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THE THERMAL INSULATION VALUE OF METAL SHIELDS

by

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Summary

Simple expressions are obtained for the temperatures of one or more metal shields surrounding simple surfaces maintained at a uniform temperature. The approximations involved render the results invalid if the lowest metal temperature is less than 90°C.

1) Introduction

In this report the thermal protection offered by metal shields enclosing hot surfaces such as furnace exteriors or flue pipes is considered. By making two approximations it has been possible to derive simple expressions relating the temperature of the shield or shields to the temperature of the heated surface in the cases of plane, cylindrical and spherical surfaces.

2) Approximations

2.1.) Transfer of heat

The transfer of heat between any two consecutive surfaces is assumed to be purely by radiation. The relative importance of radiative and convective transfers is considered in Appendix 1.

2.2.) Cooling at external shield

The expression for the cooling to the atmosphere by convection and radiation from the external shield is taken as σT^4 where σ is Stefan's constant and T is in degrees Kelvin (absolute). In Appendix 2 it is shown that this approximation, which neglects the effect of convection and the opposite effect of radiative warming by the atmosphere, ensures that the results obtained for the temperatures of the external shields are maximum values provided that they are above 90°C.

3) Assumptions

It is assumed that the emissivity of the metal surfaces approximates to unity (i.e. is higher than 0·7) and that the air between any two consecutive shields is not free to exchange with the atmosphere.

4) Infinite plane metal shields

Figure 1 represents a problem in which an infinite plane surface, maintained at a uniform temperature T_1, is succeeded by (n - 1) thin metal shields, the temperature of the (n - 1)th shield being required.

The Heat transfer per unit area between any two consecutive surfaces will be given by the following expressions:-

$$\left.\begin{aligned}
&\text{Heat transfer per unit area between surfaces (1) and (2)} = \sigma T_1^4 - \sigma T_2^4\\
&\text{Heat transfer per unit area " " (2) and (3)} = \sigma T_2^4 - \sigma T_3^4\\
&\text{Heat transfer per unit area " " (3) and (4)} = \sigma T_3^4 - \sigma T_4^4\\
&\\
&\text{Heat transfer per unit area " " (n - 1) and (n)} = \sigma T_{n-1}^4 - \sigma T_n^4
\end{aligned}\right\} \quad (1)$$

where $T_1 T_2$ etc. are absolute temperatures.

Adding the expressions and dividing by (n - 1) we have:-

Heat transfer per unit area between two consecutive surfaces

$$= \frac{\sigma T_1^4 - \sigma T_n^4}{n - 1} \qquad \ldots\ldots (2)$$

Equating heat fluxes either side of surface n

$$\frac{\sigma T_1^4 - \sigma T_n^4}{n - 1} = \sigma T_n^4 \qquad \ldots\ldots (3)$$

where σT_n^4 is the cooling per unit area to the atmosphere.

$$\therefore \; T_n = \frac{T_1}{n^{\frac{1}{4}}} \qquad \ldots\ldots (4)$$

Expression (4) gives a close approximation to the true temperature irrespective of the values of n and T_1, always providing that the resulting value of T_n exceeds 90°C.

The temperatures of intermediate surfaces are given by the expression

$$T_{n-a} = (a + 1)^{\frac{1}{4}} T_n = \left[\frac{(a + 1)}{n}\right]^{\frac{1}{4}} T_1 \qquad \ldots\ldots (5)$$

$$0 \leqslant a < n$$

5) <u>Concentric cylindrical shields</u>

Figure 2 represents a problem in which an infinitely long cylindrical surface is maintained at a uniform temperature T_1 and is surrounded by (n - 1) thin concentric metal shields. To find the temperature T_n of the external shield the method outlined is used saving that the total heat transfer per unit height is considered instead of the transfer per unit area. Configuration factors do not therefore enter into the calculation.

$$\left.\begin{aligned}
&\text{Heat transfer per unit height between surfaces (1) and (2)} = 2\pi(r_1\sigma T_1^4 - r_2\sigma T_2^4)\\
&\text{Heat transfer " " " " " (2) and (3)} = 2\pi(r_2\sigma T_2^4 - r_3\sigma T_3^4)\\
&\text{Heat transfer " " " " " (3) and (4)} = 2\pi(r_3\sigma T_3^4 - r_4\sigma T_4^4)\\
&\\
&\text{Heat transfer " " " " " (n - 1) and (n)} = 2\pi(r_n\sigma T_n^4 - r_n\sigma T_n^4)
\end{aligned}\right\} (6)$$

where $T_1 T_2$ etc. are absolute temperatures.

Adding the expressions and dividing by (n - 1) we have:-

Heat transfer per unit height between two consecutive surfaces

$$= \frac{2\pi(r_1\sigma T_1^4 - r_n\sigma T_n^4)}{(n-1)} \qquad \text{......(7)}$$

Equating heat fluxes either side of surface n

$$\frac{2\pi(r_1\sigma T_1^4 - r_n\sigma T_n^4)}{n-1} = 2\pi r_2\sigma T_n^4 \qquad \text{......(8)}$$

where $2\pi r_n\sigma T_n^4$ is the cooling per unit height to the atmosphere.

$$\therefore\; T_n = \left(\frac{r_1}{nr_n}\right)^{\frac{1}{4}} T_1 \qquad \text{......(9)}$$

Expression (9) again gives a close approximation to the true temperature irrespective of the values of n and T_1, always providing that the resulting value of T_n exceeds 90°C.

The temperatures of intermediate surfaces are given by the expression

$$T_{n-a} = \left[\frac{(a+1)}{n}\left(\frac{r_1}{r_{n-a}}\right)\right]^{\frac{1}{4}} T_1 \qquad \text{......(10)}$$

$$o \leqslant a < n$$

6) Concentric spherical shields

Figure 2 may alternatively be taken to represent a problem concerning spherical shields.

The following results may then be derived as in paragraph (5).

$$T_n = \left(\frac{r_1^2}{nr_n^2}\right)^{\frac{1}{4}} T_1 \qquad \text{......(11)}$$

$$o \leqslant a < n$$

$$T_{n-a} = \left[\frac{(a+1)}{n}\left(\frac{r_1^2}{r_{n-a}^2}\right)\right]^{\frac{1}{4}} T_1 \qquad \text{......(12)}$$

7) Examples

Temperature readings have been taken on a 6 in. metal flue pipe and an unventilated 8 in. metal shield [1] and the results may be compared with calculations made firstly by the method outlined and secondly by a tedious numerical approximation method using a more accurate formula for the cooling of the shield. Close agreement with experimental results cannot, however, be expected as the temperature of the pipe varied along its length whilst there could be mass transfer of the air from hot to cold sections of the enclosure between the shield and the flue pipe. Thus, for example, in the results shown in Table 1 the measured shield temperatures for the values 500°C and 480°C of the flue pipe temperature appear to be incompatible.

It will be seen that the results given by the method outlined approximate quite closely to the predictions by the more accurate method, being less than 22°C higher for temperatures in the range of 607°C to 256°C.

Table 1. Predictions and experimental results: temperature of an 8 in. diameter casing around a 6 in. unventilated flue pipe

Source of data (1)		Flue temp. T_1(°C)	Casing temp. T_2(°C)			Difference	
Figure number	Distance of measurement above stove		Measured	Calculated		Approx. and numerical methods	Approx. and measured results
				Accurate numerical method	Approx. method		
Fig. 6	1 foot	870°C	600°C	607°C	622°C	15°C	22°C
Fig. 6	4 feet	700°C	460°C	468°C	488°C	20°C	28°C
Fig. 6	10 feet	590°C	320°C	383°C	403°C	20°C	83°C
Fig. 6	20 feet	500°C	270°C	312°C	332°C	20°C	62°C
Fig. 9 (coke)	1 foot	630°C	370°C	413°C	433°C	20°C	63°C
Fig. 9 (coke)	4 feet	480°C	285°C	297°C	317°C	20°C	32°C
Fig. 9 (coke)	10 feet	450°C	225°C	273°C	293°C	20°C	68°C
Fig. 9 (coke)	20 feet	430°C	210°C	256°C	277°C	21°C	67°C

8) Conclusions

The approximations used in the derivations are such that predicted values of T_n, the external shield temperature, will exceed the actual temperatures (provided they are always above 90°C) and will approximate closely to predictions made by more accurate methods. Any errors will therefore not lead to an increase of the hazard where safety limits concerning combustible materials or burn injuries etc. are being considered.

Used in conjunction with the relevant graphs in a previous report (1) the minimum distances at which certain combustible materials may be located with respect to the assemblies here considered may be easily calculated.

9) References

1) F.P.E. 32/1950 "The Heating of Panels by Flue Pipes". D. I. Lawson, L. L. Fox and C. T. Webster.

10) Appendix 1. Heat transfer between two surfaces

To confirm that the principal mode of heat transfer between two surfaces is by radiation the expressions for transfer by radiation and by convection must be compared. This will be done first for small values of temperature difference and then for an example where the temperature difference is large.

10.1) Radiative transfer

The expression for radiative transfer (f) per unit area per unit time between two similar surfaces whose separation is considerably less than their shortest dimension is

$$f = \sigma\left[(\theta + T_o)^4 - T_o^4\right] \qquad \text{......(13)}$$

where T_o is the absolute temperature of the cooler surface,

θ is the temperature difference between the surfaces

and σ is Stefan's Constant ($1{\cdot}37 \times 10^{-12}$ cal/cm^2/sec/degree^4).

If θ is small compared with the absolute temperature T_o of the cooler surface, expansion by the binomial theorem gives

$$f = 4\sigma T_o^3\theta \qquad \text{......(14)}$$

The coefficient of heat transfer by radiation for unit temperature difference is therefore $4\sigma T_o^3$

10.2) Convective transfer

It has been assumed that the space between the two surfaces is not open to the atmosphere, thus the minimum value of the air temperature will be that of the cooler surface. The convective heat transfer under these circumstances cannot be greater than the rate at which the hotter surface would lose heat by convection to a free atmosphere at the temperature of the cooler surface. The expression for the latter is $H\,\theta^{1\cdot 25}$ (where $H = 4{\cdot}7.10^{-5}$ cal/cm^2/sec/$^oC^{1\cdot25}$) and for small values of θ this approximates to $H\,\theta$.

10.3) Ratio of radiative and convective transfer co-efficients

From the previous paragraphs the ratio of radiative and convective transfer co-efficients is in excess of $\frac{4\sigma T_o^3}{H}$. This expression is tabulated (see Table 2) for different values of T_o, the temperature of the cooler shield.

Table 2. Minimum value of ratio of radiative and convective transfer co-efficients

Temperature of cooler Surface above ambient (17°C)	Co-efficient of radiative transfer $4\sigma T_o^3$ (cal/cm^2/sec/°C)	$\frac{4\sigma T_o^3}{H}$
500°C	$2{\cdot}7.\ 10^{-3}$	57
400°C	$1{\cdot}8.\ 10^{-3}$	38·5
300°C	$1{\cdot}12.10^{-3}$	24
200°C	$6{\cdot}45.10^{-4}$	13·5
100°C	$3{\cdot}24.10^{-4}$	6·9
73°C	$2{\cdot}62.10^{-4}$	5·55

10.4) Higher values of temperature difference

In practice the temperature difference θ between any two consecutive surfaces might assume much higher values and the fact that the expression for the maximum value of the convective transfer depends on $\theta^{1\cdot 25}$ means that the relative effect of convection would increase. From Table 2 it can be seen that this is only of importance for low values of T_o. The lowest value of the ratio of radiative to convective transfer will occur when the outer shield is at the lowest temperature considered (90°C) and the shield nearest to it approximately 70°C higher (this value giving a heat flow which would maintain an external shield temperature of 90°C). Using expressions (13) and $H\theta^{1\cdot 25}$ for radiative and convective transfers respectively, the ratio is 2·55.

Thus in a typical problem the ratio would range between 2·55 and 57. Since the expression $H\theta^{1\cdot 25}$ is a maximum value to which the level of convective transfer could rise it can be neglected in comparison with the level of radiative transfer.

11) Appendix 2. Cooling at the external shield

The expression for the cooling per unit area of a black surface to an atmosphere is

$$\sigma(T^4 - T_o^4) + H(T - T_o)^{1\cdot 25} \qquad \ldots\ldots(14)$$

where the term $-\sigma T_o^4$ represents the radiation from the atmosphere to the surface and the term $H(T - T_o)^{1\cdot 25}$ represents the convective cooling of the surface to the atmosphere. Figures 3 and 4 are graphs of σT^4 and expression 14 for different temperature ranges. For temperatures above 90°C (73°C rise above ambient temperature) the curve σT^4 always lies below the curve of the more accurate expression and thus predictions of the external shield temperature using σT^4 in place of expression 14 will be high, which is appropriate for safety calculations.

The examples in Table 1 show that the use of the approximation does not introduce a very great error as the approximate results are only 15°C to 21°C higher than the more accurate calculations over a range of temperature of 607°C to 256°C.

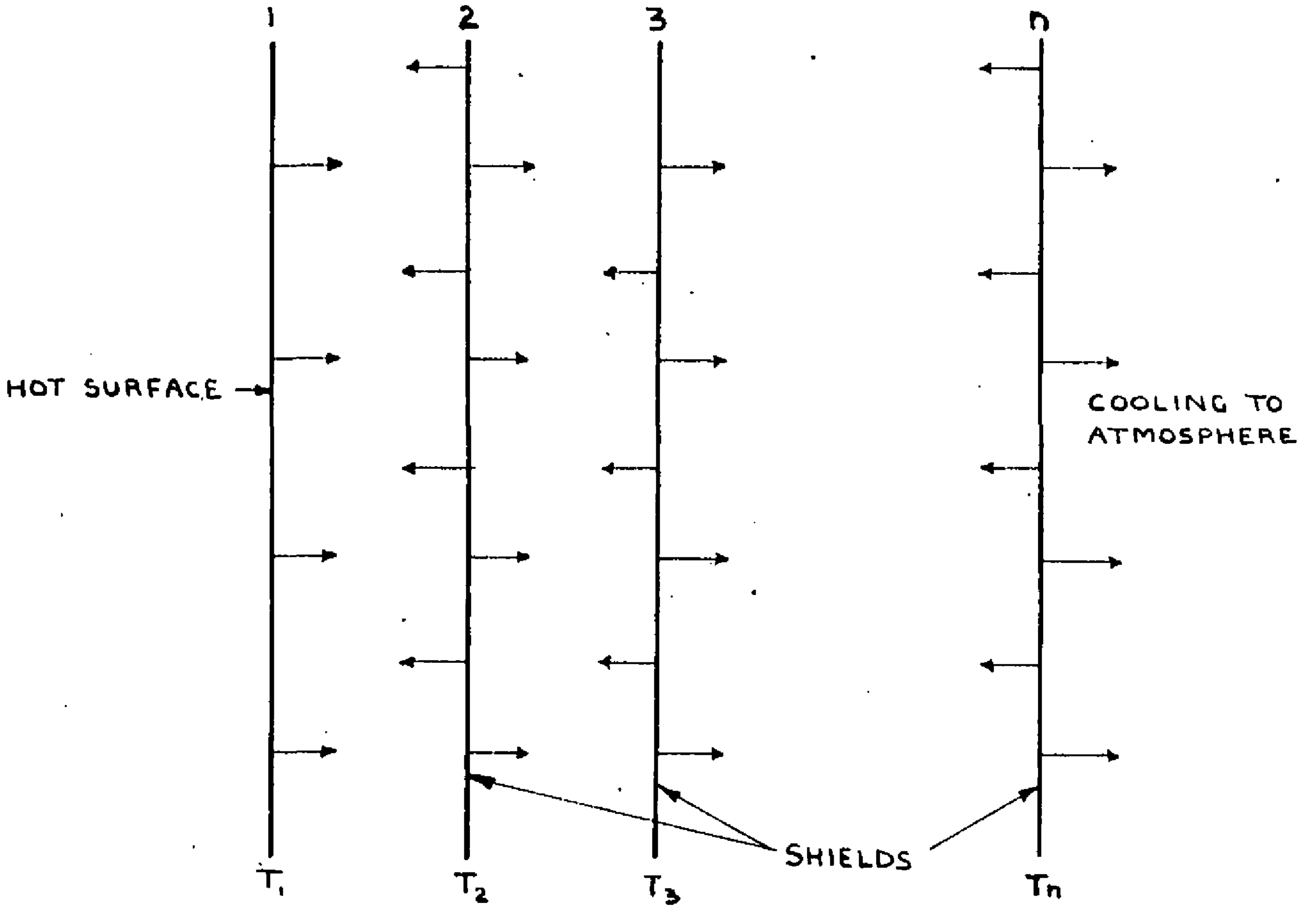

FIG. 1. PROBLEM CONCERNING INFINITE PLANE METAL SHIELDS

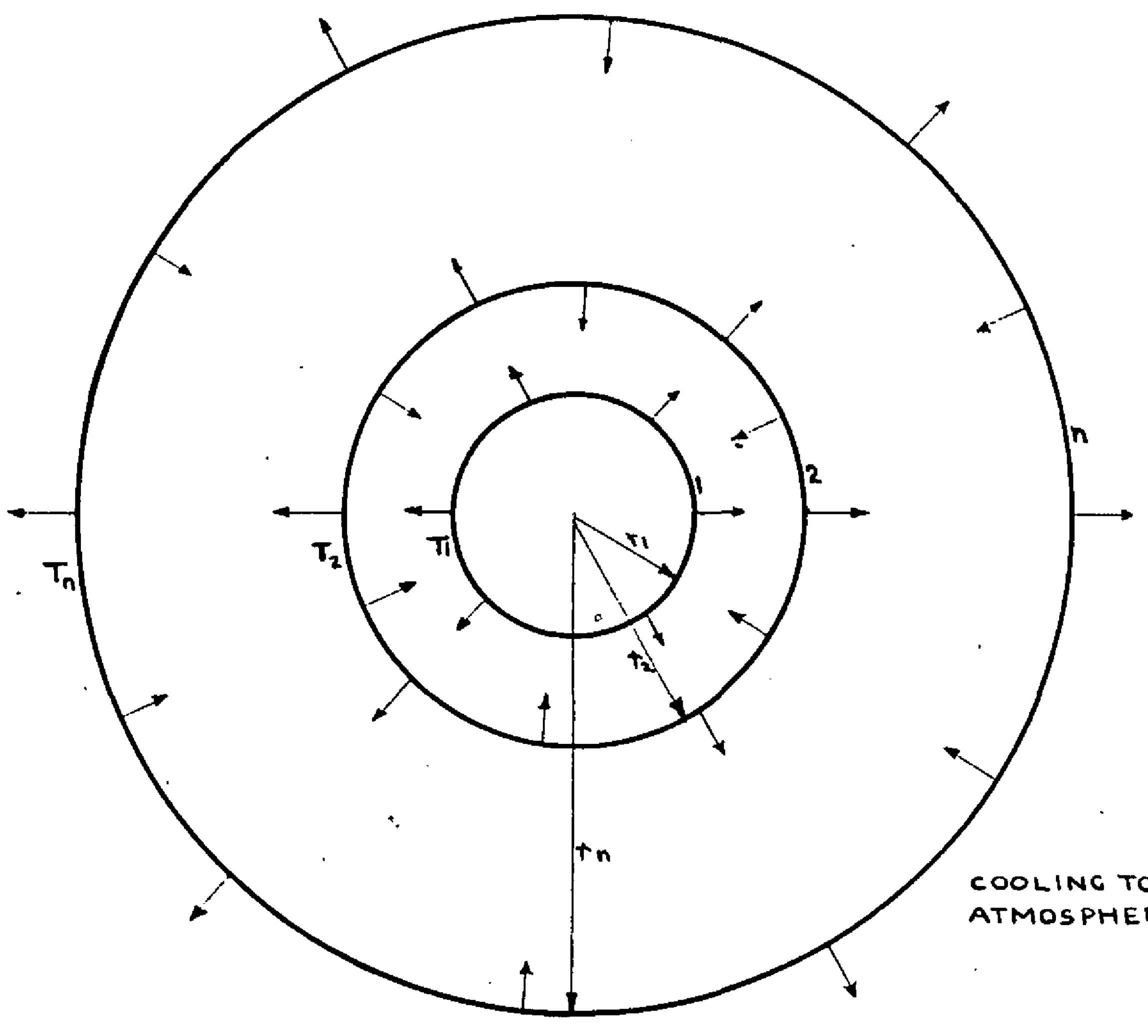

FIG. 2. PROBLEM CONCERNING CONCENTRIC CYLINDRICAL SHIELDS

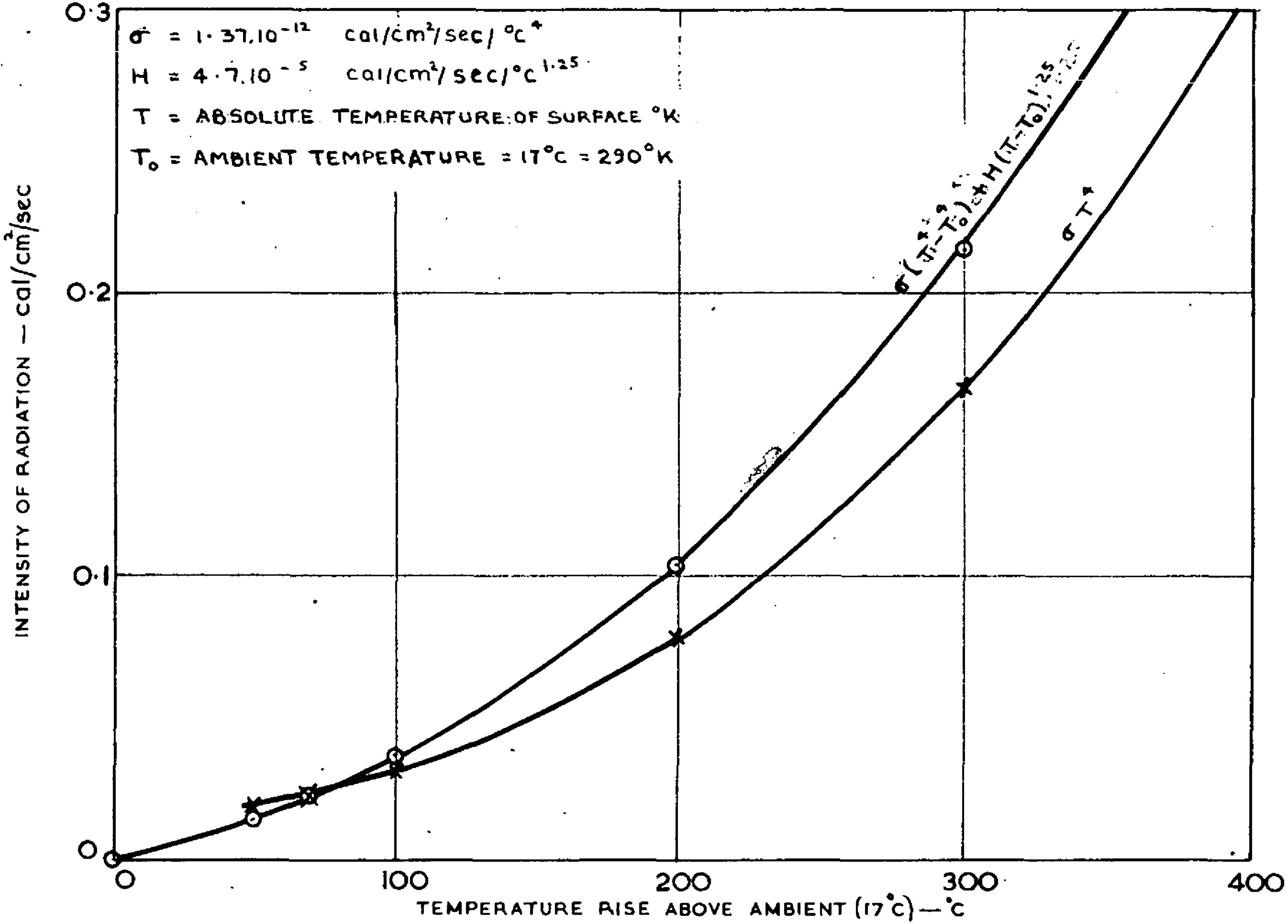

FIG. 3. THE APPROXIMATION σT^4 COMPARED WITH COOLING TO AN ATMOSPHERE (LOW TEMPERATURES)

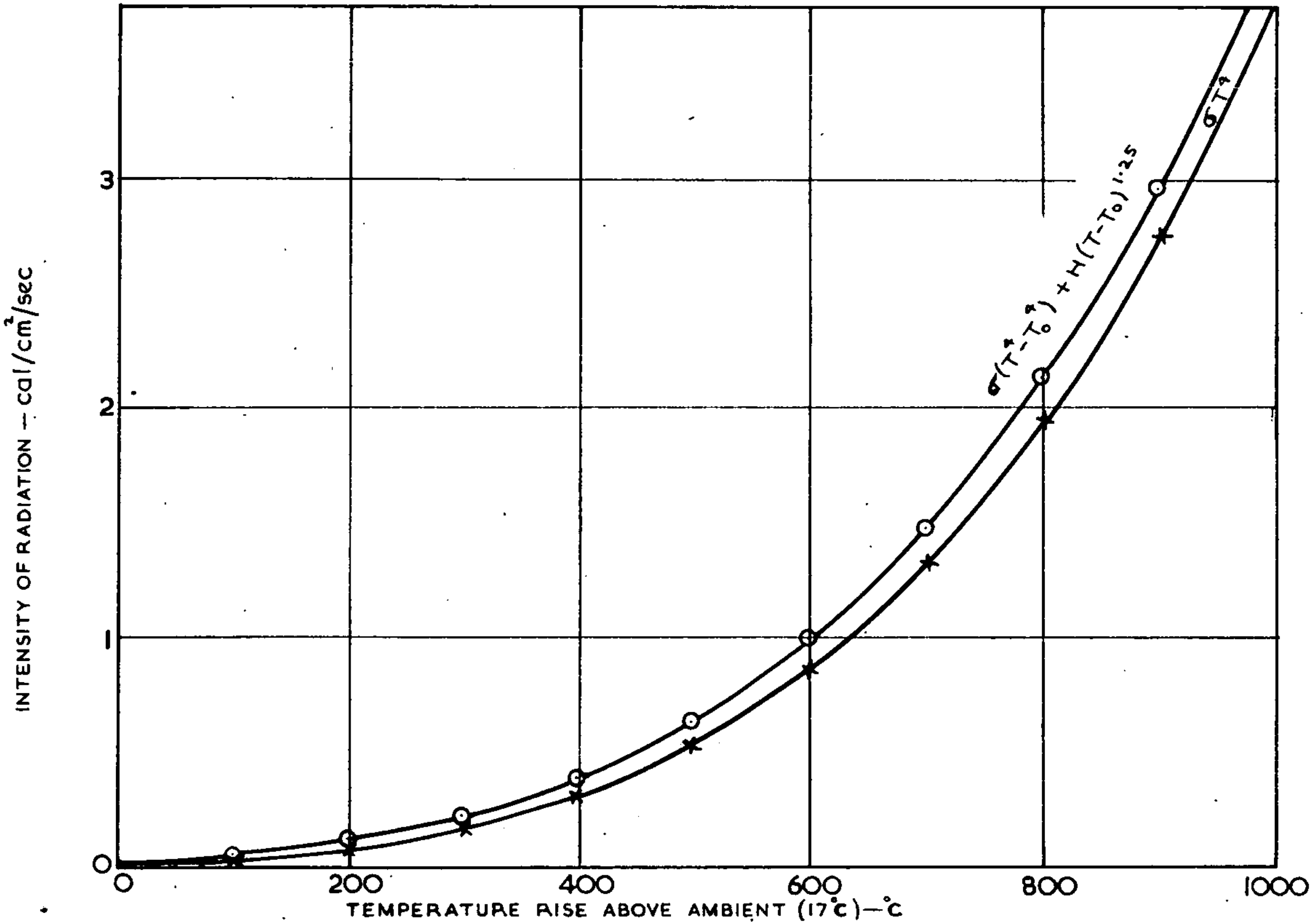

FIG. 4. THE APPROXIMATION σT^4 COMPARED WITH COOLING TO AN ATMOSPHERE (HIGH TEMPERATURES)