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A THEORETICAL MODEL OF THE RATE OF SPREAD OF FIRE IN A CONTINUOUS BED OF FUEL

by

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SUMMARY

This paper describes an approximate theory of fire spread in a fuel continuum, e.g. a long crib of wood fuel. Experiments are now in progress to test this theory.

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Introduction

This paper discusses a theoretical model of flame spread in a horizontal bed of fuel under the action of wind. This theory is physically somewhat similar to that described by Fons[1] and the relation between them is discussed in this paper. Fons's theory neglects conduction in the unburnt fuel and it neglects the effect of the size of the burning zone on the heat transfer ahead of the burning zone. Most important, it uses too simplified an analysis for the pre-heating of this unburnt fuel. His expression for the rate of spread 'R' in a horizontal bed of fuel elements standing vertically, separated by a distance L from each other is

$$R = \frac{C\left(f_c + f_r\right)\delta L}{\bar{\gamma}\,\bar{C}_p \log_e C_t\left(\frac{t_f - t_o}{t_f - t_i}\right)}$$

where C is an empirical proportionality constant (of order unity) relating a natural fuel bed to a standardised laboratory version of that bed.

$f_c + f_r$ is the overall heat transfer coefficient to unburnt fuel, the suffixes 'c' and 'r' referring respectively to convection and radiation.

δ is surface/volume ratio for fuel.

$\bar{\gamma}$ is density of moist fuel.

$\bar{C}_p$ is specific heat of moist fuel.

t_f is temperature of flame.

t_i is ignition temperature of fuel.

t_o is initial temperature of fuel.

C_t is a factor of order unity $= \dfrac{t_f - t_1}{t_f - t_o}$

where t_1 is temperature of nearest fuel element ahead of flame

and L is the distance between fuel elements in direction of spread.

Since C is of order unity it is not possible to vary C greatly to accommodate differences between calculated and experimental data so that its presence is not an objectional feature in comparing theoretical and experimental values of R.

Now for the values of t_f, t_i and t_o used by Fons, C_t varies only between 1 and 0.67. But the reciprocal of the logarithm of $\dfrac{C_t\,(t_f - t_o)}{(t_f - t_i)}$ i.e. $\left(\mathrm{Log}_e \dfrac{t_f - t_1}{t_f - t_i}\right)^{-1}$ varies from 2.5 to ∞ for this range of C_t. Thus small variation in the empirical constant C_t can accommodate large variations in R and this considerably reduces the value of the theory. We shall later in the paper discuss the relation between our theory and Fons's.

There are not yet sufficient data to compare theory and experiment.

Theory

Consider a long bed of fuel in which a burning zone is progressing in one direction at constant speed (a linear flame front) Fig. (1).

The forward and rear fronts A and B are in general inclined to the vertical but if $D \gg 2\rho \cot\theta$ where D, ρ and θ are as shown in Fig. (1) we may consider the burning zone to be effectively rectilinear. The fuel near the front B is only partially burnt and that near the rear nearly wholly burnt, but for the sake of simplicity we assume that so far as the pattern of flame and heat transfer ahead of the fire is concerned we can take the burning zone as consisting of fuel burning uniformly over the length D.

The heat transfer ahead of the burning zone

Unburnt fuel receives heat from flames and the burning zone and if $\rho \ll D$ we shall at first assume that the heat transfer rate to any portion of unburnt fuel is the same at all heights in the fuel bed and is described by

$$i = I f(\beta z) \qquad \text{.....(1)}$$

where I is a characteristic heat transfer rate conveniently taken as the maximum heat transfer rate and $f(\beta z)$ which is ≤ 1 is a distribution function in terms of the distance z ahead of the front B and a characteristic distance $1/\beta$ (see Fig. 2). We show below that we can assume as a first approximation

$$i = I f(\beta z) = I e^{-z\beta} \qquad \text{.....(1a)}$$

Again for simplicity we assume that in every element of fuel heat is conducted into it normally to its surface, i.e. we neglect lateral heat loss within the fuel element.

For simplicity we assume a cylindrical fuel element, neglecting heat conduction along the element. The surface temperature at time 't' due to an element of heat $i(\lambda)d\lambda$ striking unit surface area of the cylinder at time λ is obtainable by the methods described by Caslaw and Jaegar[2] as

$$\delta\theta = \frac{2}{a}\sum_{n=1}^{\infty} \frac{e^{-k\alpha_n^2(t-\lambda)}\, i(\lambda)\, \alpha_n^2\, d\lambda}{(h^2+\alpha_n^2)\rho c_s} \qquad \text{.....(2)}$$

where $\alpha_n a J_1(\alpha_n a) = h a J_0(\alpha_n a)$

a is the radius of the cylindrical element

h is H/K

H is the Newtonian cooling coefficient for the cylindrical surface

K is the thermal conductivity of the fuel

ρ is its density

C_s is its specific heat

k is $K/\rho C_s$

and J_0 and J_1 are Bessel functions of the first kind of zero & first order.

Equation (1A) can be written in terms of a fixed distance scale x and time λ (see Fig. (2)). x may be considered as the distance to a given element of fuel from the front end of the burning zone when the rate of spread has reached a steady value.

$$i(\lambda) = I \exp - \beta(x - R\lambda) \quad \ldots\ldots\ldots\ldots\ldots(3)$$

We shall only be concerned with $\lambda < x/R$; R is the rate of spread of the fire. We have to integrate λ from 0 to x/R during which time θ rises from θ_Z the fuel temperature at Z ahead of the flame front to θ_0 the ignition temperature above ambient at the front B, i.e.

$$\theta_0 - \theta_z = \frac{2I}{\rho c_s a} \sum_{n=1}^{\infty} \frac{d_n^2}{h^2 + d_n^2} \int_0^{x/R} e^{-k d_n^2 (t-\lambda)} e^{-\beta(x - R\lambda)} d\lambda \quad \ldots\ldots\ldots\ldots\ldots(3a)$$

Replacing λ by $t - \lambda'$ and taking t as x/R the time when the front B arrives at the element of fuel being considered gives the following

$$\theta_0 - \theta_z = \frac{2I}{\rho c_s a} \sum_{n=1}^{\infty} \frac{d_n^2}{h^2 + d_n^2} \int_0^{x/R} e^{-k d_n^2 \lambda'} e^{-\beta R \lambda'} d\lambda'$$

Now when $x \to \infty$, $\theta_Z \to 0$, because the temperature of the fuel an infinite distance ahead of the front B is the ambient.

$$\therefore \quad \theta_0 = \frac{2I}{\rho c_s a} \sum_{n=1}^{\infty} \frac{d_n^2}{(h^2 + d_n^2)} \frac{1}{(\beta R + k d_n^2)} \quad \ldots\ldots\ldots\ldots\ldots(4)$$

Equation (4) is the general equation which is functionally of the form

$$\frac{H\theta_0}{I} = F\left(ha, \frac{h^2 k}{\beta R} \right) \quad \ldots\ldots\ldots\ldots\ldots(4a)$$

and this general equation is the form of the solution whatever the form of $f(z, \beta)$ in equation (1), provided $f(z\beta) \to 0$ at $Z \to \infty$. Similar forms would be deduced if the element were not cylindrical, provided it could be described in terms of one dimension only.

Where we have the case of a fuel element being heated slowly enough for there to be no effective thermal gradients across it, K is effectively infinite. The solution can be obtained from first principles or from considering the form of equation (4) where K tends to infinity for which it may be shown that

$$\theta_0 = \frac{2I}{\rho c_s a} \frac{1}{\beta R + \frac{2H}{\rho c a}}$$

$$\text{i.e.} \quad \rho c_s \beta R a \theta_0 = 2(I - H\theta_0) \quad \ldots\ldots\ldots\ldots\ldots(5)$$

When we have the case of a fuel element so thick that its centre remaints unheated even when the surface reaches θ_0, the solution cannot involve 'a'. The limiting form of equation (4) for $a \to \infty$ can be shown to be

$$\theta_0 = \frac{I}{H} \left\{ \frac{1}{1 + \sqrt{\frac{\beta R}{h^2 k}}} \right.$$

$$\text{i.e.} \quad \frac{\beta R}{h^2 k} = \left(\frac{I}{H\theta_0} - 1 \right)^2 \quad \ldots\ldots\ldots\ldots\ldots(6)$$

Solutions to equation (4) are given in Fig. (3), but it is not possible at this stage to discuss them until it is known how β varies for different conditions.

The characteristic length $1/\beta$ of the heat transfer region

Now β is essentially a function of flame size and orientation and like flame length it should be possible to express β as a function of D, 'r' the rate of burning per unit width* of fire and U the wind speed. For a fire which is effectively infinitely wide and of length D the relevant flow parameters are expected to be $r^2/\rho_g^2 g D^3$ and $\frac{u^2}{gD}$ where ρ_g is the cold fuel gas density and g the gravitational acceleration. The former term is a modification of $\frac{Q^2}{g D^5}$ which has been used for fires of circular or square base where ($\rho_g Q$) is the total rate of burning[3] (Q being the volumetric flow of flammable vapours ~~and~~ ρ_g their density)

We thus expect

$$\beta D = G\left\{\frac{u^2}{gD}, \frac{m''^2}{\rho_g^2 g D}\right\} \qquad \text{.....(7)}$$

where G is some function

m'' is the mean rate of burning per unit base surface $= \frac{r}{D}$

The function $f(x\beta)$

Some experiments have been made with cribs of wood of large width and length D varying from 6 in to 2 ft in winds of up to 15 m.p.h.

So far only radiation heat transfer rates normal to the base of the crib have been measured at various lee distances from the crib. Plotting these rates on a log-linear graph gives the results shown in Fig. (4) and it is seen that, to a first approximation it is reasonable to regard these lines as linear, i.e. an exponential shape function $e^{-x\beta}$ can be defined. Experiments have yet to be performed in which m'' is varied (by increasing the surface of fuel per unit base area of crib) and convective heating rates have yet to be studied.

The burning zone length D

If we define a mean mass rate of burning per unit base area of fire m'' as before then if ω is the weight of material per unit base area we have from the two expressions for the burning time of an element of fuel

$$\frac{R}{D} = \frac{m''}{\omega} \qquad \text{.....(9)}$$

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*By width we mean the direction perpendicular to the plane of Fig. (1).

Leaving aside the question of $I\,\varepsilon\,m''$ being factors dependent on wind speed, etc., we have a theoretical equation (4a) and two experimental relation equations (7) and (9) to determine the three quantities R, D and β. Eliminating β we have two relations between R and D which are, in principle, sufficient. The first is equation (9) expressing R as proportional to D and the second a combination of equations (4a) and (7) which may or may not be a linear relation. The experimental data obtained so far suggest that R is proportional to some fractional power (less than 1) of D.

Consider Fig. (5); these two relations are shown in diagrammatic form. The ordinate of curve (b) increases in proportion to $1/\beta$ and increases with I.

The point X_1 gives the equilibrium value of R and D. We now show this equilibrium to be stable.

Stability

Had curve (b) in Fig. (5) been a straight line there would be no equilibrium at any finite values of R and D. We now show that only because this curve is convex upwards is the equilibrium at X_1 a stable one.

Suppose the velocity of spread R slowly decreases. If this is slow enough for R to be approximately constant during the time D/R - for any pair of values of D and R then the position of equilibrium would tend to fall to X_2. At each instant the velocity that can be sustained by the heating rates ahead of the flame is greater than the R given by curve (a) and this would tend to increase R and equilibrium would then tend to be re-established. Were curve (b) concave upwards this would not be true and the velocity R would continue to fall to zero, i.e. the equilibrium would be unstable. Indeed, the velocity R and burning zone length D could not increase from an initial condition of R and D nearly zero.

Relation between this theory and Fons's theory

If in equation (3a) λ is replaced by $t-\lambda'$ we obtain

$$\theta_o-\theta_z = \theta_o\, e^{-\beta z} \qquad \text{.....(10)}$$

i.e. the temperature ahead of the flame front B falls off exponentially, according to the same law as the heat transfer rate. Fons assumes a heat transfer rate depending on constant transfer coefficients and we can equate the two expressions for fuel temperature a distance 'L' ahead of the flame front

$$\text{i.e.}\quad t_1-t_o = (t_i - t_o)\, e^{-\beta L}$$

$$\therefore \text{Log}\, C_t \left(\frac{t_f-t_o}{t_f-t_i}\right) = \text{Log}_e \left(1 + \frac{(t_i - t_o)(1 - e^{-\beta L})}{t_f - t_i}\right)$$

The right hand side is approximately given by $\frac{(t_i - t_o)(\beta L)}{t_f - t_i}$ so that

$$R = \frac{C\,(f_c + f_r)\,\delta\,(t_f - t_i)}{\bar{\gamma}\,\bar{C}_p\,(t_i - t_o)\,\beta}$$

Now in our notation $t_i - t_o$ is θ_o and $(f_c + f_r)\,(t_f - t_i)$ is the net heat transfer to an element at t_i, i.e. $I - H\theta_o$. δ for a cylinder is $2/a$ and hence

we have

$$R = \frac{2C(I - H\theta_0)}{a\rho c_s \theta_0 \beta}$$

which for C = 1, its theoretical value, is identical with equation (5). What in fact is missing from Fons's theory is any consideration of the relation between β and the burning zone length D and the effect of thickness of fuel in producing non-uniform heating of fuel. This latter approximation is justified in his experiments with thin fuels, but is not necessarily so for tests with thicker fuel elements.

Discussion

Measurements are being made of the heat transfer rates at various points on the lee side of localised fires from which it will be possible to test the validity of equation (7) and the exponential variation of heat transfer with distance. Even if this is not exponential over the whole range of experimental conditions it may still be possible to evaluate an effective length $1/\beta$ and obtain, experimentally, a correlation of the form of equation (4a).

Measurements of the spread of fire in long cribs are to be analysed to evaluate 'm''' and correlate with wind speed.

It is hoped that this theory provides a working hypothesis for the correlation of experimental data which will permit an extension to full scale conditions. The theory provides methods of evaluating, albeit partly empirically, basic dimensionless terms which include terms such as 'm''' and I and θ_0 which can be used to obtain a method of scaling up to the condition in the field.

References

(1) FONS, W. L. "Analysis of fire spread in light forest fuels". Jour. Agricultural Research, 72 No. 3, 1946.

(2) CASLAW, H. S. and JAEGAR, J. C. "Conduction of Heat in Solids" O.U.P. 1st Edition, p.176.

(3) THOMAS, P. H. "Buoyant Diffusion Flames". Combustion and Flame, 1960. 4 381.

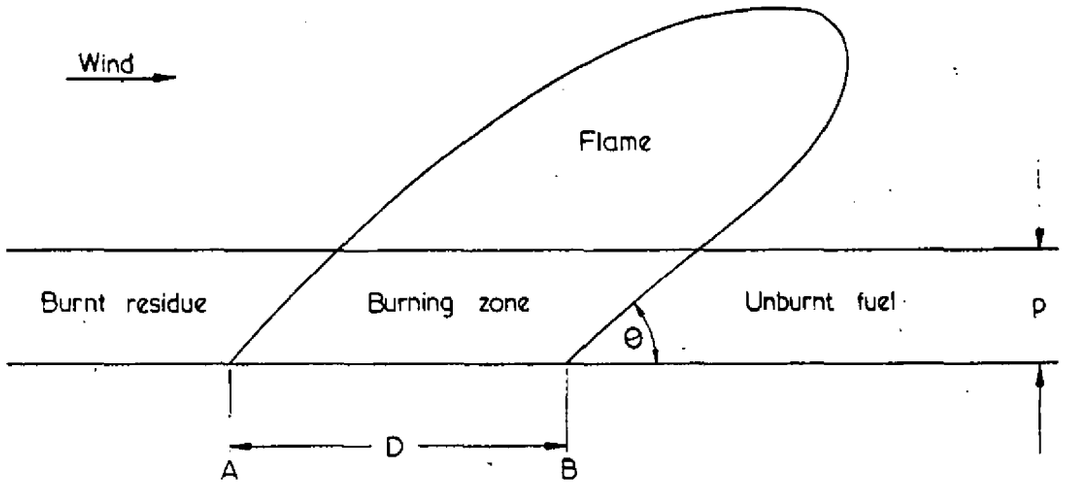


FIG. 1. DIAGRAMMATIC SKETCH OF FLAME SPREAD MODEL

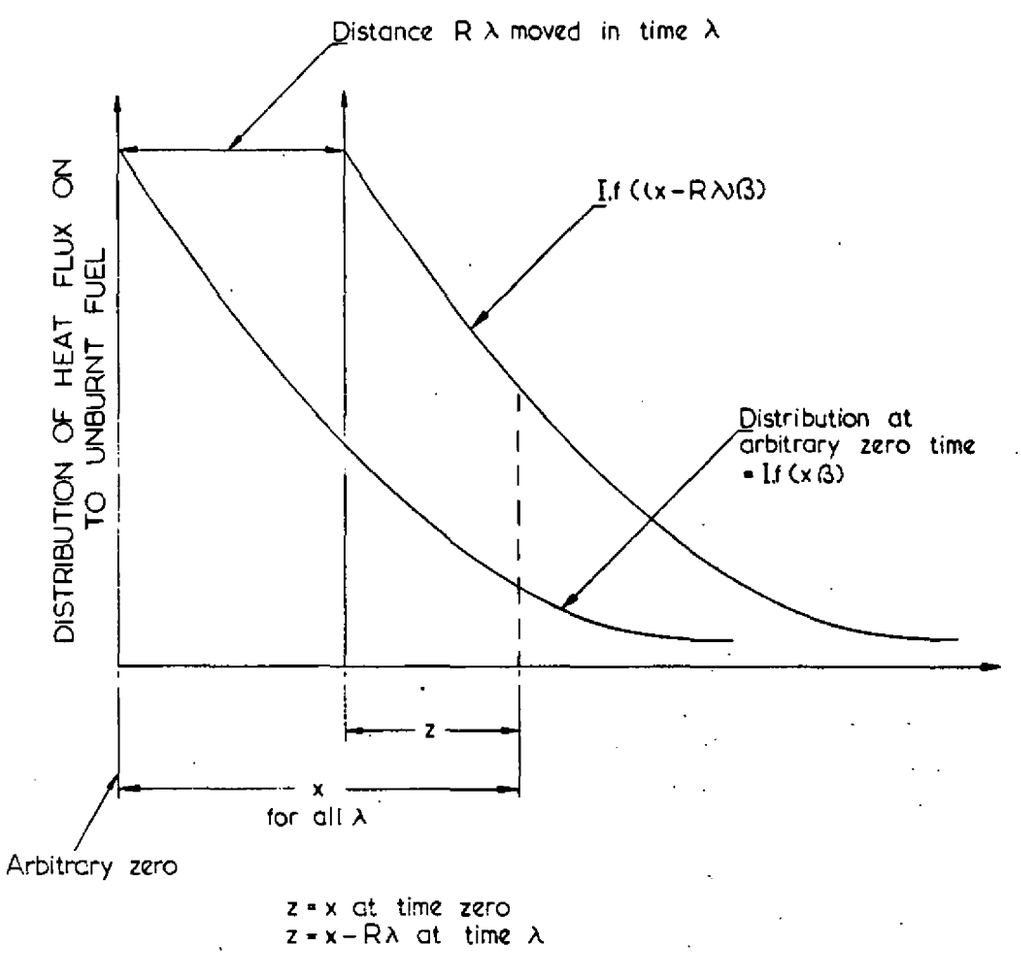
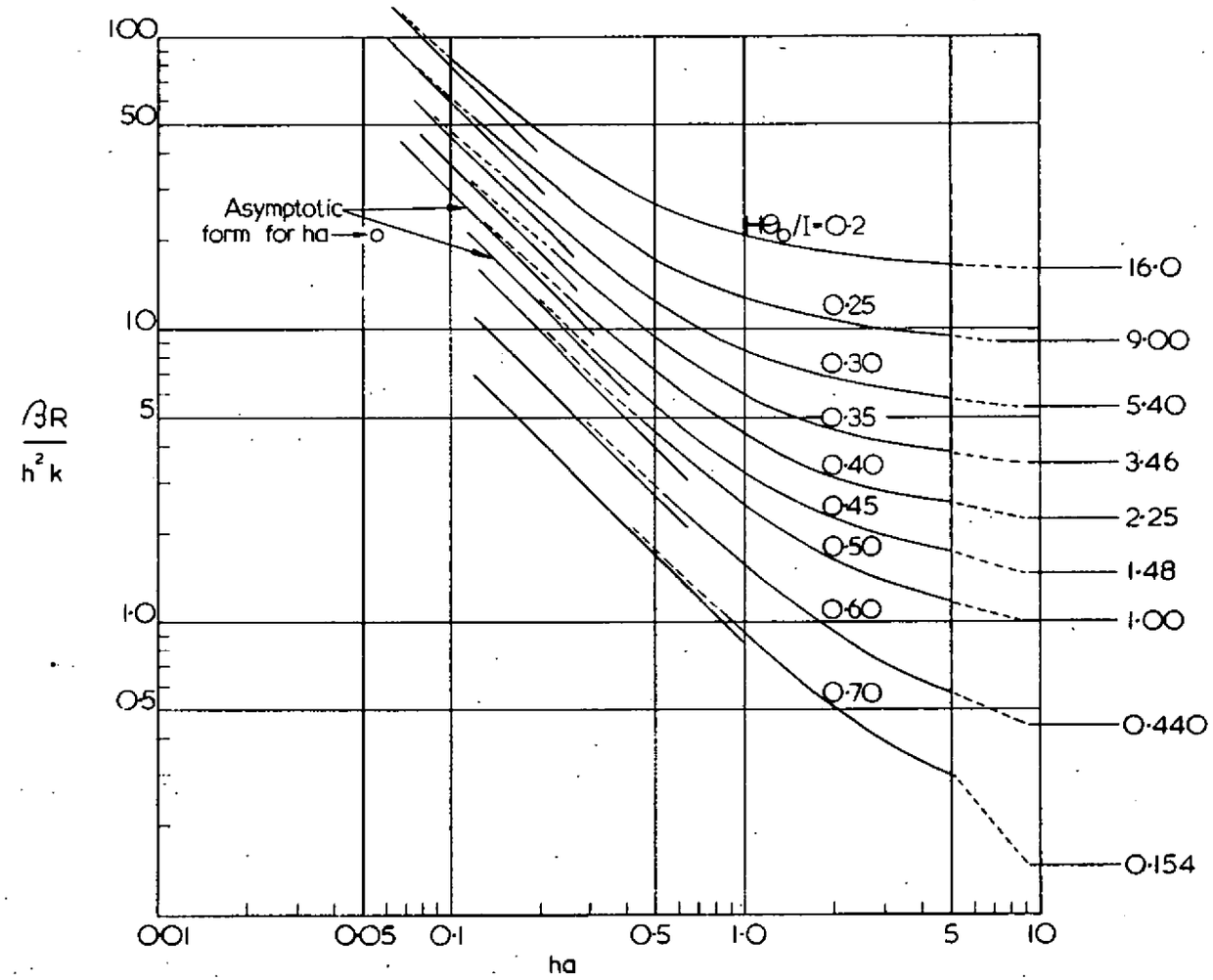


FIG. 2. DEFINITIONS OF FIXED AND MOVING DISTANCE SCALES



Asymptotic values for ha → o and ha → ∞ are given by equations (5) and (6) respectively

FIG. 3. CALCULATED PREHEATING RELATIONS

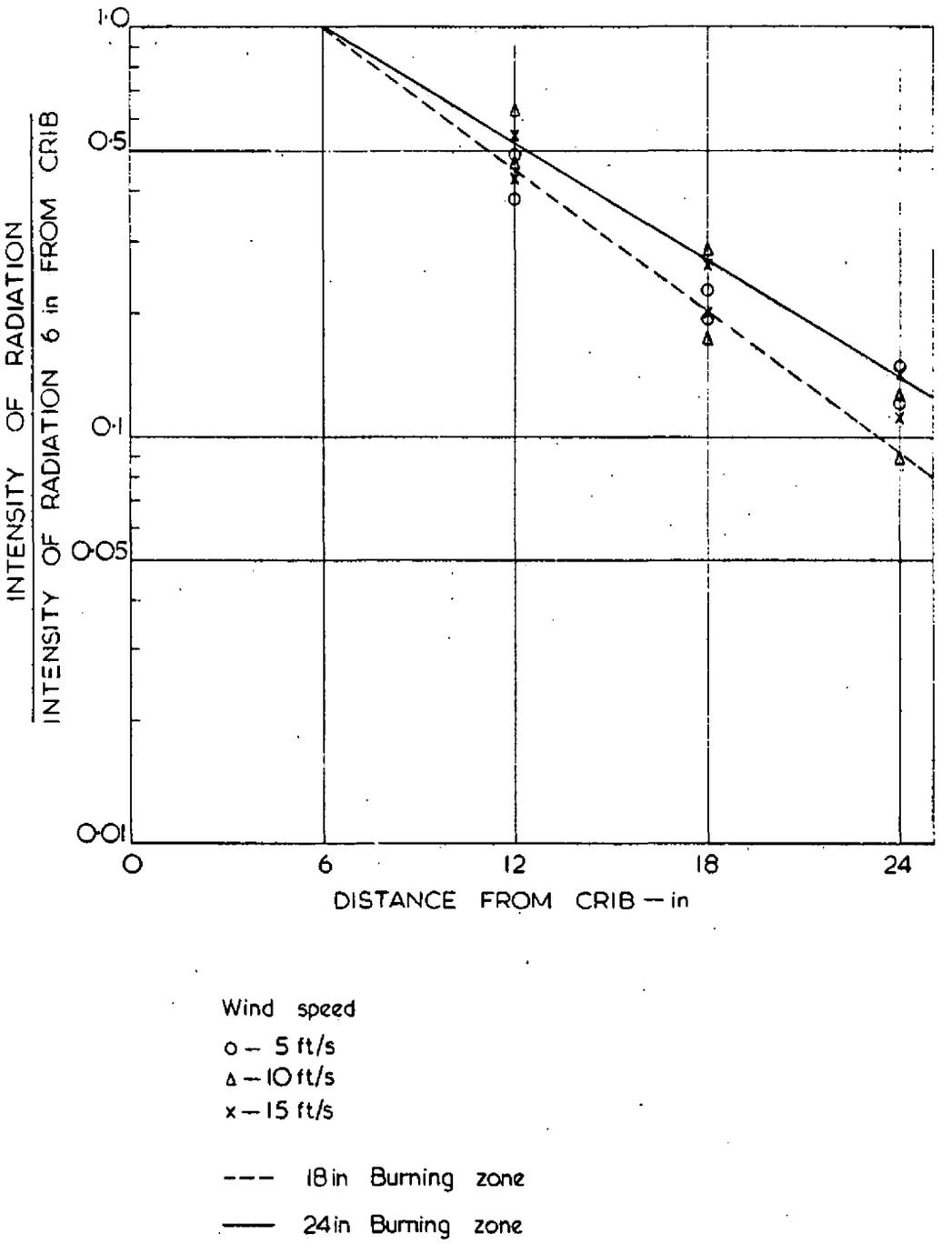


FIG. 4. APPROXIMATE EXPONENTIAL DECREASE IN INTENSITY OF RADIATION AHEAD OF FIRE

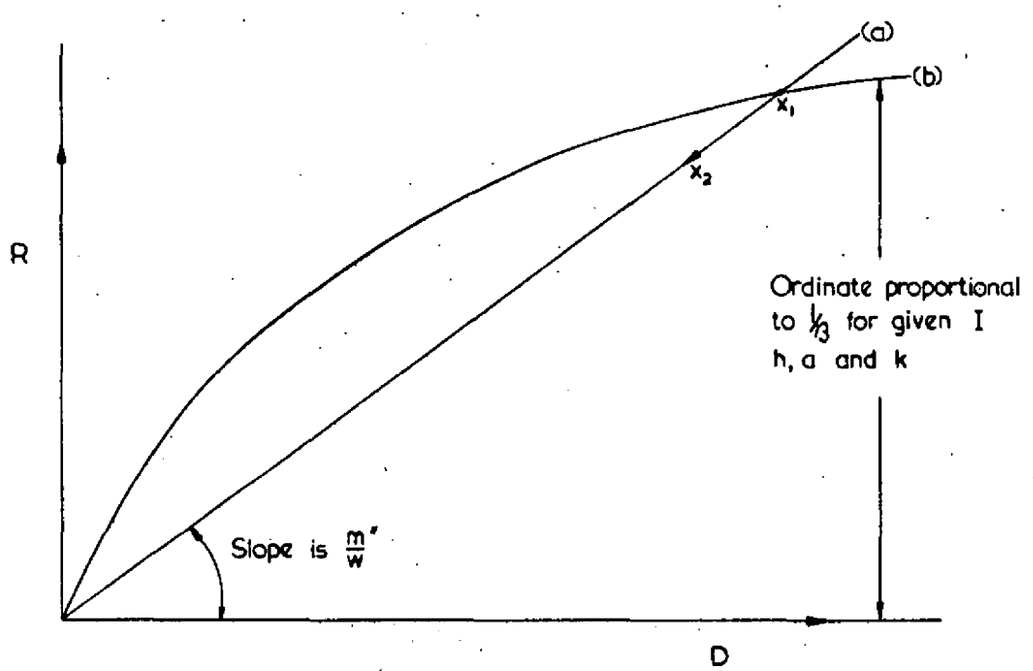


FIG. 5. DEFINITION OF EQUILIBRIUM R AND D