DEPARTMENT OF SCIENTIFIC AND INDUSTRIAL RESEARCH AND FIRE OFFICES' COMMITTEE
JOINT FIRE RESEARCH ORGANIZATION

ON THE HEIGHTS OF BUOYANT FLAMES

by

P. H. Thomas

Summary

Yokoi has recently reported extensive data on the temperature distribution in hot gases rising vertically or ejected horizontally from vertical windows and has used the distance at which a certain temperature (500°C) is reached as a criterion for the flame length. Thomas has measured flame length directly for some similar situations and the two types of correlation are compared and discussed.

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Introduction

Yokoi[1] has studied the vertical temperature distribution in the plume of hot gas above an alcohol fire and has distinguished between three domains; in the first, nearest to the fire, the temperature is approximately constant; in the second the temperature distribution is like that from a line source and, higher up, in the third it behaves like that from a point source. He has obtained these correlations for various ratios 'n' of the sides of the rectangular source.

Yokoi has also reported extensive experimental data correlated in terms of dimensionless variables for the temperature distribution and trajectories of streams of hot gas from windows in a vertical plane in the wall of an enclosure. He gives data for rectangular windows of various breadth to height ratios both for where there is a vertical wall above the window and where there is not, the latter condition being referred to by Yokoi as "free space". As a result of this work he was able to define an equivalent rectangular source on a horizontal plane to give approximately the same vertical temperature distribution as the vertical window. This was equal to the upper half of the window on a horizontal plane through the top of the window.

The author has presented dimensionless correlations of data for the heights of flames from wood fires on a square horizontal base[2] from a fire burning in a cubical enclosure with one side open[3].

Yokoi's analysis does not refer explicitly to flames, though he does define an equivalent flame height at the point where the temperature is 500°C on the argument that the radiation has largely ceased by the time the temperature falls to this value and he uses the height to this point to evaluate radiation transfer to nearby walls etc. There is thus a criterion by which his data and those of the author can be compared where these refer to similar conditions. The object of this paper is to make such a comparison.

Flames in the open

Yokoi correlates the maximum temperature rise $\Delta\theta$ at a height Z in the centre of hot gas from an alcohol fire on a rectangular base 'b' x 'a' by defining these parameters.

$$\mathcal{Z} = \frac{Z}{r_o} \qquad 1\text{ (i)}$$

$$n = \frac{a}{b} \text{ and} \qquad 1\text{ (ii)}$$

$$\textcircled{H} = \Delta\theta . r_o^{5/3} / \left(\frac{Q^2\, T_o}{c^2 \rho^2 g} \right)^{\frac{1}{3}} \qquad 1\text{ (iii)}$$

where Q is the convection heat output from the source,
c is the specific heat at constant pressure,
ρ is density of the gases,
g the accelaration due to gravity,
T_o the absolute temperature, and
r_o the characteristic dimension of the source which is defined as follows:-

$$r_o = \sqrt{\frac{a\,b}{\pi}} \qquad (2)$$

Yokoi then shows how η and (H) are related for different values of n.

For a square source of side D, such as that studied by Thomas, Webster and Raftery, n = 1 and $r_o = D$ and for this system Thomas showed that the height of flames from wood fires could be correlated in the range $2 < L/D < 10$ by

$$\frac{L}{D} = 4.4 \left(\frac{R^2 \times 10^6}{D^5} \right)^{0.305} \qquad (3)$$

where R is the rate of burning in g/s and D is in cm.

The dimensionless form of the parameter R^2/D^5 is $R^2/g\,\rho_f^2\,D^5$ where ρ_f is the density of the fuel volatiles.

The relationship between R and Q is of the form

$$C\,R = Q \qquad (4)$$

where C is the convective heat output per gm of fuel. Since R^2/D^5 is proportional to Q^2/r_o^5 the two parameters are effectively the same in this context.

Clearly, comparing a system in which there is combustion throughout a volume with a system in which combustion is assumed to occur at a plane source with heat thereafter conserved, the identification of the end of the combustion zone with a particular temperature in a non-burning system is open to some question, though it should be a reasonable approximation for the following reasons. If A is the rate of entrainment of air by volume at NTP up to the top of the flame, F the rate of supply of fuel by volume at NTP, and ρ_f the fuel density, then the mean temperature $\Delta\theta_f$ at the top of the flame is approximately given by the following equation

$$\rho_f\,F.C. \doteqdot \rho' c' \,\Delta\theta_f\,(F + A) + \text{heat loss by radiation}$$

where ρ' and c' refer to the combustion products. Hence, neglecting radiation loss

$$\Delta\theta_f \doteqdot \frac{\rho_f\,C\,F/A}{\rho' c' \left(\frac{F}{A} + 1\right)} \qquad (5)$$

Now although C varies between different fuels CF/A the calorific value with respect to unit air volume at N.T.P. varies much less. Since F/A is usually small compared with unity the value of $\Delta\theta_f$ can be approximately taken as constant to the extent that $\rho_f\ C\,F/A/\rho' c'$ is constant. Yokoi's method is a reasonable approximation for determining flame height at least for a given fuel, since the above expression for $\Delta\theta_f$ does not involve scale. Any difficulties in this approach are, however, avoided in the author's approach of a direct appeal to experimental measurements on flames for each particular fuel without invoking a temperature criterion.

The top of the flame must be as high as the point where mixing is complete, so we should expect it to be approximately at the upper limit of the first domain, which, according to Yokoi, is at Z equal to 2.5. The data he uses for a symmetrical source i.e. n=1 cover a range of 3.7 in r_o for which range Q/r_o^2 varies from 4.5 to 6.5 so that R^2/D^5 varies by 1.8 and hence, from equation (3), L/D should vary by $1.8^{0.30}$ i.e. by 1.2 which cannot easily be detected on Yokoi's log/log plot.

We shall attempt a comparison between the two correlations on the assumption that Yokoi's correlation is quite general and variations in R (i.e. Q) can be regarded as a variation in $\textcircled{H}$, $\Delta\theta_f$ being taken as constant.

Yokoi takes

$$c_{air} = 0.24 \text{ cal/gm}$$

$$\rho = 0.000456 \text{ g/cm}^{-3}$$

$$\Delta\theta_f = 500^{o}C$$

from which he obtains

$$\textcircled{H}_{500} = 1.717 \left(\frac{r_o^5}{Q^2}\right)^{\frac{1}{3}} \tag{6}$$

With C = 3560 cal/cm - a value taken by Yokoi for wood, equation (3), neglecting radiation loss from the flames, can be rewritten as

$$\frac{L}{\sqrt{\pi}\, r_o} = 4.4 \left\{ \frac{1.717^3}{\textcircled{H}^3} \times \frac{10^6}{3560^2} \times \frac{1}{\pi^{5/2}} \right\}^{0.305}$$

$$\frac{L}{r_o} = 2.5 \ \textcircled{H}^{-0.915} = 1.5 \left(\frac{Q^2}{r_o^5}\right)^{0.305} \tag{7}$$

Equation (7) is shown in Fig.(1) in comparison with Yokoi's results. We can compare the value of L from this with the value of Z where $\Delta\theta_f$ is 500°C, given by Yokoi for an open fire of 12 m square. Taking his value for Q^2/r_o^5 equation (7) gives

$$\frac{L}{r_o} = 3.28$$

compared with Yokoi's 3.96 (page 61, ref.1) and for 15 m square fire equation (7) gives 3.1 where Yokoi gives 3.43 and for a 6 m square fire 3.62 where Yokoi gives 4.2 (pages 68-9 ref.1). The estimates from equation (7) are lower than Yokoi's but a somewhat larger value for $\Delta\theta_f$ would make the agreement better. A better fit is obtained with $\Delta\theta_f$ equal to 650°C.

It is necessary to point out here that the form of Yokoi's correlation in the upper domain, being theoretically that for the plume from a point source

i.e. $$\frac{Z}{r_o} \propto \frac{1}{\textcircled{H}^{3/5}}$$

takes no account of the inertia of the fuel, i.e. his correlation is appropriate to flames which are entirely buoyancy controlled.

Thomas has pointed out the dual significance of the variable R in his correlation as a measure of air required for combustion and (in the form R^2/D^4) as a measure of fuel momentum. At high values of R^2/D^5, L/D should become constant as for a fuel jet. Thomas obtained a constant index between L/D and R^2/D^5 over a wide range of L/D. In the same range of Z/r_o there is a marked curvature in Yokoi's correlation. A more detailed analysis of the constant temperature criterion would be necessary to discuss the significance of this difference, which, however, is not large in view of the other approximations that have to be made in applying these correlations to practical problems.

Flames from windows

In the experiments by Yokoi on gases emerging from windows there is not a well defined transition between the first, second or third domain (see Figs. 8.5, 8.6 and 8.7 ref.1), and Yokoi's conclusion No.7 (p.102, ref.1) states that the temperature distribution law fails if there is significant flaming outside the window.

Because entrainment into a horizontal hot stream is weak, one can conceive of the stream of hot combustion products not losing temperature until the stream departs from the horizontal towards the vertical direction. The transition between the first and second domain can, therefore, be the end of the flame zone or the point where the trajectory reaches a certain inclination and since this is not a well defined point the transition is not sharply defined either, that is, it is not possible to define the end of the flame zone by any discontinuity. In view of Yokoi's conclusion, referred to above, one must assume his flames were short and therefore a comparison between his data and Thomas's means generalising Thomas's relation to regions where in fact Yokoi's experimental data were obtained in the cooled stream of combustion products.

Yokoi's use of Q for heat output is not entirely analogous to Thomas's use of R for quantity of fuel; Yokoi's data cannot be used where the flow of hot gas from the window has momentum arising from a buoyancy greater than the window height itself, i.e. his trajectories apply only to fires on the one storey.

Yokoi states that the temperature along the trajectory of a stream of gas from a window of breadth b and height h, is describable by the same parameters as described above, namely Z/r_o, (H) and n, where here

$$r_o = \sqrt{\frac{bh}{2\pi}} \qquad (8)$$

$$n = \frac{2b}{h} \qquad (9)$$

In Yokoi's correlations Z is the distance along the trajectory.

Flames from windows without a wall above.

For the case of 'free space', as Yokoi describes this condition, he gives the results that are shown in Fig.(2). In his case, Q refers to the heat content of gases leaving the window and Z the distance along the trajectory above the top of the window. He does not correlate the effects of n and it is seen that the experimental results for different values of n lie on different curves. Thomas[2] has shown that the results obtainted by Webster and Raftery for flames emerging from the open side of a cube of dimension D could be given by

$$\frac{L}{D} = 400 \left(\frac{R^2}{D^5} \right)^{\frac{1}{3}} \qquad (10)$$

Here, however, L was measured from the base of the cube and R, being the total rate of weight loss, includes the burning that actually takes place within the cubical enclosure. It is easy to correct for the difference in definition between L and Z, if use is made of Yokoi's correlation of Z and its vertical projection Z_o, but it is not easy to correct for the difference between R and Q. In practical situations R is the more usable parameter, though if this is done it is strictly necessary to separate the two components of this burning in any scaling or to obtain correlations for different shapes of enclosure as well as different windows.

Admittedly Yokoi's definitions in terms of external distance and external heat output Q are more fundamental, but in an actual experiment with flames it is not practical to divorce the internal and external parts of R. Thus, the L-R correlation is a practical one, though in fact it may depend on the shape of the enclosure to some extent as well as on the window. This is the penalty for being unable to split the effects outside the box from those inside the box, but assuming that one does define R as the total, then on the same arguments as have been used by both writers, the L-R correlation should, in principle, depend on compartment window geometry in addition to the other dimensions involved.

We define a rate of burning per unit width, R' $(= R/b)$, and then if we assume that increasing the width of the window has no effect on the flame height if R' is constant, it follows that

$$\frac{L}{D} = 400 \left(\frac{R'^2}{D^3} \right)^{\frac{1}{3}}$$

since for these experiments $b = D$.

Hence $$L = 400 \, R'^{\frac{2}{3}} \qquad (11)$$

where L is in centimetres and R' is in g cm^{-1} s^{-1}. In view of the fact that R and R' are based on experiments where the total rate of burning is measured it is probably best for the sake of consistency to include the total flame height and retain the definition in terms of L, otherwise we shall be including features of the behaviour inside the enclosure on one side of the equation and neglecting them on the other. For simplicity we shall identify L with Z and R with Q - the "errors" in both processes, though not equal, tend to compensate each other. Equation (11) may now be put into the form used by Yokoi with the above values of $\sigma \rho C$ and $\Delta\theta_f$ using equations (6), (8) and (9)

$$\mathcal{Z} = \frac{Z}{r_o} = \frac{400}{r_o} \left(\frac{Q^2}{6^2 \, c^2} \right)^{\frac{1}{3}}$$

$$= \frac{400}{n^{\frac{1}{3}}} \left(\frac{Q^2}{3560^2 \, \pi \, r_o^5} \right)^{\frac{1}{3}}$$

$$= \frac{2.02}{n^{\frac{1}{3}} \, Ⓗ} \qquad (12)$$

Thus we have Ⓗ Z = constant as has Yokoi in the middle range of his data.

Fig.(3) shows the data in Fig.2 recalculated as $n^{\frac{1}{3}} Z/r_o$ against Ⓗ from which it is seen that the presence of the factor $n^{\frac{1}{3}}$ correlates the data in the middle region better than originally presented. The line drawn assuming $\Delta\theta_f$ is 500°C with the specific heat and density of air at 500°C compares the flame height data with Yokoi's data on a common basis. The difference is here in the opposite direction to that obtained for an "open fire" and for better agreement a value of $\Delta\theta_f$ less than 500°C is necessary but the difference in the curves is too large for a small, realistic adjustment in $\Delta\theta_f$ to accommodate the discrepancies. Yokoi, in a personal communication, has suggested that this discrepancy is related to the discrepancy to which he drew attention in his report (page 95-6 and figure 8.5), i.e. if one calculates the temperature distribution along the trajectory from the "equivalent" horizontal source, equal to half the window one gets lower temperatures from the vertical window than from the horizontal equivalent.

This he suggests is due to a loss of heat vertically from the main stream while this is moving with a horizontal velocity component. This discrepancy is equivalent to the main stream when it is fully deflected having only 0.5 - 0.6 of the heat content Q at the source.

Allowing for this equation (12) becomes

$$n^{\frac{1}{3}} \frac{Z}{r_o} = \frac{2.02}{ⒽH} \times (0.55)^{\frac{2}{3}} = \frac{1.35}{ⒽH} \qquad (13)$$

and if this is plotted in Fig.(3) agreement is much better.

If this is the explanation, or a large part of it, then it is clear that the loss of heat makes a difference to the temperature distribution of heated gases but not to the position of the flame tip.

Yokoi states that the law, Z Ⓗ = constant, which corresponds to a line source should be usable up to a value of

$$= Z/r_o = 11.75\sqrt{n}$$

and the points that lie above this criterion are shown with a line through them and should be excluded in any comparison between the plotted points and the line derived from Thomas's data.

This criterion is equivalent to

$$Z/\ell = 6.6$$

which is more clearly seen to be the appropriate form of criterion for the upper point of the region where the source behaves as a source of infinite width, i.e. the correlation is independent of ℓ up to a value of Z proportional to ℓ. The value of Z along the trajectory is greater than the vertical projection of this distance so the discrepancy between the points shown in Fig.(3) and the line based on equation (11) is slightly greater than appears in Fig.(3) but this effect has been ignored here.

Flames from window - vertical wall above window

If heat loss to the wall is assumed to be the same as the radiation loss to "free space" an argument used by Yokoi can be employed to provide an approximate comparison between flames in "free space" and near a vertical wall. For wide windows under a vertical wall the air only enters the flame on one side and not on two sides as in "free space". The flame height would be the same as for "free space" with R' half as large. Therefore, in the presence of a wall, equation (11) is written as

$$L = 400\,(2R')^{\frac{2}{3}} \qquad (14)$$

$$= 640\,R'^{\frac{2}{3}} \qquad (15)$$

i.e. it is $2^{\frac{2}{3}}$, i.e. 1.6 times taller than for the "free space" condition.

Figs.(4) and (5) are drawn as before, the latter converting Yokoi's data to $n^{\frac{1}{3}}\frac{Z}{r_o}$ and the line from equation (13) drawn as

$$\frac{n^{\frac{1}{3}} Z}{r_o} = \frac{1.6 \times 2.02}{\textcircled{H}} = \frac{3.23}{\textcircled{H}} \qquad (16)$$

Here we see that these points for different values of n are no longer on a single correlation and the line derived from Thomas's data lies above all of them as it did in the case of the "free space" data. The agreement is best for the data for high "n" as indeed it should be since these most nearly correspond to the assumption implicit in generalising Thomas's data that the flames are effectively from an infinite strip source for which the correlation should be independent of n. Taking Thomas's data, even allowing for the fact that Yokoi's Z is measured along the trajectory, over-estimates the vertical projection of flame length as defined by Yokoi.

It has not been possible to bring together Yokoi's data for different "n" by making any allowance for the extra heat loss to the wall or for the fact that the trajectories differ for different "n". Making the same allowance for the loss of heat as before equation (16) becomes

$$n^{\frac{1}{3}} Z/r_o = \frac{3.23}{H} \times (0.55)^{\frac{2}{3}} = 2.16/\textcircled{H} \qquad (17)$$

which is in better agreement with Yokoi's temperature data.

The assumption that the correlation in terms of $n^{\frac{1}{3}}Z/r_o$ should be independent of $\mathcal{J}$ in the second domain is applicable over a smaller range of "n" when there is a vertical wall than when there is not, because of the effect of the wall on the trajectory.

Difference between correlations

The error introduced by equating L and Z which, even if Z is regarded as vertical distance, differ by the window height, should become small when Z/r_o becomes large. Likewise the relative importance of the fraction of R' burning inside the enclosure will become smaller as Z/r_o increases and this error should not appear as a constant error over the whole range which is in fact the case. In other words Thomas's correlation is of a form $Z\textcircled{H}$ = constant, which constant, irrespective of the origin of Z and of R, should be asymptotic to Yokoi's data. It should also be borne in mind that the correlation given by Thomas for open fires is based on photographs of flames while that for flames from windows is based on visual observation. It has since been found that this may introduce a 10-30 per cent difference (visual measurements being larger than photographic measurements).

Yokoi does imply (conclusion (7) p.102) that his correlation fails when the gases continue to burn after emerging from the window but this would apply to his data for "open" fires as well. However, the correlations agree reasonably well if allowance is made for a vertical loss of heat from the main stream while moving with a horizontal component. This loss of heat was taken as 45% an empirical estimate by Yokoi on the basis of a comparison between vertical streams of hot gas and streams from windows.

Protection from flames

Open fires

Yokoi applies his data from open fires to the protection of television towers by evaluating the height to the point at which $\Delta\theta_f$ = 500°C on the grounds that steel does not lose strength below this temperature. This is equivalent, on his definition of flame length, to protecting only to the top of the flames.

Protection spandrel

Yokoi gives arguments for taking the 500°C point as the highest point at which windows above a burning room can be broken, i.e. he states the spandrel must extend up to the top of the flames. Thus in both these cases it is sufficient to consider flame length only and therefore a correlation made directly in terms of flame height could be employed to evaluate the necessary protection.

Protection by projections

Thomas has no data for the flame length when there are horizontal projections above the window but within the range of conditions examined Yokoi states that the temperature at any distance along the trajectory is between the value for "free space" and when there is a vertical wall. However, the size of the projection alters the trajectory markedly. Yokoi again takes the 500°C point along the trajectory, irrespective of the position of the trajectory with respect to the wall.

Simplification of Yokoi's calculations

Yokoi's trajectories are shown as multiples of H" the distance between the neutral pressure plane and the top of the window. Yokoi gives values of H"/h for a wide range of temperature fire load conditions but for temperatures above 400°C these ratios do not vary by more than ± 5 per cent from 0.64 (Table 7.1 and 9.7).

Taking a constant H"/h should simplify the labour of calculation without sacrificing too much accuracy.

To a first approximation we could regard projections in the range examined by Yokoi as producing a variation in temperature along the trajectory (albeit this has changed position) midway between the "free space" and the vertical correlations. Thus with a 60 per cent factor of safety one need only use Yokoi's trajectory data for the case where there are horizontal projections.

Conclusions

A comparison has been made between Yokoi's experiments where Yokoi's dimensionless temperature range corresponds mainly to a range of actual temperature from point to point and Thomas's correlations which in terms of Yokoi's dimensionless temperature correspond to a fixed temperature and a range of values of Q^2/r_o^5. They are similar in form, but for flames from windows Thomas's data correspond to twice the height obtained by Yokoi if $\Delta\theta_f$ is taken as 500°C. Another way of expressing this is that Yokoi suggests that there is a loss of heat from the main stream of hot gas which affects the temperature distribution but this would not appear to affect the position of the flame tip to the same extent. Some simplifications in Yokoi's procedure are probably permissible for the purpose of developing fire regulations.

References

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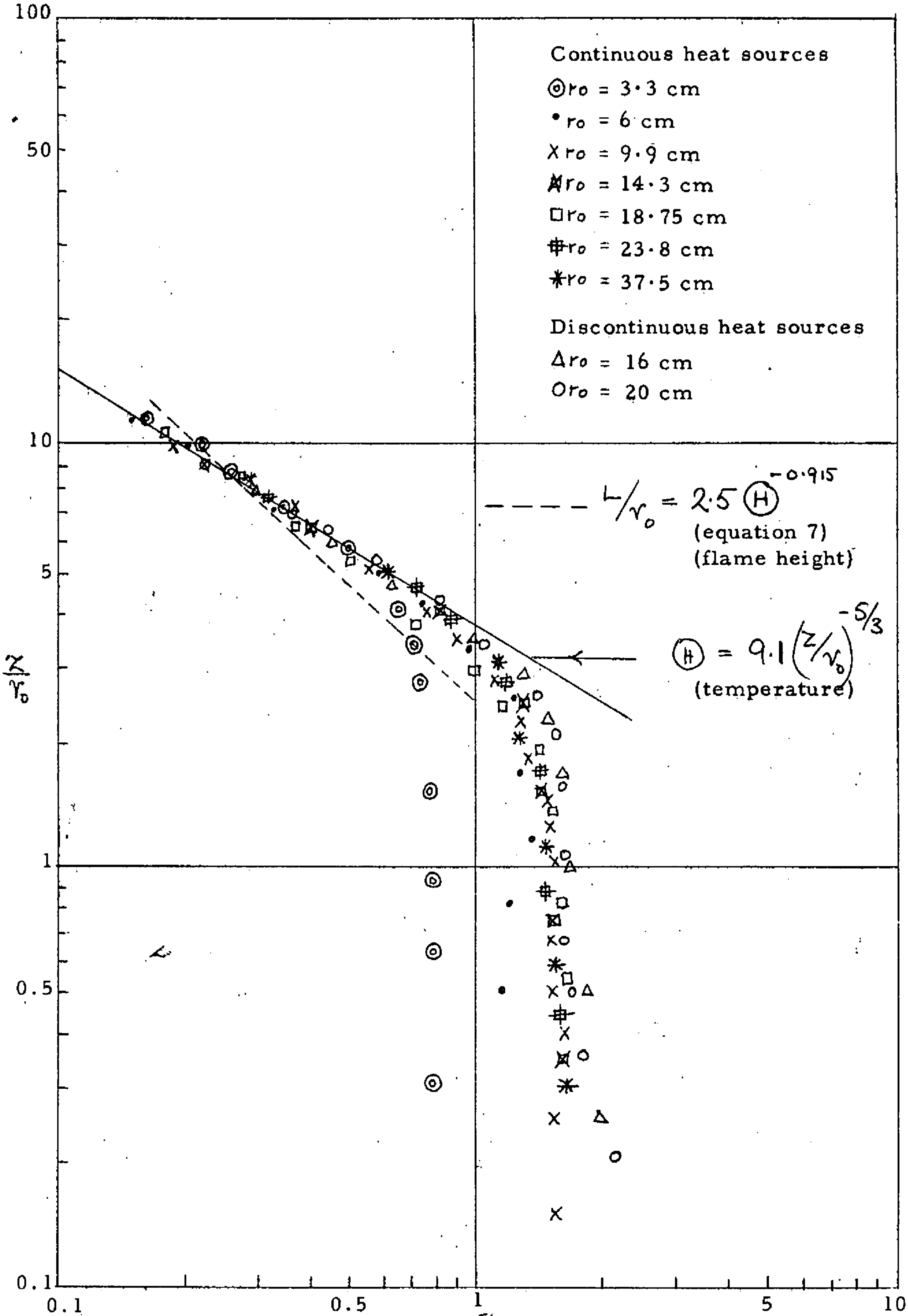
3. THOMAS, P. H. V.F.D.B. 1960. December Special Issue No.3 p.96.



$$\textcircled{H} = \frac{\Delta\theta r_0^{5/3}}{\left(\frac{Q^2\theta_0}{C_p^2\rho^2 g}\right)^{1/3}}$$

FIG. 1. CORRELATION FOR HORIZONTAL, CIRCULAR AND SQUARE HEAT SOURCES

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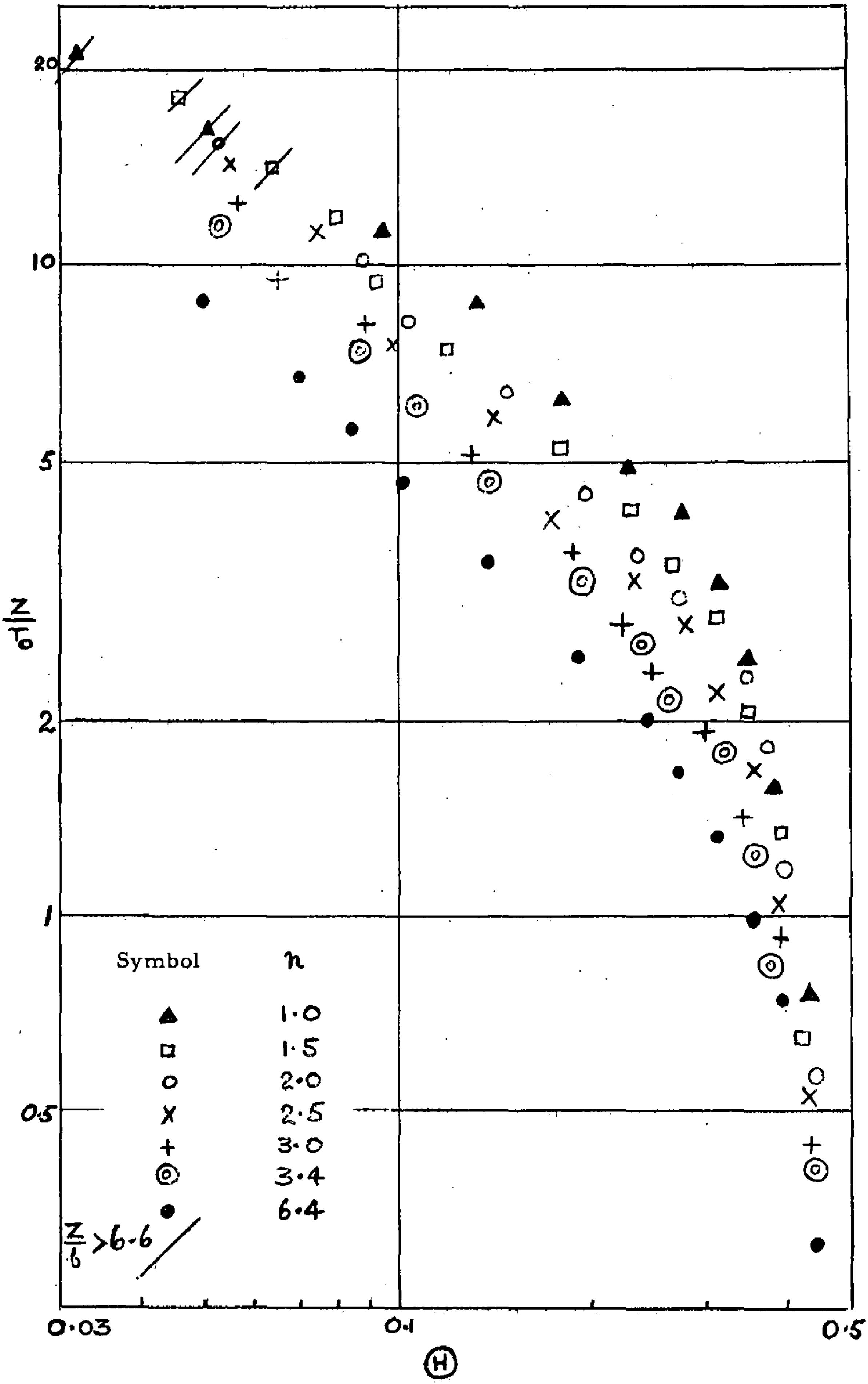

FIG. 2. YOKOI'S DATA FOR WINDOW BELOW FREE SPACE

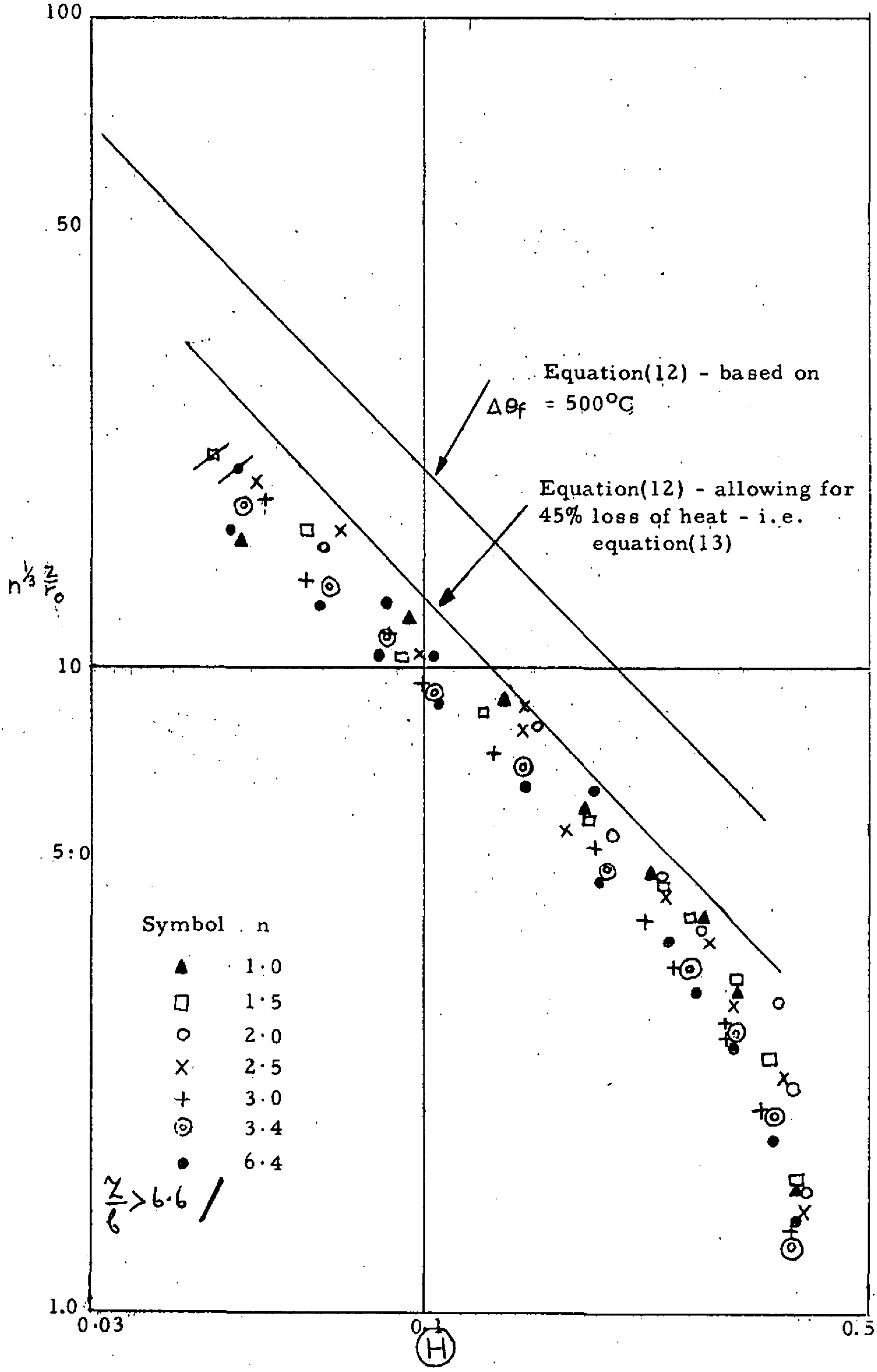

FIG. 3. YOKOI'S DATA (FIG. 2) REPLOTTED AS $\frac{z}{r_o} n^{1/3}$

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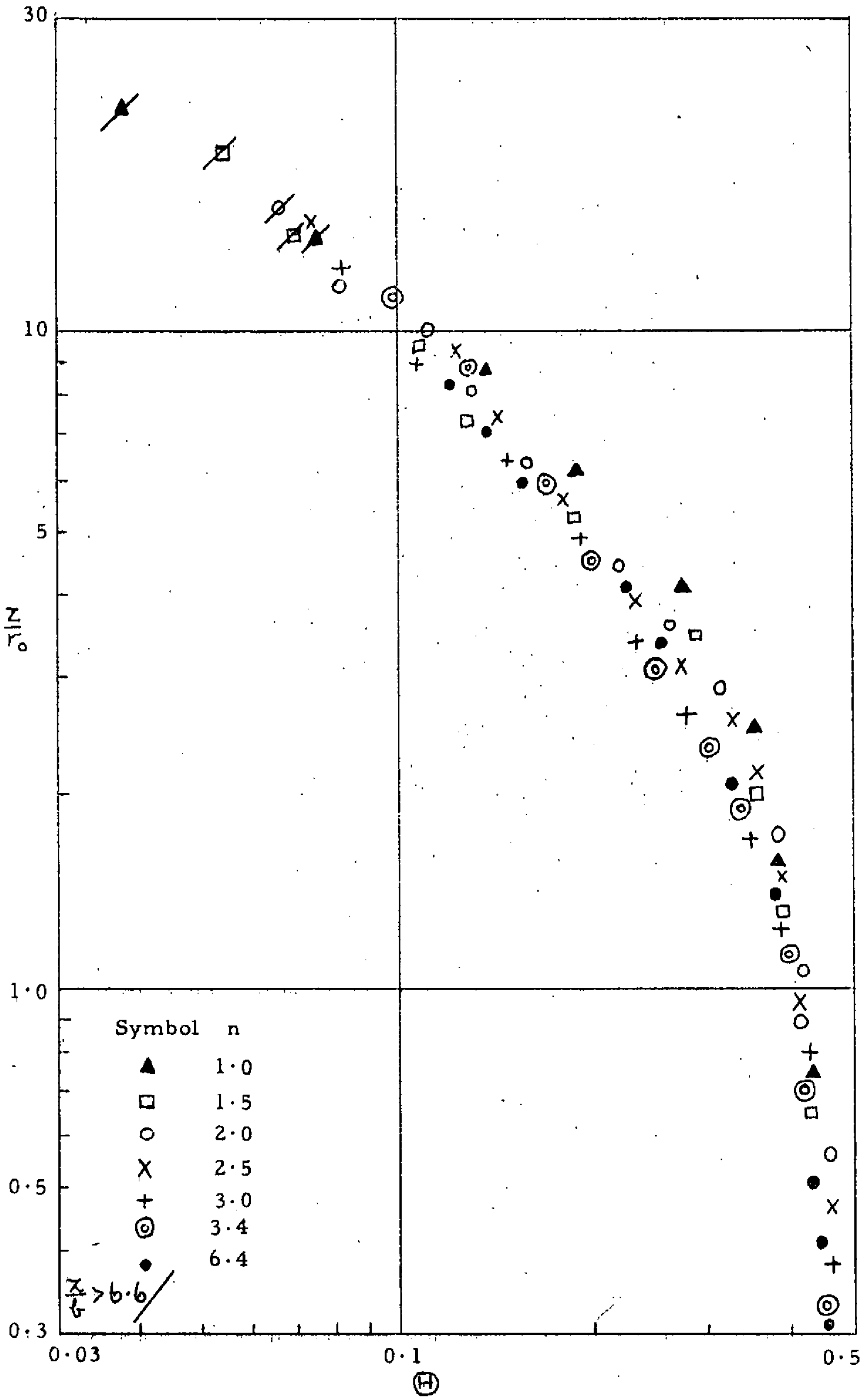

FIG. 4. YOKOI'S DATA FOR WINDOW BELOW VERTICAL WALL

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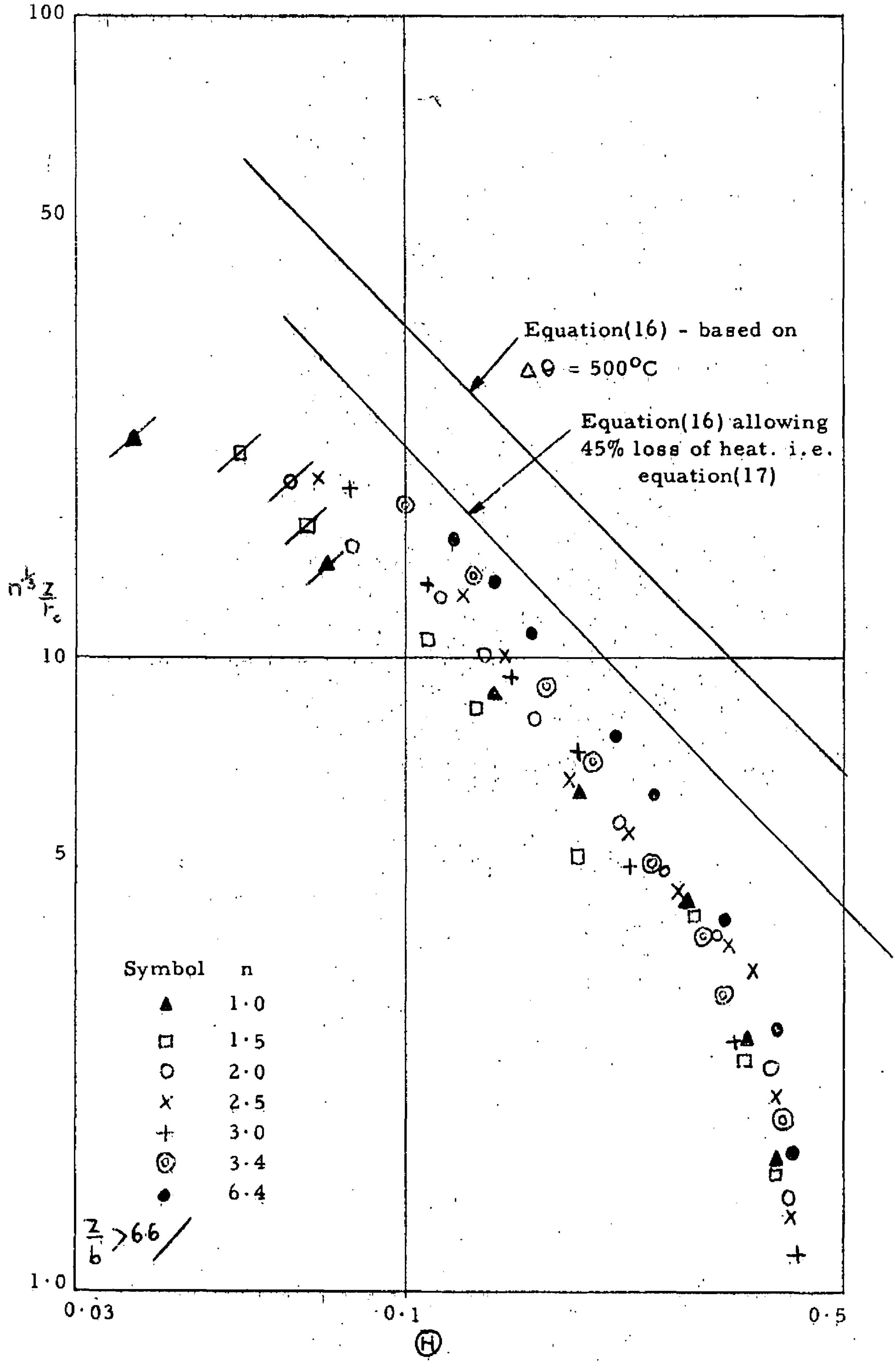

FIG. 5. YOKOI'S DATA (FIG.4) REPLOTTED AS $\frac{z}{r_o} n^{1/3}$

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