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THE THEORY OF A SIMPLE DOSAGE METER FOR THERMAL RADIATION

by

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Summary

A radiation dosage meter has been described and calibrations given in a previous note. This note describes some theoretical aspects of its design and calibration.

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Introduction

This note describes the theory of a simple device, suggested by D.I. Lawson, for measuring the total amount of heat radiation falling on a given area in a short time. If thermal radiation is allowed to fall on a restricted area on the front of a sheet of material, heat will subsequently spread outwards. If the unexposed surface of this sheet is painted with a paint which melts or changes colour at a definite temperature, then the area of paint melted can be taken as a measure of the dose applied.

The apparatus can also be used to measure the value of the flux if this is constant for some time, but we shall consider here only the use as a dosemeter. It is worth noting that in principle one could make use of radial flow or linear flow - the first from a circular area and the second from a long strip. The theory for the linear flow is somewhat easier than for a radial flow and the calibration will be approximately linear between dose and distance of spread instead of between dose and area. Circular radiometers have, in fact, been made (1)
and it is with these that this report is concerned. Practical difficulties arise in calibrating strip type radiometers because of the small areas of the available high intensity sources but being linear they may for some uses be more suitable though the range of any instrument will be less than for a radial flow instrument of equal size and aperture.

Dosemeter Theory

The general radial conduction problem which is involved in this instrument is not amenable to analytic solution - except in terms of infinite integrals - but nevertheless it is possible to establish quite simply some useful criteria to be observed in its design and to derive the approximate form of its calibration.

To consider the idealised problem of a sheet across which the temperature remains uniform when heated and allowed to cool, we assume that the pulse of energy is delivered to the sheet over a circular area of radius R.

We have

$$\frac{d^2\theta}{dr^2} + \frac{1}{r}\frac{d\theta}{dr} = \frac{1}{k}\frac{d\theta}{dt} + \frac{2H\theta}{\Delta k} - \frac{Q'_m f(t/t_m)}{\Delta K} \qquad (1)$$

where θ is the temperature above ambient at radius r and time t

K is the thermal conductivity
k is the thermal diffusivity
H is the surface cooling coefficient
Δ is the thickness of the sheet
Q'_m is the peak value of the incident flux
t_m is a time characteristic of the duration of the pulse, e.g. the time to maximum flux and 'f' describes the pulse shape in time.

The boundary condition is

$$Q'_m = 0 \qquad r > R$$
$$\quad = Q_m \qquad r < R$$

For θ equal to zero time we have the solution

$$\theta \; \frac{Q_m}{\Delta\rho c} \int_0^\infty J_1(z) \;\; J_0(z^r/R)\, dz \int_0^t f\left(\frac{\lambda}{t_m}\right) e^{-\left\{\left(z^2 + \frac{2hR^2}{\Delta}\right) \cdot k \frac{t-\lambda}{R^2}\right\}} \, d\lambda \qquad (2)$$

where $h = {}^{H}/K$

and J_0 and J_1 are Bessel functions of the first kind of zero and first order.

We can write quite generally

$$\theta \; \frac{Q_m t_m}{\Delta\rho c} \quad F\left\{\frac{r}{R}, \frac{t}{t_m}, \frac{kt_m}{R^2}, \frac{hR^2}{\Delta}\right\} \qquad (3)$$

where F denotes 'a function of'

The energy dose is

$$E = Q_m \int_0^\infty f(^{\lambda}/t_m)\, d\lambda \; = \; A_1, Q_m, T_m, \text{ say}$$

where A depends on the pulse shape only and not on its magnitude of duration, so that in the above equation Q_m t_m can be replaced by $^E/A$.

The value of E is to be measured by the radius, r_1, of the area of paint melted at a temperature above ambient of θ_p. This means that the maximum temperature in time at radius r_1 is θ_p.

$$\text{Hence} \quad \frac{E}{\Delta\rho c\theta_p} \;=\; F_1\left\{\frac{r}{R}, \quad \frac{kt_m}{R^2}, \quad \frac{hR^2}{\Delta}\right\} \qquad (4)$$

This must be the general form of the calibration.

It should be noted that as $\frac{kt_m}{R^2}$ approaches zero equation (1) tends to

$$\theta_m = \frac{E}{\rho c\Delta} \exp\left(\frac{-2hR^2}{\Delta} \; \frac{kt}{R^2}\right) \int_0^\infty J_1(z) \, , \;\; J_0(z^r/R) e^{-z^2\frac{kt}{R^2}} \qquad dz \qquad (5)$$

$$\text{and} \; \frac{E}{\Delta\rho c\theta_p} \;=\; F_2\left(\frac{r}{R} \cdot \frac{hR^2}{\Delta}\right) \qquad (6)$$

This suggests that only if $\frac{kt_m}{R^2}$ is small can the calibration be truly that for a dosemeter and independent of pulse shape and duration. In fact the condition is not as stringent as this; it is shown below that the conditions required are $\frac{kt_m}{r^2} \ll 1$ and $\frac{hr_1^{\,2}}{\Delta} < < 1$. We shall consider now the limiting form of the function F_2 when $\frac{hR^2}{\Delta}$ is zero and when $\frac{kt_m}{r^2}$ is effectively zero. The heated plate behaves as if over the exposed area the temperature were instantaneously raised to $\frac{E}{\rho c\Delta}$. If we consider the change in the temperature distribution at small times after this heating the maximum temperature just beyond r equal to R is $\frac{E}{\rho c\Delta}$ (see Fig. 1).

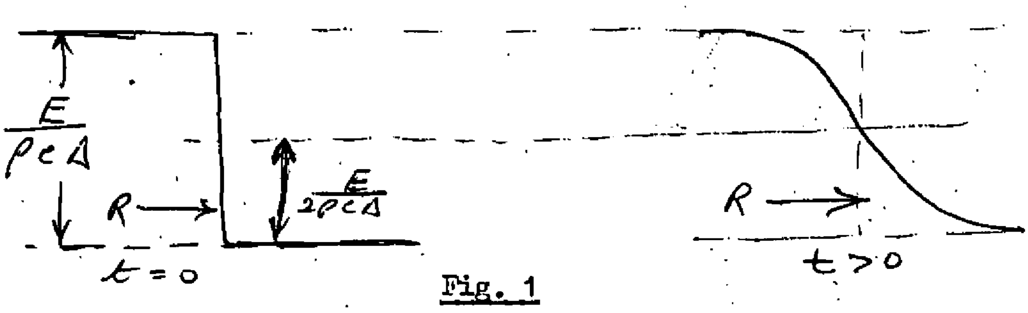

Fig. 1

Since this result is for $t \rightarrow o$ it must be independent of cooling we have therefore

$$\frac{E}{\rho c\Delta\theta_p} \rightarrow 2 \qquad\qquad r \rightarrow R$$

We now consider high values of E so that the value of $\frac{r_1}{R}$ is large

Equation (3) gives, for $\frac{hR^2}{\Delta} = 0$

$$\frac{\rho c \Delta\theta_p \, r_1^2}{ER^2} = \frac{r_1^2}{R^2} \int_0^\infty J_1(z)\, J_0(z^r/R) \; e^{-z^2 \frac{kt}{R^2}} \, dz = P_{(say)}$$

where $\frac{kt}{R^2}$ is such as to make P a maximum.

Assume for the time being that $\frac{kt}{r_1^2}$ remains finite as $r_1 \to \infty$ so that

$$P \to \frac{r_1}{R} \int_0^\infty J_1\left(\frac{R}{r_1}x\right) J_0(x)\, e^{-\tau x^2}\, dx$$

where $\tau = \frac{kt}{r^2}$

We consider the limiting value of P for infinite r_1. In the region $\sqrt{r_1/R} < z < \infty$ the contribution to P tends to zero while in the region $0 < z < \sqrt{r_1/R}$ $J_1\left(\frac{Rz}{r_1}\right)$ can be replaced by $\frac{R}{2r}$ z so that

$$P \to \frac{1}{2} \int_0^\infty x\, J_0(x)\, e^{-\tau x^2}\, dx = \frac{1}{4\tau} \exp\left(-\frac{1}{4\tau}\right)$$

The maximum value of P occurs when $\tau = \frac{1}{4}$ - which does not contradict the assumption made above, and the maximum of P is therefore

$$P_{max} = \frac{1}{e}$$

Hence

$$\frac{ER^2}{r_1^2 \rho c \Delta\theta_p} \to 2.72 \qquad \text{as} \to r_1$$

The value of $\frac{ER^2}{r_1^2 \rho c \Delta\theta_p}$ at infinite r/R therefore differs only by 36% from its value at r/R equal to one and suggests that a "square law" calibration would be a first approximation.

The above result does not take cooling or finite times of exposure into account. For the special case of a large distance of spread and a short but finite application time it is possible to take the theory further allowing for these effects without undue complexity. We write $f = 1$ for $\lambda < t_m$ in equation (2) i.e. we discuss a square pulse of duration t_m. As above, we replace

$J_1\left(\frac{R}{r}\right)$ by $\frac{R}{2r_1}$ for $\frac{r}{R} >> 1$ and it can be shown that

$$\frac{\rho c \Delta\, r_1^2}{ER^2} = \exp\left(-\frac{2hr_1^2\tau}{\Delta}\right) \int_0^\infty y\, J_0(y)\, e^{-\tau y^2} \left\{ 1 + \frac{T_m y^2}{2!} + \frac{T_m^2 y^4}{3!} + \text{etc} \right\} dy \tag{7}$$

where $\tau = \frac{kt}{r_1^2}$ and $T_n = \frac{kt_m}{r_1^2}$

All integrals involved in this series are convergent for $\tau > 0$ and, from Watson[2]

$$\frac{\rho c \Delta\theta \, r_1^2}{ER^2} = e^{-\frac{2hr_1^2\tau}{\Delta}} \sum_{n=0}^{\infty} \frac{T_m^n}{4\,(n+1)} I_n$$

$$\text{where } I_n = \frac{1}{\tau^{n+1}} e^{-1/4\tau} \left\{ 1 - \frac{n}{4\tau} + \frac{n(n-1)}{(4\tau)^2 (2!)^2} \ldots \text{etc.} \right\} \tag{8}$$

To discuss how the maximum value of θ varies with τ when τ is small, we consider only the first two terms of the above series m = o and 1 and eventually obtain

$$\frac{ER^2}{\rho c \Delta r_1^2} = \frac{2e^{\sqrt{1+\frac{2hr^2}{\Delta}}}}{1+\sqrt{1+\frac{2hr^2}{\Delta}}}\left\{1+\frac{2Ht_m}{\rho c \Delta}+\text{higher powers of } t_m\right\} \tag{9}$$

This gives the correction for a finite value of t_m. The equation is plotted in Fig. (2) for zero t_m and is discussed below.

Discussion

Two radiometers - a copper sheet 0.016 cm thick and a steel sheet 0.046 cm thick having an aperture of 1.9 cm diameter were both calibrated by exposure to a constant intensity. The value of τ was approximately constant and for the steel was about 0.09 for exposure at 12 cals $cm^{-2}s^{-1}$ and twice this for exposure at 6 $cal.cm^{-2}s^{-1}$. For the copper meter the values of τ were respectively 0.031 0.015. These values are all small compared with unity. For steel the largest value of $\frac{2Ht_m}{\rho c \Delta}$ was 0.17 and for copper 0.12, also small compared with unity. The conditions for the approximate theory above may therefore be regarded as being fulfilled.

In Fig. (2) experimentally determined points are shown together with the theoretically determined calibrations for zero exposure time. The effect of the finite exposure time is to increase the value of the ordinate for a given r/R by 17.5% at the largest value of r_1/R. Increasing the ordinate of the calculated line would bring the calculated and experimental points even closer together at large values of r_1/R where the calculations should be most accurate. The practical difference between exposure at 12 and 6 $cal.cm^{-2}s^{-1}$ is small and the calibrations at 12 $cal.cm^{-2}s^{-1}$ both for the steel and the copper have been taken as the best calibrations. According to equation (9) the difference between the two copper calibration curves made out intensities of 12 and 6 cal/sq.cm/sec. should be less than that between the corresponding calibrations using steel sheets and the copper calibration should lie below the steel on Fig. (2), because of the smaller value of $\frac{h}{\Delta}$. This was not found to be so and there is an unexplained discrepancy between the measured and calculated results in the case of the copper plate.

It has been shown that the direct effect on the calibration of sideways conduction during the exposure is assessed by $\frac{kt}{r_1^2}$ which is small. Sideways conduction may however markedly reduce the peak temperatures attained. In the field where exposure times are short, the temperatures more nearly approach the theoretical limits of $\frac{E}{\rho c \Delta}$. As will be seen from Fig. (2), this means that the field performance of the radiometers is limited by the melting of the metal. Experiments have shown that the burning of the carbon does not noticeably affect the performance of the radiometers if these are roughened and oxidised before blackening. It would, in principle, be possible to reduce this effect by increasing the sideways conduction during exposure, but this would require exposure areas which are too small (c.1 m.m. diameter) for the measure of spread to be accurate.

Reference

(1) SMITH, P. G. "A simple radiation dosage meter". Department of Scientific and Industrial Research and Fire Offices' Committee, Joint Fire Research Organization, F.R. Note No. 363/1958

(2) WATSON, G. N. "Theory of Bessel Functions. C.U.P. 2nd edit. p. 393.

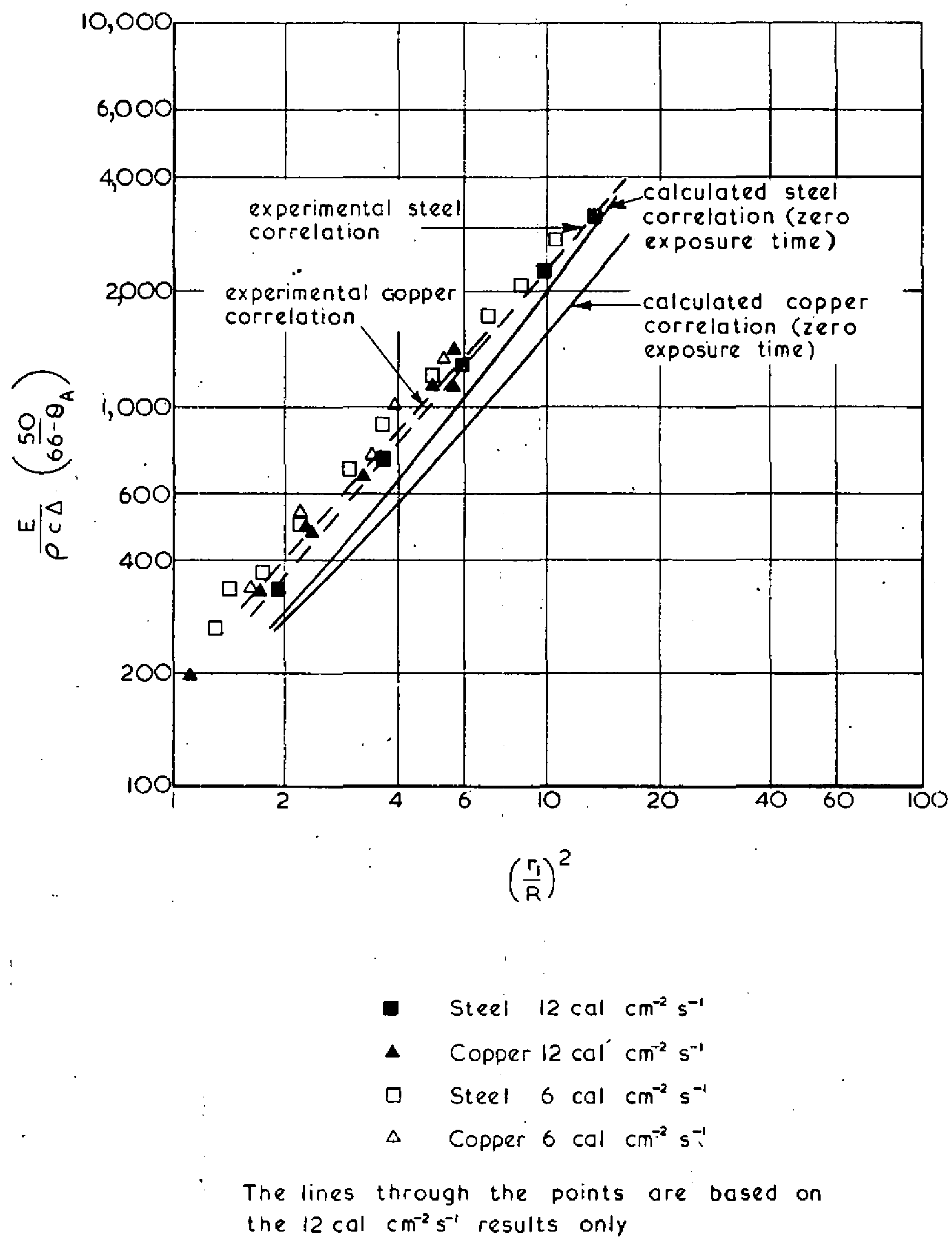

FIG. 2. CORRELATION OF DATA FOR CALIBRATION OF DOSEAGE METERS