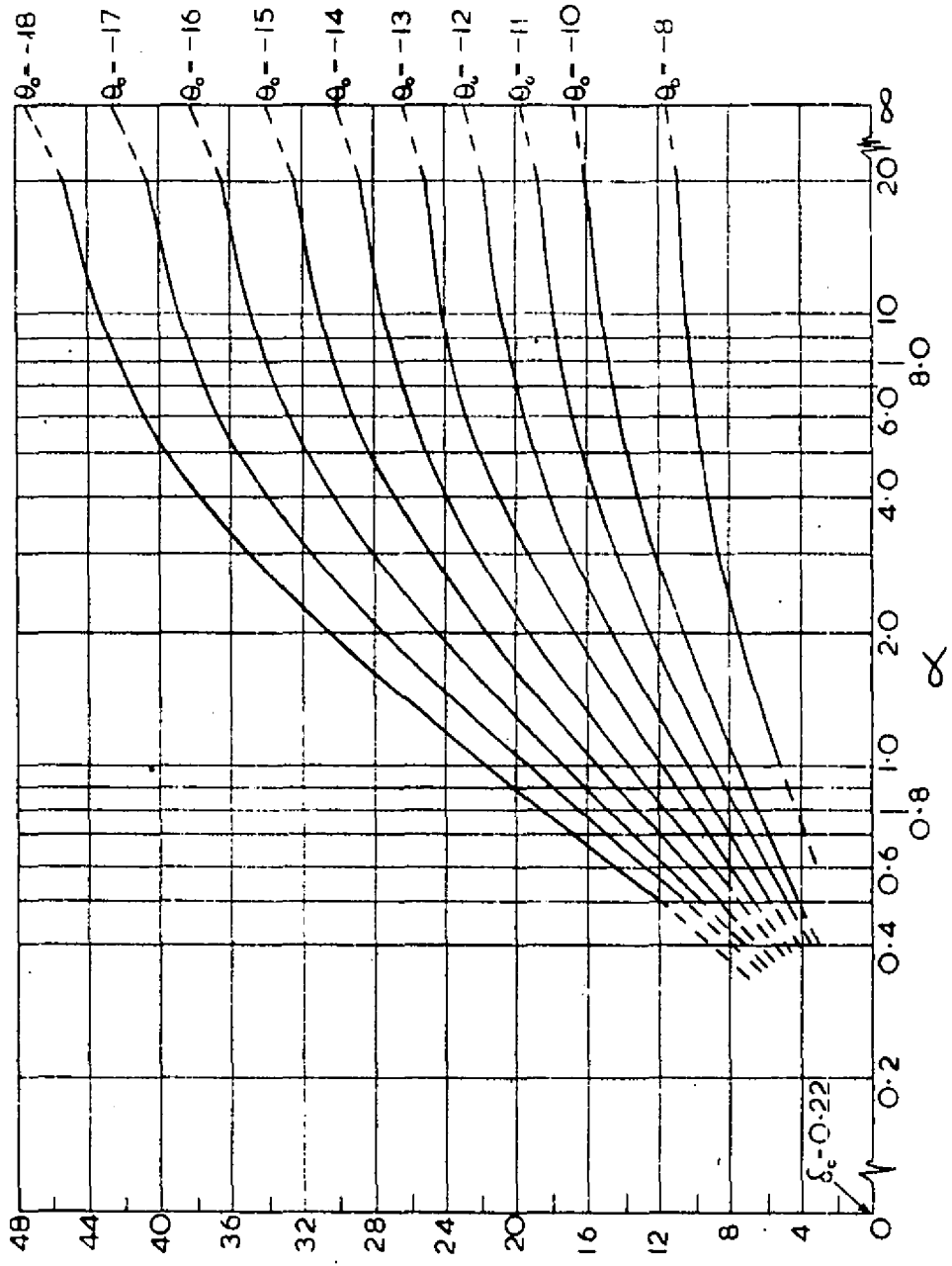FIG I δ_c AS FUNCTION OF $\propto$ AND θ_o