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A NOTE ON THE OPERATION OF HEAT SENSITIVE LINE DETECTORS

by

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SUMMARY

The temperature distribution in the element of a heat-sensitive line detector has been analysed. A method of assessing the performance of this type of detector has been suggested.

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1. Introduction

Heat sensitive fire detectors depend for their operation on the increase in temperature of a sensitive element, due to heat transfer from the hot gases rising from a fire. This increase results in a change in some physical property of the element which causes the alarm to be given. In many designs of detector the element is small and is affected only by air temperature changes in its immediate vicinity. Such detectors are described as 'point' or 'spot' detectors. In other detectors the element extends linearly over a large ceiling area and is, therefore, subject to the varying air temperature conditions over this area.

The cumulative effect on the element under these conditions will differ from that on a point detector, which has already been studied(1)(2) mainly in two respects. First the rate of rise of air temperature to which the element is subjected will vary over its length(3) and secondly the temperature rise at any point in the element may depend amongst other factors on the conduction of heat along the element.

In the following analysis the importance of these factors is examined in relation to possible methods of assessing the performance of this type of detector.

2. Theoretical

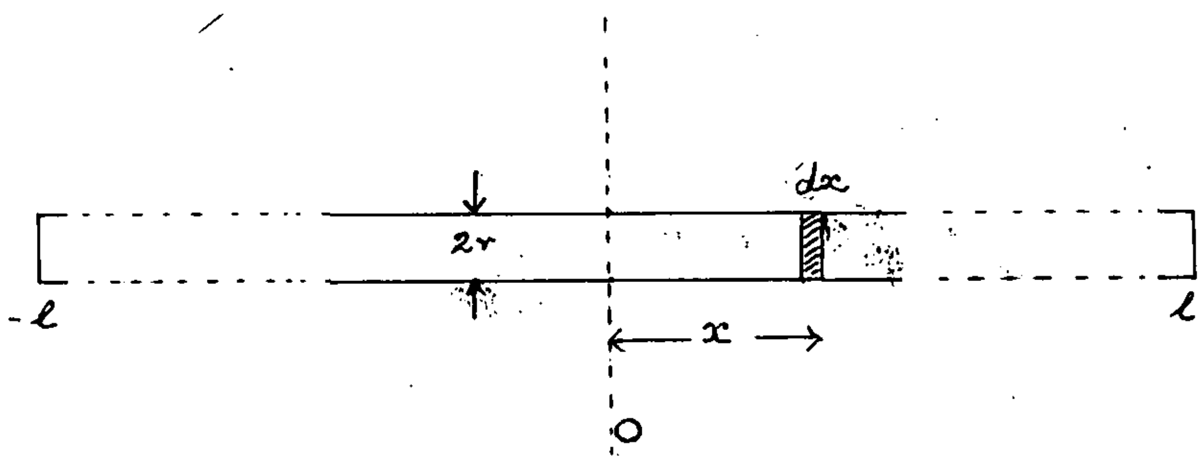

FIG. 1

Fig. 1 represents a section of a line detector element of length 2ℓ, diameter $2r$, constructed of a material with thermal conductivity K, density ρ and specific heat c. We shall assume that the ends of the element are maintained at a constant temperature and that the air temperature rise, to which any elemental section of the element is subjected, varies linearly with time(1)

The rate of rise of air temperature α_x at any point x will be assumed to be a function of x only, this function being symmetrical about the point $x = 0$. Neglecting radiation and convection losses, the conduction equation may be written

$$H.2\pi r\left(\alpha_x t - \theta\right) + K\pi r^2 \frac{d^2\theta}{dx^2} = \pi r^2 \rho c \frac{d\theta}{dt} \quad \text{........(1)}$$

where θ is the temperature rise of the element at any point x and time t,

and H is the heat transfer coefficient between the air and the element.

Hence

$$2H\left(\alpha_x t - \theta\right) + Kr\frac{d^2\theta}{dx^2} = r\rho c \frac{d\theta}{dt} \quad \text{........(2)}$$

Applying the Laplace transformation to Equation (2) we obtain, with the usual notation

$$2H\left(\frac{\alpha_x}{p^2} - \bar{\theta}\right) + Kr\frac{d^2\bar{\theta}}{dx^2} = r\rho c p\bar{\theta} \quad \text{........(3)}$$

where $\bar{\theta}$ is the Laplace transform of θ

To solve Equation (3) it is necessary to insert the form of α_x. Experiments have shown that if the point $x=0$ is vertically above a fire then the air temperature rise at any instant can be expected to decrease as x increases to $\pm \ell$. A form of α_x between $x = 0$ and $x = \ell$, which fits the experimental results approximately and enables Equation (3) to be solved is

$$\alpha_x = \alpha_0 e^{-\beta x} \quad \text{........(4)}$$

where α_0 is the rate of rise of air temperature directly above a fire

and β is a constant which depends on the height and configuration of the ceiling near which the element is mounted.

Substituting this form of α_x in Equation (3) gives

$$\frac{d^2\bar{\theta}}{dx^2} - a^2\bar{\theta} + b e^{-\beta x} = 0 \quad \text{........(5)}$$

where $a^2 = \dfrac{2H}{Kr} + \dfrac{\rho c p}{K}$

and $b = \dfrac{2H\alpha_0}{Krp^2}$

The solution of Equation (5) with the boundary conditions that $\bar{\theta} \to 0$ as $x \to \ell$ and $\dfrac{d\bar{\theta}}{dx} \to 0$ as $x \to 0$, is

$$\bar{\theta} = \frac{b e^{-\beta x}}{(a^2-\beta^2)} - \frac{b\beta \sinh a(\ell - x)}{a(a^2-\beta^2)\cosh a\ell} - \frac{b e^{-\beta\ell}\cosh ax}{(a^2-\beta^2)\cosh a\ell} \quad \text{........(6)}$$

The expression for Θ derived from Equation (6) is very complex and we shall therefore consider a simpler set of conditions, which shows that a number of valuable simplications may be made to the theory.

First we shall assume that the rate of rise of air temperature to which the element is subjected is constant over its whole length by putting $\beta = 0$ in Equation (6). This then reduces to

$$\bar{\Theta} = \frac{b}{a^2}\left\{1 - \frac{\cosh ax}{\cosh al}\right\} \qquad \text{........(7)}$$

Expanding $1 - \frac{\text{Cosh } ax}{\text{Cosh } al}$ in a cosine series and using known inversion formulae (4) the expression for Θ follows

$$\Theta = \alpha_0 t\left\{1 - \frac{\cosh\sqrt{\frac{2H}{K r}}\,x}{\cosh\sqrt{\frac{2H}{K r}}\,l}\right\} -$$

$$- \sum_0^\infty \frac{(-1)^n 4\alpha_0 \Upsilon \cos(2n+1)\frac{\pi x}{2l}}{(2n+1)\pi\left[(2n+1)^2\pi^2\frac{K r}{8Hl^2} + 1\right]^2}\left(1 - e^{-\left[(2n+1)^2\pi^2\frac{K r}{8Hl^2} + 1\right]t/\Upsilon}\right) \qquad \text{........(8)}$$

where $\Upsilon = \frac{r\rho c}{2H}$

Fig.2 shows the temperature distribution to be expected in a detector element 300 cms long after being subjected to a rate of rise of air temperature of 30°C per minute for two minutes. The value of Υ is taken as 20 records and the thermal diffusivity $K/\rho c$ as 1 $cm^2\ s^{-1}$. It can be seen, therefore, that under this simplified condition, the temperature rise of a detector element with these dimensions and physical properties is sensibly uniform over almost the entire length of the element.

If the operation of the detector occurs when a given physical property of the element undergoes a given change, which varies linearly with temperature, then it is the mean temperature rise which will determine the overall change in this property. The mean temperature rise Θ_m is given by

$$\Theta_m = \frac{1}{l}\int_0^l \Theta\, dx \qquad \text{........(9)}$$

Hence

$$\Theta_m = \alpha_0 t\left\{1 - \frac{\tanh\sqrt{\frac{2H}{K r}}\cdot l}{\sqrt{\frac{2H}{K r}}\cdot l}\right\} -$$

$$- \alpha_0 t \sum_0^\infty \frac{8\left(1 - e^{-\left[(2n+1)^2\frac{\pi^2 K r}{8Hl^2} + 1\right]t/\Upsilon}\right)}{(2n+1)^2\pi^2\left[(2n+1)^2\frac{\pi^2 K r}{8Hl^2} + 1\right]^2} \qquad \text{.......(10)}$$

In the example taken $\pi^2 K r/8Hl^2$ is equal to 2.2×10^{-3} and in most practical cases will be very much less than unity. Thus Equation (10) reduces to

$$\Theta_m = \alpha_0\left\{t\left(1 - \frac{\tanh\sqrt{\frac{2H}{K r}}\,l}{\sqrt{\frac{2H}{K r}}\cdot l}\right) - \Upsilon\left(1 - e^{-t/\Upsilon}\right)\right\} \qquad \text{.......(11)}$$

Further, if the value of $\sqrt{\frac{2H}{K\gamma}}\cdot \ell$ exceeds 10, which is likely for most elements longer than about 100 cms, then the expression for Θ_m becomes

$$\Theta_m = \alpha_o\left\{t - \gamma\left(1 - e^{-t/\gamma}\right)\right\} \qquad \text{.........(12)}$$

Equation (12) is identical with the expression which would be obtained were the element considered as a point detector element with the same value of Υ (1). It is thus possible to neglect the effect of thermal conductivity in most cases where the length of element exceeds a given value, if it is subjected to a uniformly rising air temperature over its whole length.

It is now necessary to consider whether the above simplifications are permissible if the element is subjected to a rate of rise of air temperature which varies over its length in the form suggested at the outset of the analysis.

Equation (6) gives the Laplace transform of the temperature rise at any point and the transform of the mean temperature $\bar{\Theta}_m$ is given by

$$\bar{\Theta}_m = \frac{1}{\ell}\int_0^{\ell} \bar{\Theta}\, dx \qquad \text{.........(13)}$$

i.e. $$\bar{\Theta}_m = \frac{b\left(1 - e^{-\beta\ell}\right)}{\beta\ell\left(a^2 - \beta^2\right)} - \frac{b\beta\left(1 - \frac{1}{\cosh a\ell}\right)}{a^2\ell\left(a^2 - \beta^2\right)} - \frac{b e^{-\beta\ell}\tanh a\ell}{a\ell\left(a^2 - \beta^2\right)} \qquad \text{.......(14)}$$

Now the previous values of Υ, ℓ and $k = \frac{K}{\rho c}$ used suggest that in many cases $\frac{1}{\cosh a\ell} \ll 1$ and $\frac{\tanh a\ell}{a\ell} \rightarrow 0$. With these approximations Equation (14) reduces to

$$\bar{\Theta}_m = \frac{b\left(1 - e^{-\beta\ell}\right)}{\beta\ell\left(a^2 - \beta^2\right)} - \frac{b\beta}{a^2\ell\left(a^2 - \beta^2\right)}$$

This gives

$$\Theta_m = \frac{\alpha_o}{1 - \Upsilon k\beta^2}\left\{t - \frac{\Upsilon}{\left(1 - \Upsilon k\beta^2\right)}\left(1 - e^{-\left(1 - \Upsilon k\beta^2\right)t/\gamma}\right)\right\}\left\{\frac{1 - e^{-\beta\ell}}{\beta\ell}\right\} -$$

$$- \frac{\Upsilon k\beta^2}{\left(1 - \Upsilon k\beta^2\right)}\frac{\alpha_o}{\beta\ell}\left\{t - \gamma\left[\frac{2 - \Upsilon k\beta^2}{1 - \Upsilon k\beta^2} - \frac{\left(1 - \Upsilon k\beta^2\right)e^{-t/\gamma}}{\Upsilon k\beta^2} + \frac{e^{-\left(1 - \Upsilon k\beta^2\right)t/\gamma}}{\Upsilon k\beta^2\left(1 - \Upsilon k\beta^2\right)}\right]\right\} \qquad \text{...(15)}$$

Some temperature distributions near ceilings due to a fire have been determined experimentally (3) and these show that the value of β is probably of the order 10^{-3} cm^{-1}. Thus $\Upsilon k\beta^2 \ll 1$ and we may neglect the second expression in Equation (15) which then reduces to

$$\Theta_m = \frac{\alpha_o}{\beta\ell}\left(1 - e^{-\beta\ell}\right)\left\{t - \gamma\left(1 - e^{-t/\gamma}\right)\right\} \qquad \text{........(16)}$$

Equation (16) shows that even when subjected to a varying rate of rise of air temperature, the effect of thermal conductivity in most line detector elements may be neglected and they can be treated as an infinite number of elemental detectors extended over a length 2ℓ.

3. The thermal testing of line detectors

Ideally, detectors should be tested under the conditions which occur in practice. However, it is difficult to achieve these in a laboratory apparatus and the following method is suggested, which can be carried out in an apparatus already in use (5) for testing point detectors. This apparatus enables a length of detector to be subjected to a uniform rate of rise of air temperature.

It is important to note that the length of detector element used in any given situation may vary widely and thus any test procedure in which a standard length of detector is examined must yield results from which the performance of any length may be assessed.

We will assume for example that a line detector is designed to operate when its total resistance has changed by a given value ΔR. This is equivalent to the "setting" (6) of a point detector. If the length of detector which can be accommodated in the apparatus is 2L and its resistance per unit length at ambient temperature is R_0, then the response time t when subjected to a rate of rise of air temperature α_0 will be given by Equation (12) as

$$\alpha_0 \left[t - \tau\left(1 - e^{-t/\tau}\right) \right] 2 R_0 \gamma L = \Delta R \qquad \text{.........(17)}$$

where γ is the temperature coefficient of resistance of the element.

The response times of detectors are generally long enough to make $e^{-t/\tau}$ small compared with unity and we obtain

$$t = \frac{\Delta R}{2 R_0 \gamma L \alpha_0} + \tau \qquad \text{.........(18)}$$

Equation (18) shows that the response time of the detector will depend markedly on the length of detector for any given value of ΔR as well as the rate of rise of air temperature to which it is subjected. Thus, if the response time of a detector is limited to values such as have been suggested for point detectors (5), this suggests that the length of a line detector with a given "setting" should lie between limits which give these response times., If one of the critical values of response times at a rate of rise of air temperature α_0 is t_c then it follows from Equation (18) that the length of detector $2L_c$ which will give this response time under the same conditions is given by

$$L_c = L \frac{(t - \tau)}{(t_c - \tau)} \qquad \text{.........(19)}$$

The value of τ may be deduced, using Equation (18), by determining the response time of a given length of detector over a range of rates of rise of air temperature. If in practice the rate of air temperature rise varies exponentially from the centre of the element, as assumed in the analysis in Section 2 then the permissible length of detector $2L_c'$ can be deduced from Equations (16) and (18) which give

$$1 - e^{-\beta L_c'} = \beta L_c \qquad \text{.........(20)}$$

Fig.3 shows the relation between Lc and L_c' for values of β between 5×10^{-4} cm^{-1} and 2×10^{-3} cm^{-1}.

The previous arguments must be modified slightly if the detector to be tested is of the "rate-of-rise"(2) type. Here the detector consists of two elements with values of τ of τ_A and τ_B and operates when, we shall assume, the difference in resistance between the elements, δR has reached a given value. It has been shown (2) that the response time will be given by

$$t = \tau_B \log_e \frac{\alpha \tau_B}{\alpha(\tau_B - \tau_A) - \theta_0} \qquad \text{........(21)}$$

where $\theta_0 = \dfrac{\delta R}{2R_0 \eta L}$

if the detector is subjected to a uniform rate of rise of air temperature.

In general $\tau_B \gg \tau_A$ and thus the response time is given approximately by

$$t = \frac{\delta R}{2R_0 \eta L \alpha} \qquad \text{........(22)}$$

Hence, with the same notation as previously we deduce

$$L_c = L \frac{t}{t_c} \qquad \text{........(23)}$$

whence 2Lc may be derived from Equation (20) as before.

4. Conclusions

An analysis of the temperature distribution to be expected in the element of a line detector has shown that in many cases the problem may be reduced to one of an infinite number of elemental point detectors, the effect of thermal conductivity on the distribution being negligible.

A method of assessing the thermal response of line detectors has been suggested which could be carried out using an apparatus at present designed for the testing of point detectors. It is further shown that the maximum and minimum lengths of a detector with a given setting may be derived, such that its response time will be within prescribed limits.

References

(1) PICKARD, R. W., HIRD, D. and NASH, P. The thermal testing of heat sensitive fire detectors. Joint Fire Research Organization F.R.Note No. 247/1956.

(2) PICKARD, R. W. A study of the operation of "rate-of-rise" heat sensitive fire detectors. Joint Fire Research Organization F.R.Note No. 248/1957.

(3) HIRD, D., BIGMORE, R. H. and PICKARD, R. W. Siting of heat sensitive fire detectors in buildings. Joint Fire Research Organization F.R. Note No. 475.

(4) Dictionary of Laplace Transforms. Part 3a. S.R.E. Dept. Admiralty Nov. 1945.

(5) British Standard 3116 : 1959. Heat sensitive detectors for automatic fire alarm systems in buildings.

(6) HIRD, D., PICKARD, R. W. and ROSS, W. Thermal tests on some heat sensitive fire detectors. Joint Fire Research Organization F.R. Note No.275/1957.

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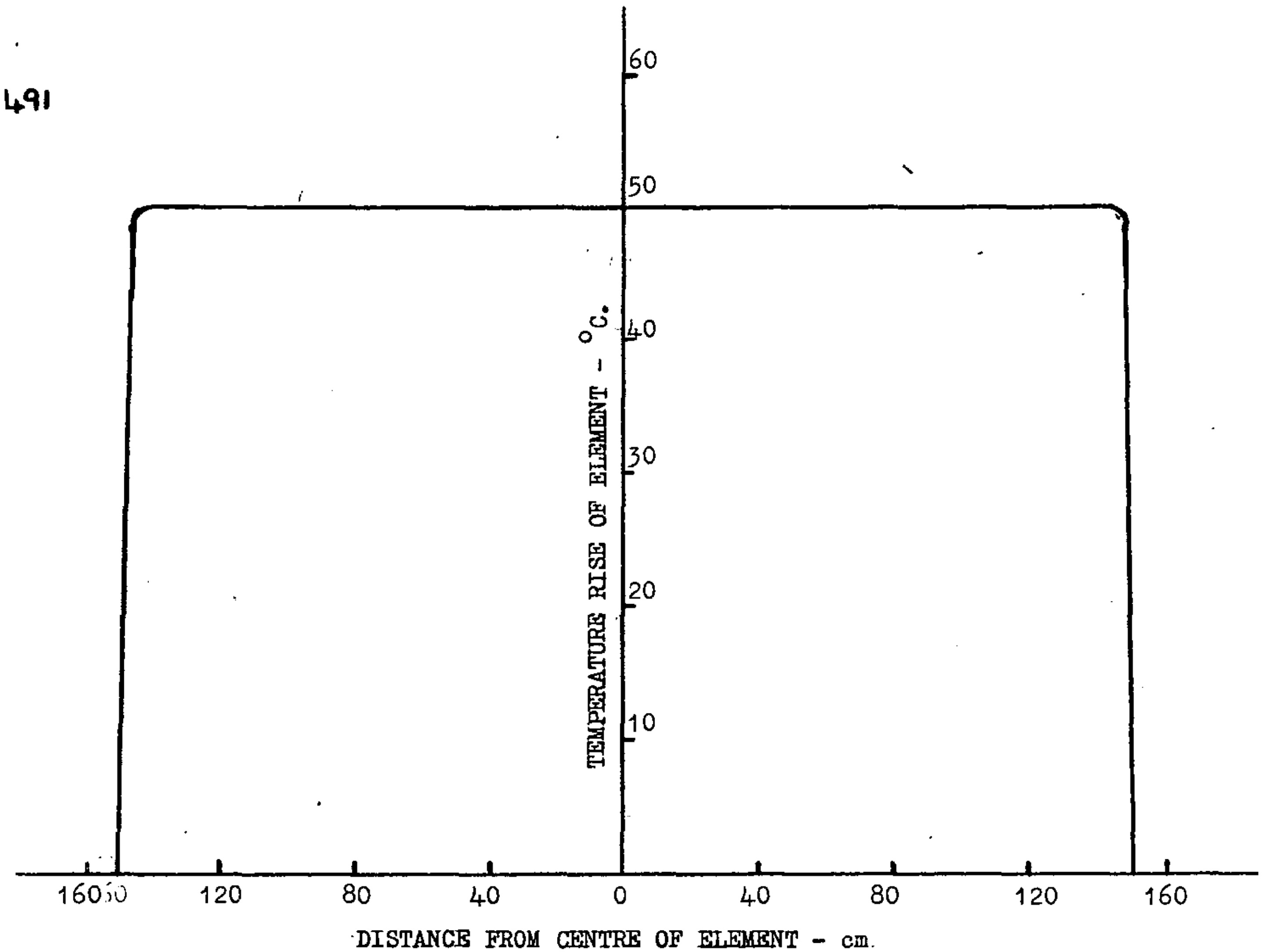

FIG. 2. TEMPERATURE DISTRIBUTION IN LINE DETECTOR ELEMENT

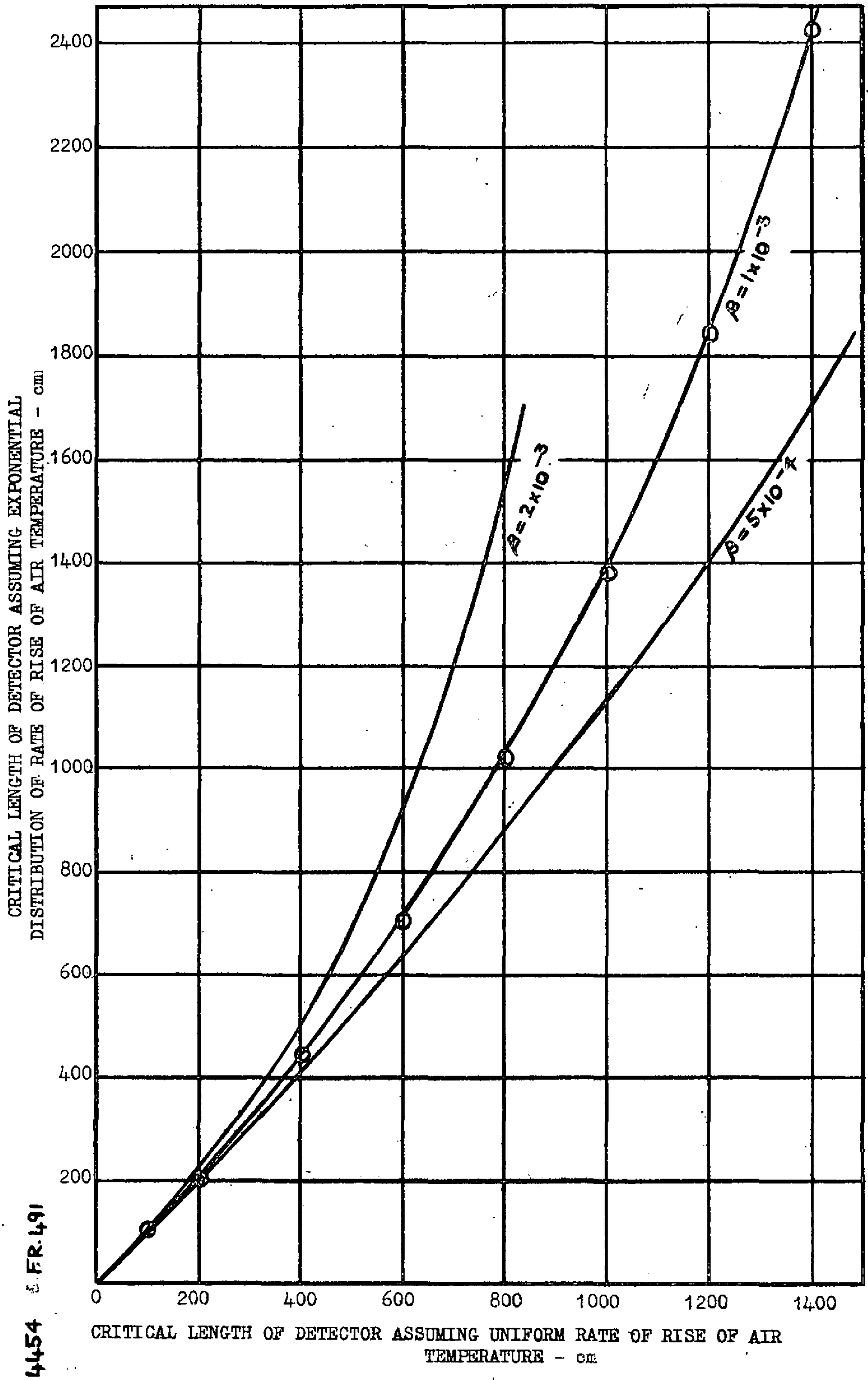

FIG. 3. RELATION BETWEEN CRITICAL LENGTHS OF DETECTOR