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SOME OBSERVATIONS OF THE EFFECT OF WIND ON LINE PLUMES

BY

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December, 1962.

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Summary

The prediction of the temperature rise downwind from a long fire front advancing in the open is of interest in the study of fire spread. This report presents correlations in terms of dimensionless variables of experimental data published by Rankine on the effect of the heat output from a line burner and the wind speed on the downwind temperature rise. It is shown that, despite some variations of temperature rise with the absolute values of the wind speed and heat output, apart from those accounted for by the dimensionless variables, the correlations are on the whole very satisfactory.

The experimental data are briefly discussed in the light of a theory developed by Sir Geoffrey Taylor.

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Introduction

In studying the effect of wind on the spread of grass and undergrowth fires one needs to estimate the heating and drying of fuel ahead of the fire by the hot gas driven ahead of the fire by the wind.

Measurements of the temperature rise downwind from a long line burner emitting convected heat at a constant rate are in fact available and this report is concerned with presenting and evaluating these data. Although this report is not directly concerned with an application of these data it is necessary to point out that in the experiments referred to the burner was stationary. The wind speed relative to the burner will in an application need to be replaced by the velocity relative to the advancing flame front.

During the last war the possibility was examined of clearing airfields of fog by using line sources of heat to set up a horizontal flow of air near the ground (1) and as part of the investigation Rankine(2) studied the effect of wind on the flow from a long line burner in the Empress Hall, Earls Court, London. Air was drawn by a large fan across a long line burner and extensive velocity and temperature measurements were made. In addition some full scale tests were undertaken in the open and satisfactory agreement was obtained between the field and laboratory experiments. In this paper the experimental results tabulated by Rankine are correlated by dimensionless variables and some observations are made on the theoretical aspects of the problem which have been studied by Sir Geoffrey Taylor (3) (4).

Results obtained by Rankine (2)

Fig 1 shows diagrammatically the experimental arrangement and also defines the measured quantities. The ranges of the independent variables and the positions at which the temperature rise and velocity were measured are given in Table 1.

TABLE 1

THE RANGE OF EXPERIMENTAL VARIABLES

Gross heat output per unit length of burner	Wind speed U_o	Distances downwind x	Heights above ground z
0.04 - 0.08 Therms per hour per yard in steps of 0.04 1 Therm = 100,000 Btu	$1\frac{1}{2}$ - 5 ft/s in steps of $\frac{1}{2}$ ft/s	3' 9", 7' 6" and 15'	1.6" to 80"

In order not to introduce any effect of temperature other than on the buoyancy no temperature rise above 30°C was recorded. The net convection flux was estimated by Rankine from a heat balance to be 80 per cent of the gross, 15 per cent of the heat appeared as thermal radiation and 5 per cent was lost to the ground. This constant factor of 80 per cent is used throughout this paper to convert the tabulated gross values to net values of convected heat. The results are given in Rankine's report in tables for each value of distance downwind x from the line burner and the height above the ground z, each table giving either the temperature or velocity for all heat flows and wind speeds.

Scaling Laws

Let Q be the net convective flux per unit length of line per unit time, (i.e. 0.8 of the tabulated value), u the local horizontal velocity, U_0 the applied wind, T_0 the absolute ambient temperature, ρ the density of air, c the specific heat of air, θ the local rise in temperature, x the distance downwind from the line burner, z the height above ground, and g the gravitational acceleration.

The following assumptions are made:

1. The temperature rise is small so that effects of changes in density other than those on the buoyancy are negligible.

2. The relevant scale and intensity of turbulence is determined by thermal instability and not by friction with the ground. Hence the scale and intensity of turbulence are determined by quantities included in those listed above.

3. The gases obey the ideal gas laws so that the effect of buoyancy is a force per unit mass of $g\,\theta/T_0$.

4. Viscous force is neglected since the friction at the ground is a very weak function of Reynolds number.

Either from the differential equations governing the heat balance and the momentum, or from a direct dimensional analysis of the above terms it follows that the solution takes the form

$$\phi = \frac{\theta x}{\left(\frac{Q^2 T_0}{\rho^2 c^2 g}\right)^{1/3}} = G\left(\frac{z}{x}, \frac{U_0}{\left(\frac{gQ}{\rho c T_0}\right)^{1/3}}\right) \tag{1}$$

ϕ is a dimensionless temperature rise and we shall define the dimensionless wind speed as

$$\Omega = \frac{U_0}{\left(\frac{gQ}{\rho c T_0}\right)^{1/3}} \tag{2}$$

Were the effect of Reynolds number significant one would include the group $\frac{U_0 x}{\nu}$ in G. u/U_0 is similarly a function of z/x and Ω.

These results are somewhat more general than Rankine's own statement regarding scaling which amounts to saying that if z, x and Ω are kept constant and it is desired that θ is the same in two experiments Q must be varied as U_0^3 and as $x^{3/2}$. Rankine showed that this was indeed so for a number of cases but here we shall use the argument more generally. We shall use all the data to obtain ϕ and u/U_0 each as functions of z/x for different values of Ω or alternatively as functions of Ω for different values of z/x.

Variation in temperature

Rankine's data have been evaluated in these dimensionless forms with ρ = 0.08 lb/ft^3, C = 0.24 Btu $lb^{-1}\,{}^oF^{-1}$, T_o=540°R. Some typical temperature results are shown in Figs (2), (3) and (4). Values based on temperature rises equal to or less than 0.3°F have been omitted. It is seen that except at the edge of the plume where the temperature is very low (see Fig. (4)) and the absolute experimental error of most consequence and at the ground where heat loss will influence the correlation the different sets of data are correlated together. The one graph of Fig. (4) includes all the data in three of Rankine's tables. The results in Fig. (3) show a slight tendency for absolutely higher windspeeds to give absolutely higher temperatures than expected on the basis of the preceding argument but the lack of correlation, although sometimes systematic, is slight. If we take a series of values of Ω throughout the range of Ω and plot ϕ against z/x we obtain the smooth curves in Fig. (5). It will be noticed that above z/x equal to 0.15-0.20 increasing the wind reduces the temperature; below z/x = 0.15-0.2 it increases it.

The variation in local velocity

In precisely the same way graphs of which Figs (6), (7) and (8) are given as examples of u/U_o against Ω were obtained for various values of z/x. It is seen in Figs (7) and (8) that there are departures from what appears to be the main correlation (shown as a full line). The higher the velocity and the higher the heat flux the greater the departure. It will be seen that the local velocity increases relative to the applied wind as Ω decreases. Rankine remarked that this wind enhancement is "emphasized both by a reduction of imposed windspeed and an increase in the degree of heating" and this is shown by these graphs to be a consequence of u/U_o increasing as Ω decreases. Rankine does not offer an explanation of this enhancement which is more than would be expected from a constancy of mass velocity during a temperature change.

It should be noted that in still air a horizontal flow is induced into a line plume equal[5] to $0.29\left(\frac{gQ}{\rho c T_o}\right)^{1/3}$ If it is assumed that this can, as a first approximation, be added to the wind present in the absence of any heat we can write

$$\frac{U}{U_o} = \lambda + 0.29/\Omega$$

where λ is less than 1 as a result of ground friction and is taken here as the reported value of u/U_o for the case of no heating.

This equation shown in Figs (6), (7) and (8) for the relevant λ has much in common with the experimental data but there is a systematic discrepancy, and as Ω decreases the value of U inside the plume tends to be about twice the horizontal flow outside a vertical plume ($\Omega = 0$). This suggests there is a discontinuity between the two regimes. The entrainment constant for this type flow can be calculated from these data and in the Appendix is shown to be an order or more larger than in vertical plumes.

Fig (9) shows u/U_o against z/x for various values of Ω.

Theoretical discussion

Sir Geoffrey Taylor[3][4] considered the theoretical interpretation of this problem by taking the velocity as uniform and in one direction as a simplifying approximation. From this the heat balance equation becomes

$$\rho c U_o \frac{\partial \theta}{\partial x} = \frac{\partial}{\partial z}\left(K \frac{\partial \theta}{\partial z}\right) \qquad \ldots\ldots\ldots\ldots\ldots(3)$$

Only transport of heat in the z direction was allowed for but two forms for K, the effective conductivity, were considered viz:

$$K_1/\rho c = C_1 l \sqrt{g \frac{\theta l}{T_0}} \qquad \text{...............(4A)}$$

$$K_2/\rho c = C_2 l^2 \sqrt{\frac{g}{T_0} \left| \frac{\partial \theta}{\partial z} \right|} \qquad \text{...............(4B)}$$

where C_1 and C_2 are constant. l is a mixing length assumed proportional to the height of the plume i.e.

$$l = x \operatorname{Tan} \alpha \qquad \text{...............(5)}$$

if the proportionality constant is absorbed into C_1 and C_2. α is the as yet unknown angle of the plume. He assumed that there was a similarity solution such that the distribution of temperature across any section of the plume was the same though the scale both of the temperature distribution and the height of the plume were assumed functions of the wind speed and the heat output. The only independent parameter that can determine these scales is the dimensionless windspeed and he assumed that the dependence of the scales on this term took the form of a power law. Equation (1) then becomes

$$\phi \propto \Omega^m F\left(\frac{z}{x \operatorname{Tan}\alpha}\right) \qquad \text{...............(6)}$$

where tan

$$\operatorname{Tan} \alpha \propto \Omega^n \qquad \text{...............(7)}$$

The indices m and n have yet to be determined. If equations (6) and (7) are substituted into equations (3) and (4) we obtain whichever form we use for K

$$\operatorname{Tan} \alpha \propto \Omega^{m-2} \qquad \text{...............(8)}$$

The heat balance is

$$\int_0^{x \operatorname{Tan}\alpha} u \theta \, dz \propto Q \qquad \text{...............(9)}$$

and using the above equation (6).

we obtain

$$\operatorname{Tan} \alpha \propto \Omega^{-1-m} \qquad \text{...............(10)}$$

Equating the indices of Ω in equations (8) and (10) we obtain

$$m = 1/2$$

The tangent of the plume angle should therefore vary as$_1$ $Q^{1/2}/U^{3/2}$ and the maximum dimensionless temperature ϕ_0 at z = o as Ω^{2}. A more detailed description of the theory is given in the Appendix. These results have recently been discussed again by Sir Geoffrey Taylor(4) and he emphasized that they are a consequence of assuming that the turbulence is produced by thermal instability. If the turbulence were assumed to originate from friction then Tan α would be independent of the dimensionless windspeed. In fact the experimental observations(2) show that

$$\operatorname{Tan} \alpha \propto \left(Q/U^3\right)^{0.286} \text{ i.e. } \Omega^{-0.86}$$

and he commented that this is intermediate between the two results and suggests that shear and thermal instability are of comparable importance. An objection to this view is that similar results were obtained in the field and in a laboratory when ground friction and the scale of turbulence to windward of the heat source would be the same only fortuitously.

However there is an interesting alternative view. Equations (4a) and (4b) are both based on the view that the characteristic mixing length is proportional to the local height of the plume. This is conventional and is based on the view that plumes (and jets) are in local equilibrium. Maczynski(5) has recently shown that this view fails for a jet injected into a stream moving in the same direction but he showed that his results were in accordance with the view that the characteristic mixing length was proportional to the distance from the source. In still air where at long distances from the source the jet or plume angle is a constant for all conditions, there is no mathematical distinction between these views except the perhaps philosophic one that the single unknown constant between mixing length and distance from the origin is in the one case a single unknown constant and in the other the product of two constants, one a proportionality constant between mixing length and jet width and the other a proportionality constant between jet width and distance from the source. However jet and plume angles are not independent of the velocity of a moving ambient fluid and Rankine's results will now be shown to lend support to Maczynski's contention.

If K is written as

$$K_1/\rho c = C_1 x \sqrt{\frac{g \theta l}{T_0}}$$

$$\text{or} \quad K_2/\rho c = C_2 x \, l \sqrt{\frac{g}{T_0} \left| \frac{\partial \theta}{\partial z} \right|} \qquad (11)$$

we obtain on substituting equations (6) and (7) into equations (3) and (11)

$$\mathrm{Tan}^{3/2} \alpha \propto \Omega^{\frac{m}{2}-1}$$

With the heat balance condition this gives

$$m = -1/4$$

Hence Tan α is proportional to $\Omega^{-\frac{3}{4}}$ i.e. the tangent of the plume angle varies as $Q^{1/4}/U_0^{3/4}$ and this is very close to the experimental result of $\left(Q^{1/3}/U_0\right)^{0.86}$ In accordance with this we obtain

$$\phi = \Omega^{-1/4} F\left\{ \frac{z}{x} \cdot \Omega^{3/4} \right\}$$

It can be shown that $\phi_3 \Omega^{\frac{1}{4}}$ and $(z/x)\,\Omega^{\frac{3}{4}}$ correlate the data somewhat better than do $\phi\,\Omega^{-\frac{1}{2}}$ and $(z/x)\,\Omega^{3/2}$ but in fact no pair of variables of the form $\phi\,\Omega^{m}$ and $(z/x)\,\Omega^{1-m}$ is satisfactory. A possible reason for this is that the theory as referred to above and given in the Appendix assumes a uniform velocity and this is not the case. For low values of Ω the value of U/U_0 is noticeably larger than 1. If we assume the distribution of temperature with z is unaffected, ϕ at a given value of $(z/x)\Omega^{\frac{3}{4}}$ would be expected to vary less with Ω than does $\Omega^{-\frac{1}{4}}$ Fig. (10) shows that except near the peak, the correlation between ϕ and $(z/x)\Omega^{\frac{3}{4}}$ is very good while Fig. (11) shows an empirical correlation employing Rankine's estimation of $\Omega^{0.86}$ rather than $\Omega^{0.75}$ for the plume angle.

A conclusion from the arguments given above would seem to be that the values of Ω although sufficiently high to make the plume bend over are not sufficiently high for it to be considered as a fully horizontal plume in the sense demanded by the assumption of a uniform horizontal velocity. This would require values of Ω of at least 4. Very high values of Ω would be associated with high turbulence in the wind so that there would be an upper and a lower limit to the range of Ω for which the thermal instability model proposed by Sir Geoffrey Taylor could apply.

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(1) "The Dispersal of Fog from Airfield Runways" ed. Walker R. G. and Fox D. A. Min. of Supply. London, 1946.

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(3) TAYLOR, G. I. - see Appendix E of ref. (1).

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(5) ROUSE HUNTER, YIH, C. S. and HUMPHREYS, H. W. "Gravitational Convection from a Boundary Source". Tellus (1952) 4, 210.

(6) MACZYNSKI, J. F. J. "A round jet in an ambient co-axial stream". J. Fluid Mechanics (1962) 13 p.597.

APPENDIX

Distribution of temperature in horizontal plumes

The treatment by Sir Geoffrey Taylor[3] is followed except for the one feature concerning the mixing length already referred to. His detailed theory is given here for convenience.

The velocity within the plume is assumed to be horizontal and everywhere equal to the applied wind. The heat transfer equation is then

$$\rho c U_0 \frac{\partial \theta}{\partial x} = \frac{\partial}{\partial z}\left(K \frac{\partial \theta}{\partial z}\right) \tag{1.1}$$

where ρ is the density, assumed constant,

c is the specific heat

θ is the temperature rise

x is the distance downwind of the source

z is the height above ground

and K is the effective conductivity

The boundary conditions are taken as

$$\frac{\partial \theta}{\partial z} = 0 \quad \text{at} \quad z = 0$$
$$\theta = 0 \quad \text{at} \quad z = x \operatorname{Tan} \alpha \tag{1.2}$$

where α is the angle of spread defining the plume boundary. Diffusion in the x direction is neglected by comparison with diffusion in the z direction and we assume that the characteristic velocity in the vertical z direction is that due to buoyancy. We can write the characteristic velocity in K as $\left(\frac{g\theta x \operatorname{Tan}\alpha}{T_0}\right)^{1/2}$ where g is the acceleration due to gravity; T_0 the absolute ambient temperature. Any constants can be incorporated in the one constant of proportionality between $K/\rho c$ and the product of the characteristic length and velocity. It is conventional to take the mixing length as proportional to the plume width i.e. to $x\operatorname{Tan}\alpha$ and with this assumption Sir Geoffrey Taylor obtained results at variance with experimental ones. We shall consider an alternative view that the mixing length is proportional to x, a result claimed by Maczynski[6] to apply to a momentum jet in fluid moving parallel to it. We thus have

$$\frac{K}{\rho c} = C_1 x (\operatorname{Tan} \alpha)^p \sqrt{g \frac{\theta}{T_0} x \operatorname{Tan} \alpha} \tag{1.3}$$

where C_1 is a constant, and p is 0 or 1 according to whether the mixing length is proportional to x or $x\operatorname{Tan}\alpha$ respectively.

An alternative second form of the characteristic velocity was proposed by

$$\frac{K}{\rho c} = C_2 x (\operatorname{Tan} \alpha)^p \sqrt{\frac{g}{T_0} \left|\frac{\partial \theta}{\partial z}\right| (x \operatorname{Tan} \alpha)^2} \tag{1.4}$$

We assume that θ can be expressed in the form

$$\theta = \frac{A}{x} F\left(\frac{z}{x \operatorname{Tan} \alpha}\right) \tag{1.5}$$

where A is a constant with respect to x and z.
With the expression for K in equation (1.3) we have from equation (1.1)

$$U_0\left(F_1 + \lambda \frac{dF_1}{d\lambda}\right) + C_1 \frac{d}{d\lambda}\left(F_1^{1/2} \frac{dF_1}{d\lambda}\right)\sqrt{\frac{gA_1}{T_0}\frac{\operatorname{Tan}^{2p}\alpha}{\operatorname{Tan}^3 \alpha}} = 0 \tag{1.6}$$

where $\lambda = z/x \operatorname{Tan} \alpha$ and the suffix 1 denotes the form of K given by equation (1.3).
We now put for convenience

$$\sqrt{\frac{gA_1}{T_0}} = \frac{U_0}{C_1}(\operatorname{Tan} \alpha)^{3/2 - p} \tag{1.7}$$

and obtain

$$F_1 + \lambda \frac{dF_1}{d\lambda} + \frac{d}{d\lambda}\left(F_1^{1/2} \frac{dF_1}{d\lambda}\right) = 0 \tag{1.8}$$

Equation (1.8) integrated with the boundary condition (1.2) which is

$$\frac{dF}{d\lambda} = 0 \quad \text{at} \quad \lambda = 0$$

gives

$$\lambda F_1^{1/2} + \frac{dF_1}{d\lambda} = 0 \tag{1.9}$$

The upper boundary of the plume is defined as $\lambda = 1$ and here the temperature must be zero, so that from the integration of equation (1.9) we obtain

$$F_1 = \frac{1}{16}\left(1 - \lambda^2\right)^2 \tag{1.10}$$

From equations (1.5) and (1.7) it follows that

$$\theta = \frac{T_0}{gx}\frac{U_0^2}{C_1^2}\frac{(\operatorname{Tan} \alpha)^{3-2p}}{16}\left(1 - \left(\frac{z}{x \operatorname{Tan} \alpha}\right)^2\right)^2 \tag{1.11}$$

The conservation of heat is expressed by

$$Q = \rho c \int_0^{x \operatorname{Tan} \alpha} \theta U_0 \, dz \tag{1.12}$$

and this, with equation (1.11), gives after rearrangement

$$\operatorname{Tan} \alpha = \left(30 C_1^2\right)^{\frac{1}{4-2p}} \left(\frac{gQ}{\rho c T_0 U_0^3}\right)^{\frac{1}{4-2p}} \tag{1.13}$$

Equations (1.11) and (1.13) give

$$\phi_1 = \frac{15}{8}\,\Omega^{\frac{2p-1}{4-2p}}\left(1-\lambda^2\right)^2\left(30C_1^2\right)^{-\frac{1}{4-2p}} \tag{1.14}$$

With the second form for K (equation 1.4) and the suffix 2 on A, F and C we obtain, following the same arguments as above

$$F_2 = \frac{1}{125}\left(1-\lambda^{5/3}\right)^3 \tag{1.15}$$

$$\operatorname{Tan}\alpha = \left(312\,C_2^2\right)^{\frac{1}{4-2p}}\,\Omega^{-\frac{3}{4-2p}} \tag{1.16}$$

and $$\phi_2 = \frac{312}{125}\,\Omega^{\frac{2p-1}{4-2p}}\left(1-\lambda^{5/3}\right)^3\left(312\,C_2^2\right)^{-\frac{1}{4-2p}} \tag{1.17}$$

Equations (1.14) and (1.17) may be written respectively as

$$\phi_1\,\Omega\,\operatorname{Tan}\alpha = \frac{15}{8}\left(1-\lambda^2\right)^2 \tag{1.18}$$

and $$\phi_2\,\Omega\,\operatorname{Tan}\alpha = \frac{312}{125}\left(1-\lambda^{5/3}\right)^3 \tag{1.19}$$

So long as the flow is assumed constant the indices a and b in the general expression

$$\phi \propto \Omega^{-a}\,F\!\left(\frac{z}{x}\,\Omega^{b}\right)$$

must be related by $$a + b = 1$$

Such a combination does not apply to the experimental data. This may be seen by noting Rankine's finding that the plume slope was proportional to $\Omega^{-0.86}$ so that a should be 0.14 yet the results in Fig. (5) show that the maximum ϕ increases with Ω. If we plot ϕ against $(z/x)\Omega^{\frac{3}{4}}$ we find the data correlate well for $\frac{z}{x}\Omega^{\frac{3}{4}} \nless 0.3$. The distinction between $\frac{3}{4}$ and 0.86 is slight in view of the scatter of the data. We note that the velocity enhancement is most pronounced in the region $(z/x)\Omega^{\frac{3}{4}} < 0.3$ near where the peak temperature lies and its effect is greatest for low values of Ω.

The order of magnitude of the enhancement is less than 35 per cent over the range examined and this is consistent in direction and in order of magnitude with the differences in the maxima of the curves in Fig. (1). The curves for the larger Ω, which show the least enhancement are the ones expected to fit best the simple theory assuming a constant velocity.

The absence of a suitable reference point by which to normalize the distribution to a value at zero z/x means that instead of one disposable constant C_{b2} , we have used two in fitting the data. We have assumed a maximum value of ϕ of 3. The distribution appears to be close to those calculated above, slightly closer to the first rather than the second but either, or indeed a Gaussian distribution would do as well, but in view of the complications arising from the non-uniformity of the velocity distribution the comparison between experiment and the present theory is not pressed further.

Entrainment into the plume

The rate of increase of mass in the plume is by definition

$$\frac{dm}{dx} = \frac{d}{dx}\int_0^{z_{max}} \rho\, U\, dz$$

The upper limit z_{max} is taken as $x \tan\alpha$ and variations of density will be neglected. Since U/U_0 is theoretically a function of z/x and Ω but not otherwise of x we have

$$\frac{dm}{dx} = \rho U_0 \int_0^{\tan\alpha} \left(U/U_0\right) d\left(z/x\right) \qquad (1.20)$$

We now define a conventional entrainment constant E such that

$$\frac{dm}{dx} = E\rho \left| U_c - U_0 \right| . \qquad (1.21)$$

where U_c is a characteristic horizontal velocity in the plume. If U_c is chosen as the maximum velocity we shall obtain a low value for E rather than a high one.

Combining the equations (1.20) and (1.21) we have

$$E = \frac{1}{\left|U_c/U_0 - 1\right|} \int_0^{\tan\alpha} \left(U_c/U_0\right) d\left(z/x\right) \qquad (1.22)$$

There is no way of expressing entrainment in terms of a maximum or minimum plume velocity for $\Omega = 3$ (see Fig (9)) so a more general definition of entrainment is strictly necessary. However, for the purpose at hand we shall consider only the three curves $\Omega = 1$, 1.5 and 2.0. $\tan\alpha$ is taken as $0.8/\Omega^{0.86}$ (see Fig (11)). At this value E will be underestimated because the temperature still exceeds $\frac{1}{100}$ of the maximum, which is often taken as defining the edge of plumes.

For $\Lambda = 1$ the integral obtained from Fig (9) is 0.93, for $\Lambda = 1.5$, 0.6 and for $\Lambda = 2$, 0.44.

The corresponding values of $\left(\frac{u}{u_0} - 1\right)$ can only be obtained approximately from Fig (9) and this gives 0.45, 0.17, 0.08 respectively, for $\Lambda = 1$, 1.5 and 2. The values of E obtained from equation (1.22) are then 2, 3.5 and 5.5 approximately. The smaller the value of Λ the larger the importance of buoyancy compared with the inertia of the wind so that one would expect a higher entrainment constant and this is so here. Despite the approximation and assumptions it would seem possible to draw two conclusions. E as defined in this way is not a constant; it is sensitive to Λ. E is at least in order of magnitude larger than the value for vertical plumes.

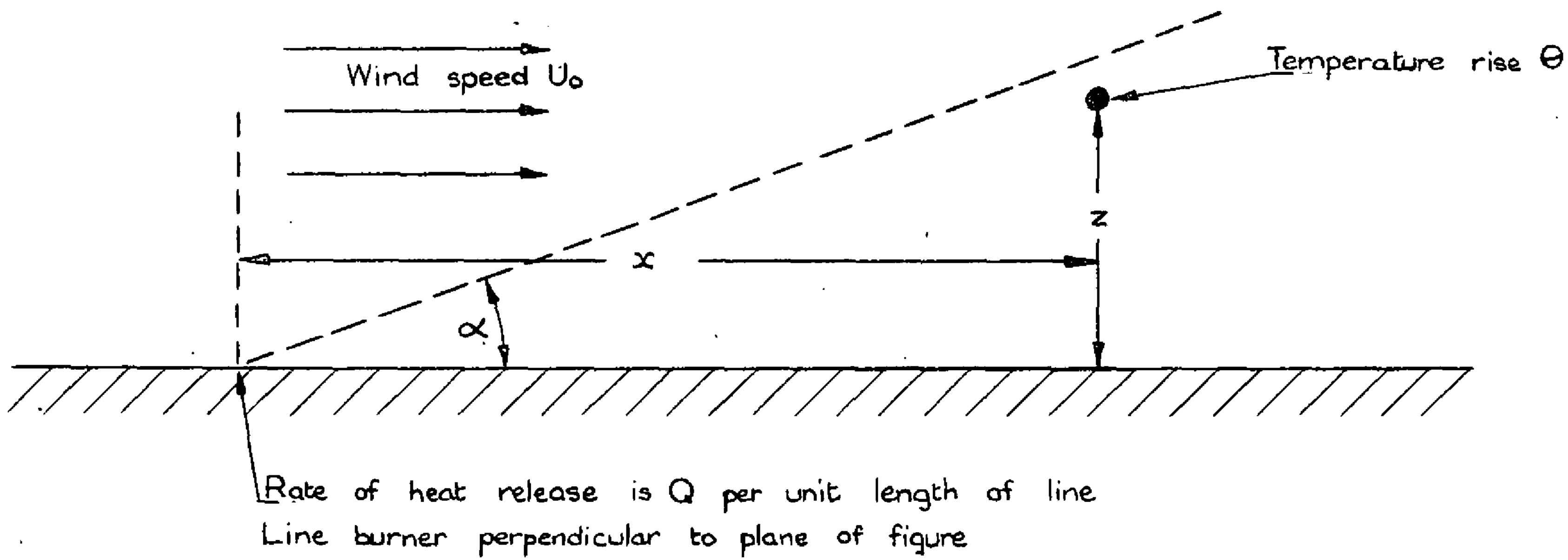
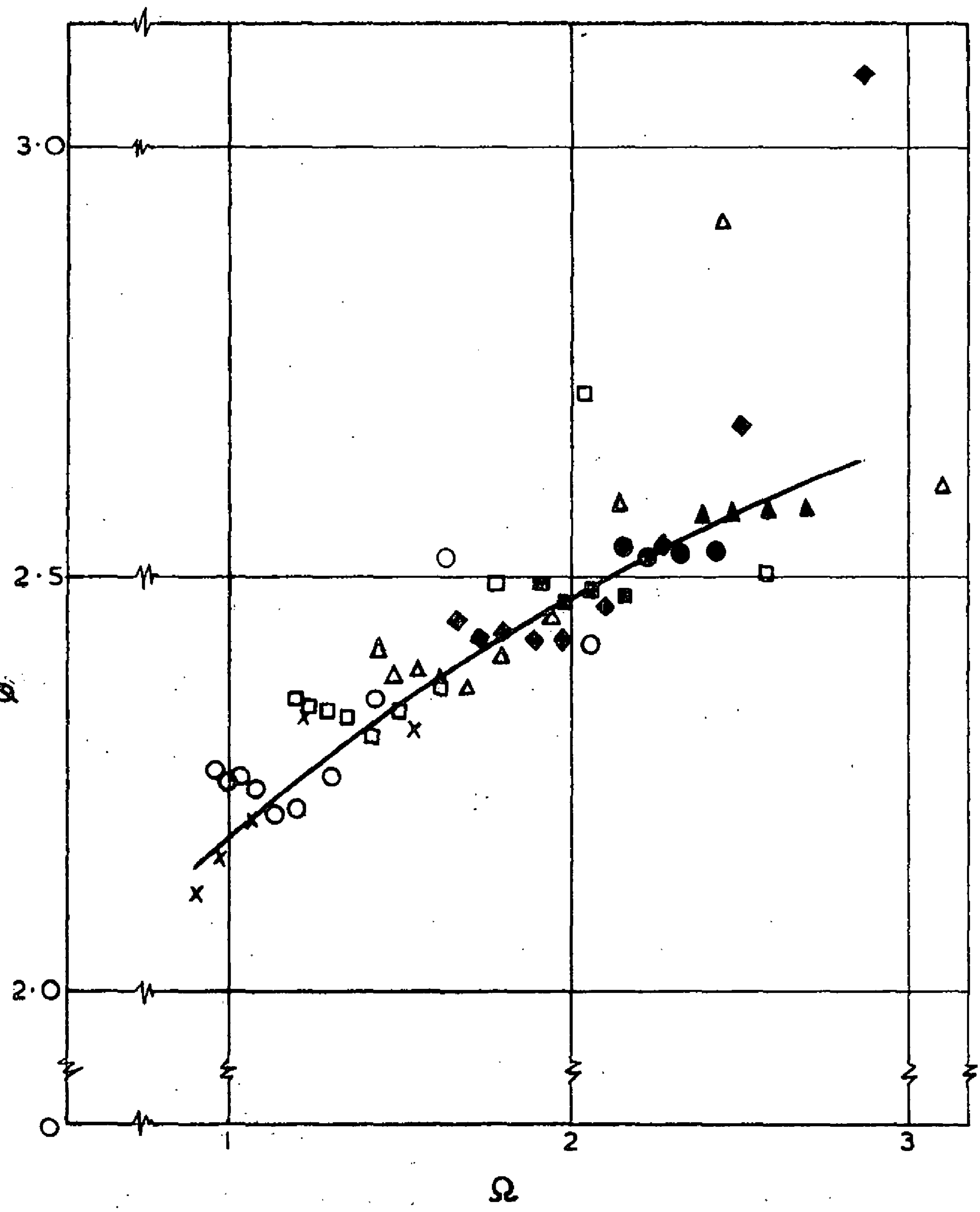


FIG. 1. DIAGRAMMATIC REPRESENTATION OF FLOW SYSTEM



Symbols	Wind speed ft/s
X	1·5
○	2
□	2·5
△	3
◆	3·5
■	4
◉	4·5
▲	5

FIG. 2. DIMENSIONLESS TEMPERATURE AT $z = 1\cdot6''$ AND $x = 3'$-$9''$ i.e. $z/x = 0\cdot0356$

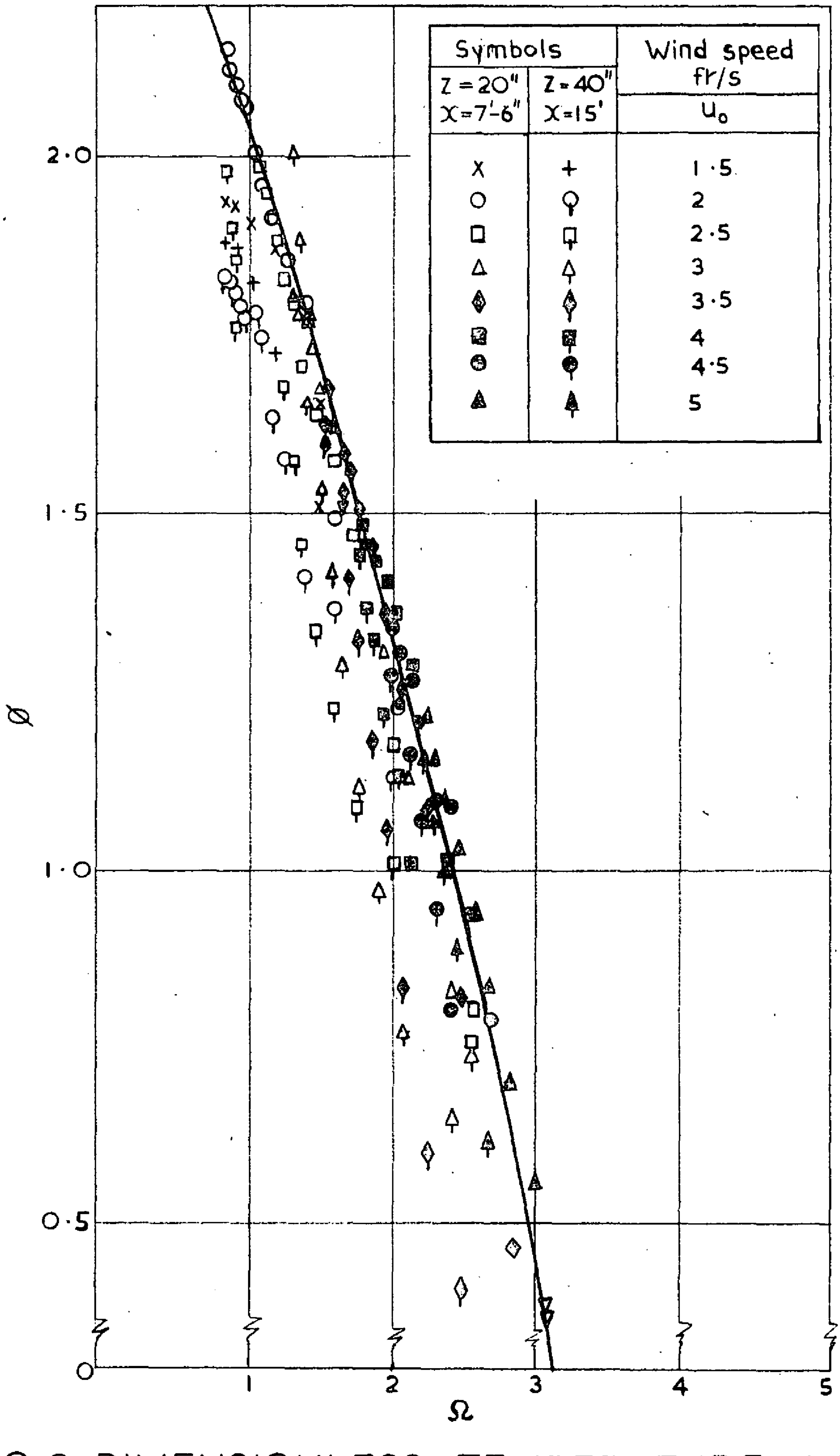


FIG. 3. DIMENSIONLESS TEMPERATURE AT $z = 20''$, $x = 7'$-$6''$ AND $z = 40''$ AND $x = 15'$ i.e. $z/x = 0 \cdot 222$

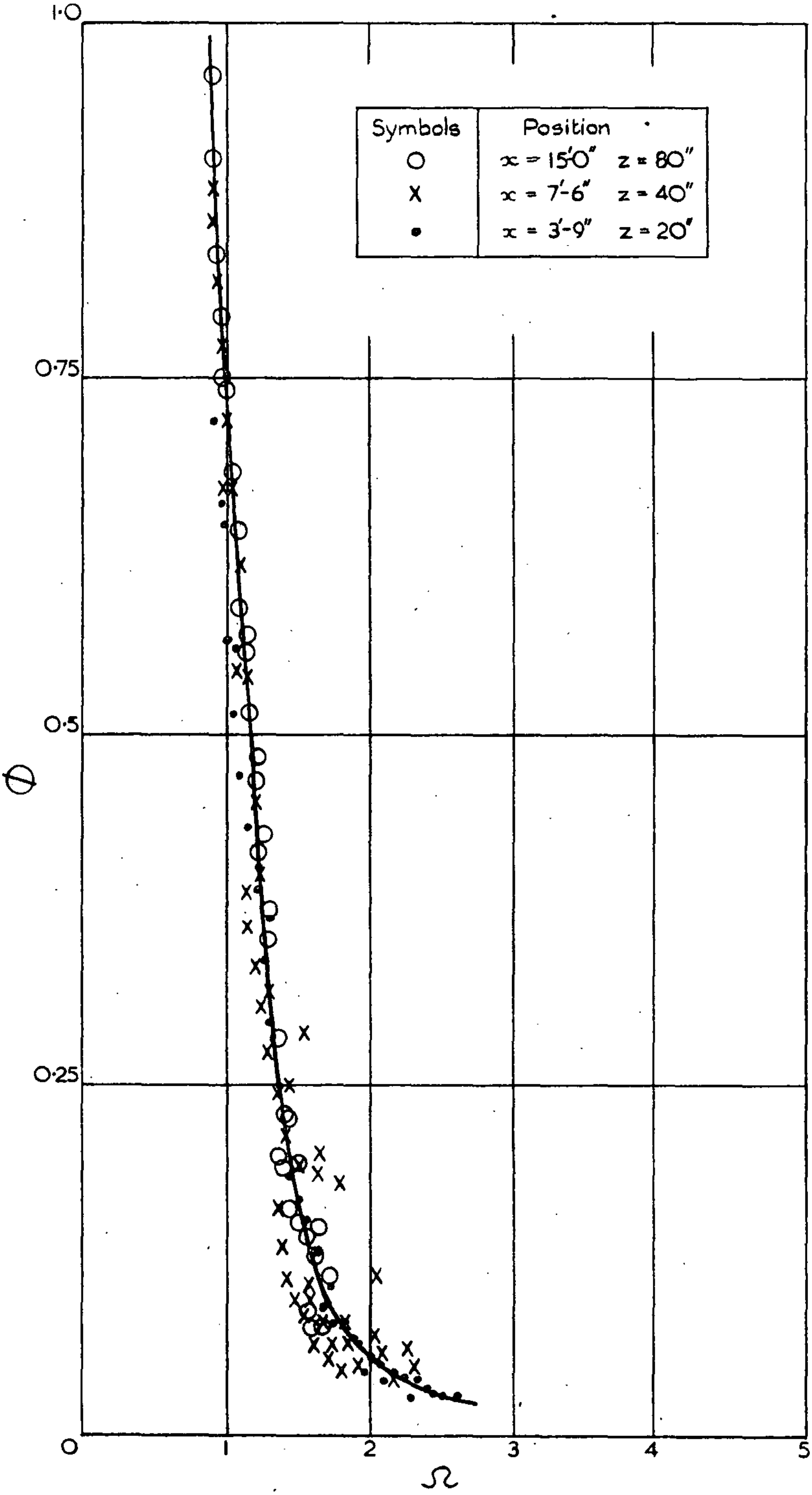
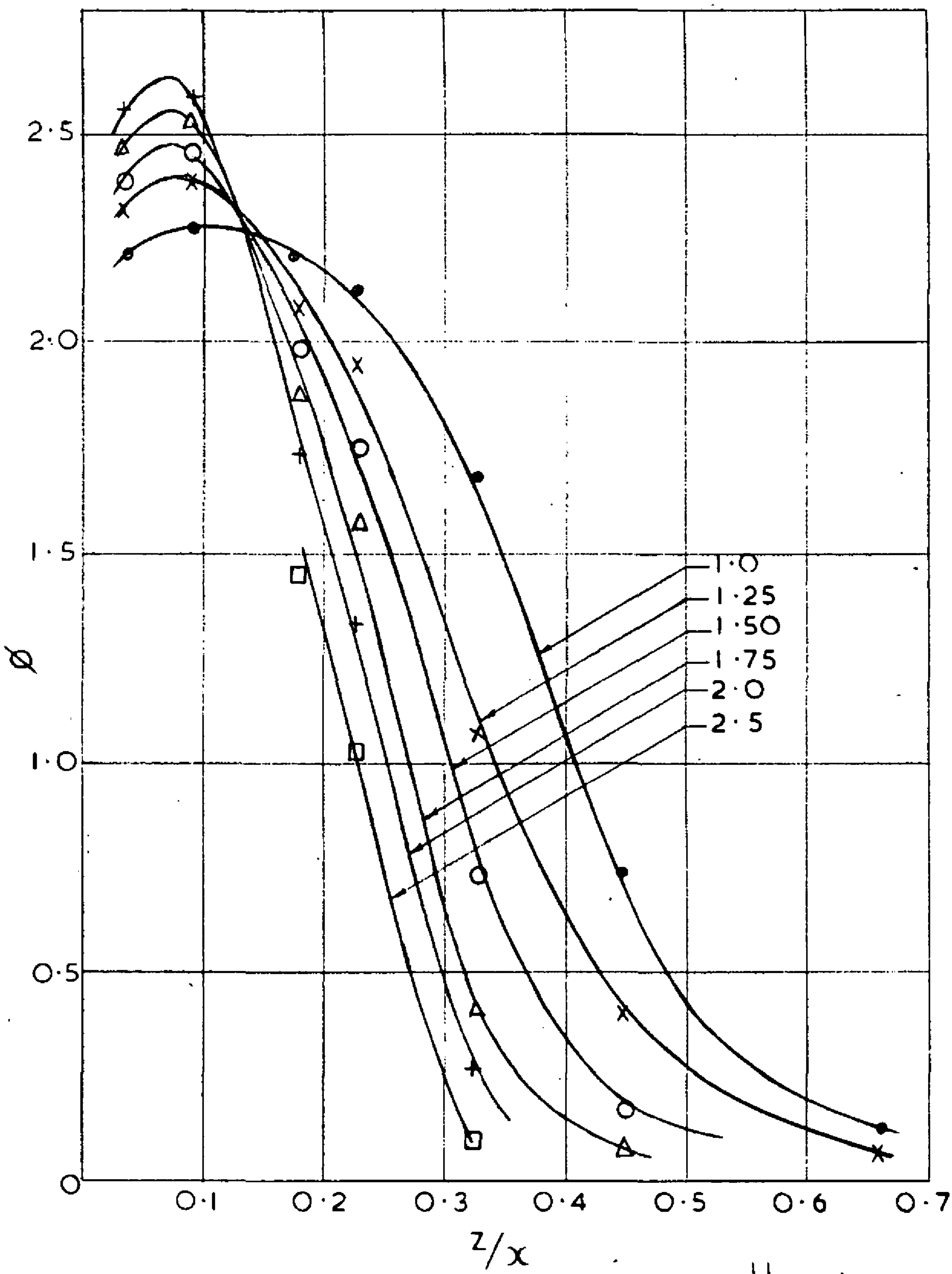


FIG. 4. DIMENSIONLESS TEMPERATURES FOR $z = 20''$ $x = 3'\text{-}9''$, $z = 40''$ $x = 7'\text{-}6''$, AND $z = 80''$ $x = 15'$ i.e. $z/x = 0{\cdot}445$



The numbers refer to values of $\left[\dfrac{U_o}{\frac{gQ}{\rho c T_o}}\right]^{\frac{1}{3}}$

Plotted points taken from best lines in figs 2, 3, 4 & similar graphs

FIG.5. DIMENSIONLESS TEMPERATURE PROFILES

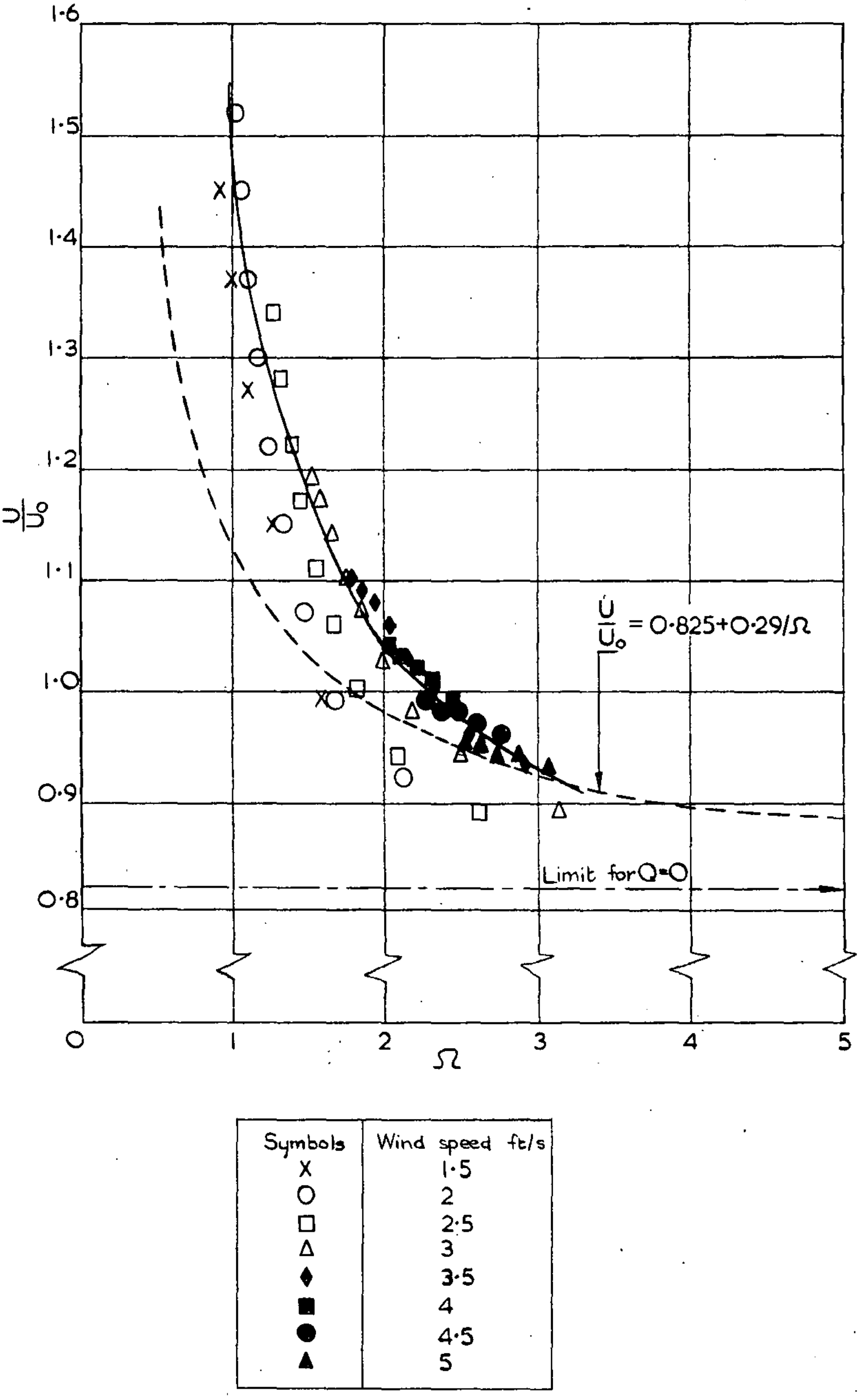
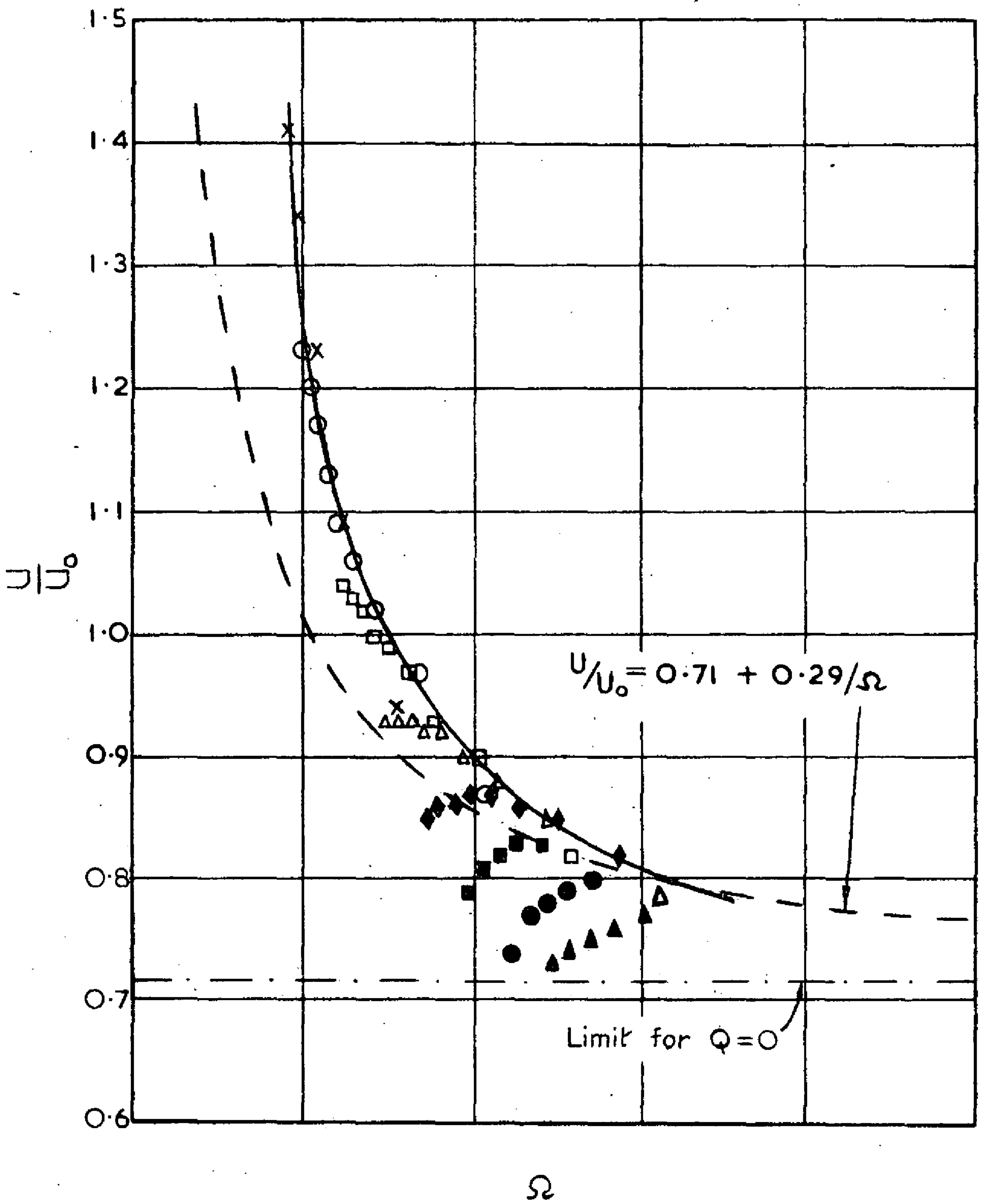


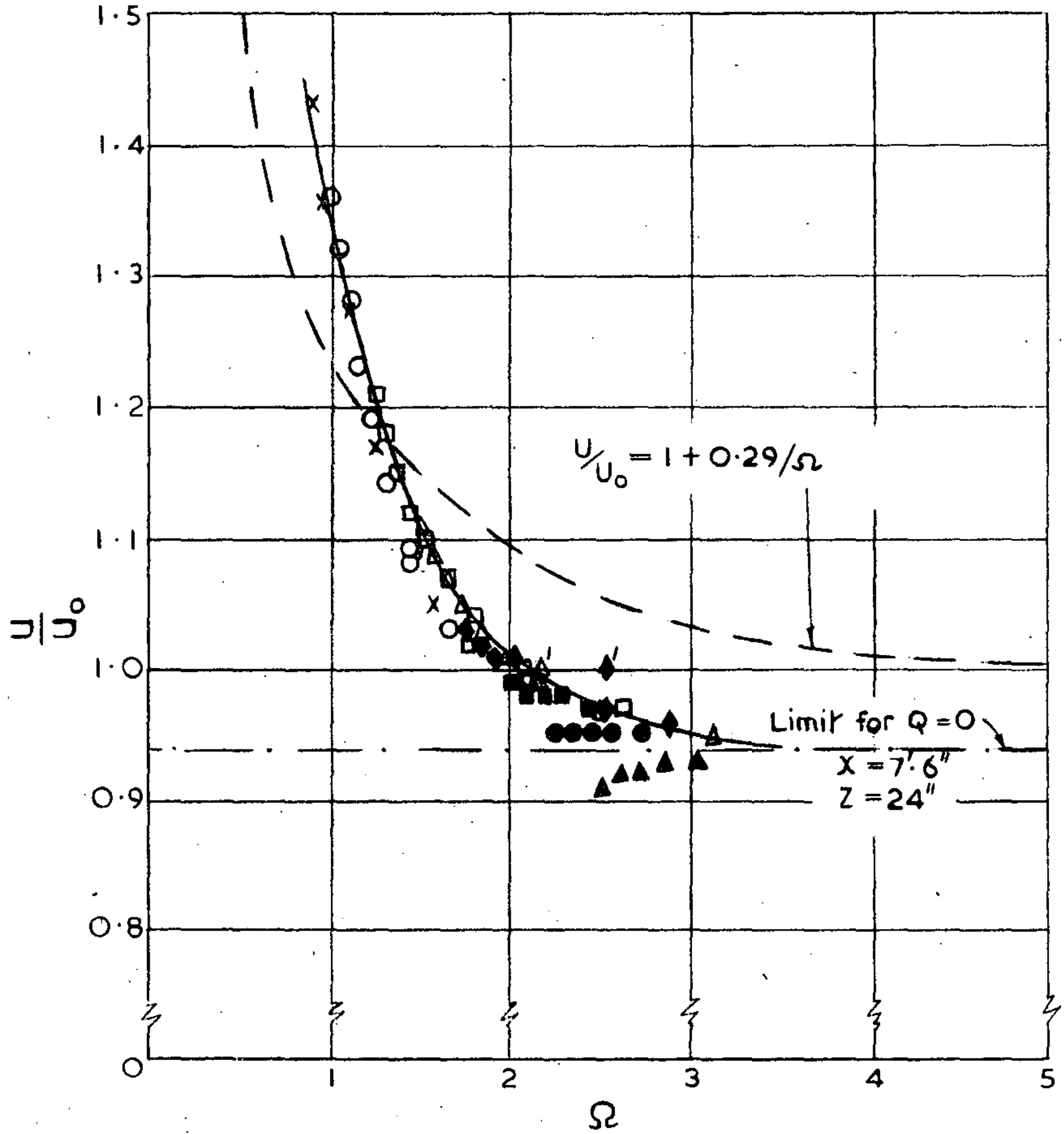
FIG. 6. DIMENSIONLESS LOCAL VELOCITY
$z = 10''$, $x = 3'\text{-}9''$, $z/x = 0{\cdot}222$



Symbols	Wind speed ft/s
x	1·5
○	2
□	2·5
△	3
◆	3·5
■	4
●	4·5
▲	5

FIG. 7. DIMENSIONLESS LOCAL VELOCITY

$z = 4''$, $x = 3'-9''$, $z/x = 0{\cdot}089$



Symbols	Wind speed ft/s
×	1·5
○	2
□	2·5
△	3
◈	3·5
▩	4
●	4·5
▲	5
Z = 24"	X = 7'-6"

Symbols	Wind speed ft/s
○'	2
□'	2·5
△'	3
◆'	3·5
Z = 48"	X = 15'

FIG.8. DIMENSIONLESS LOCAL VELOCITY
$z = 24''$, x $7'\text{-}6''$ AND $z = 48''$, $x = 15'$
i.e. $z/x = 0 \cdot 267$

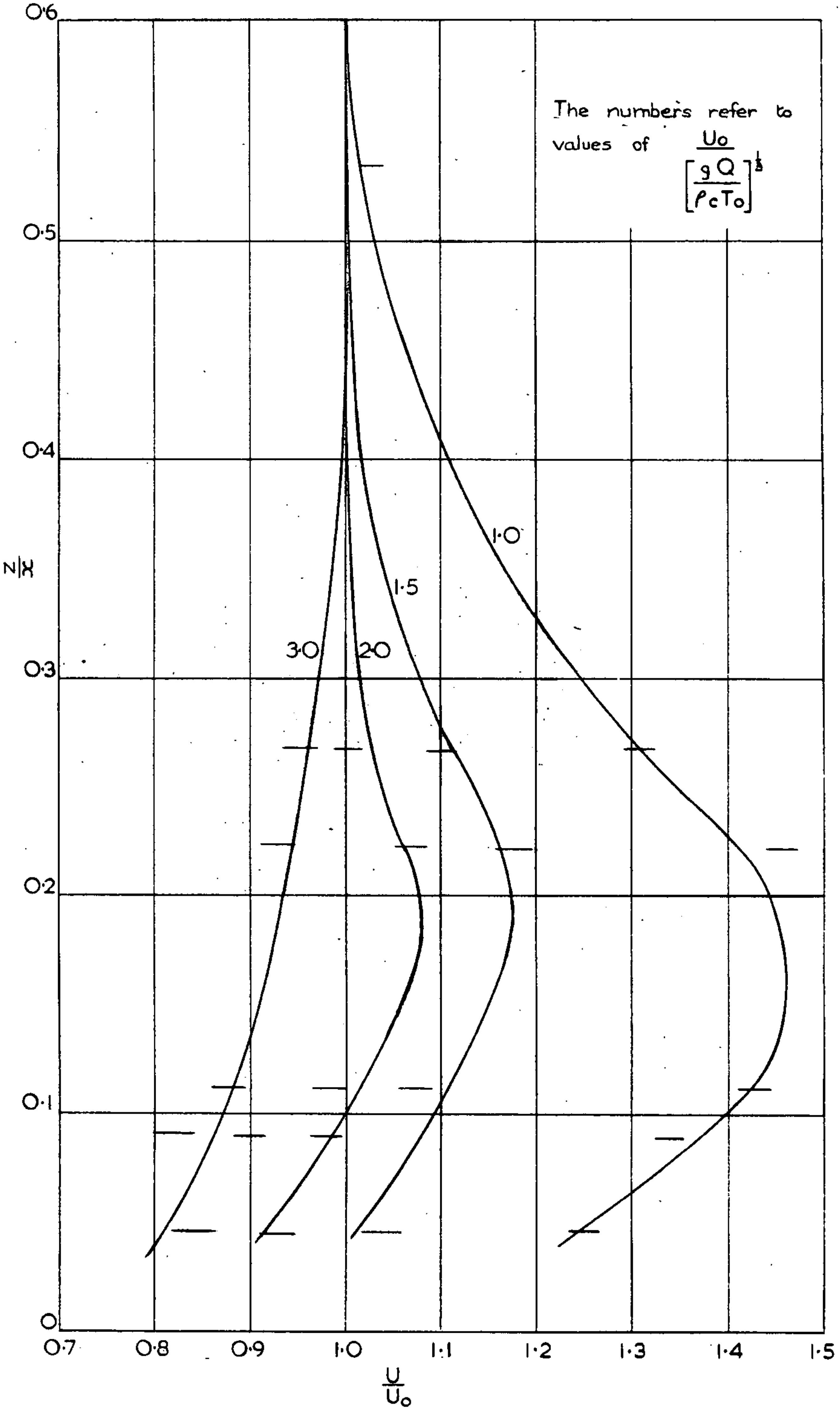
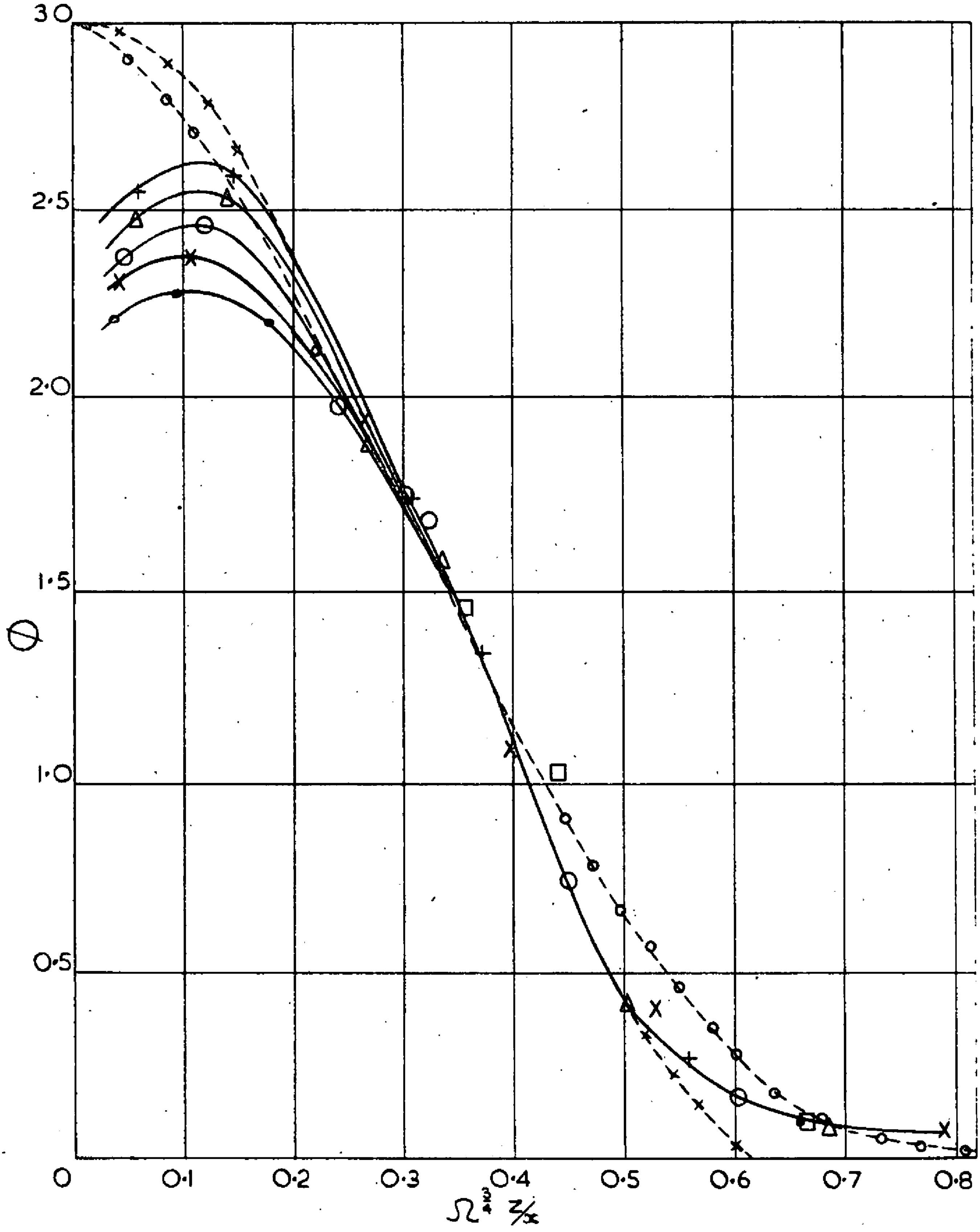


FIG. 9. VELOCITY PROFILES



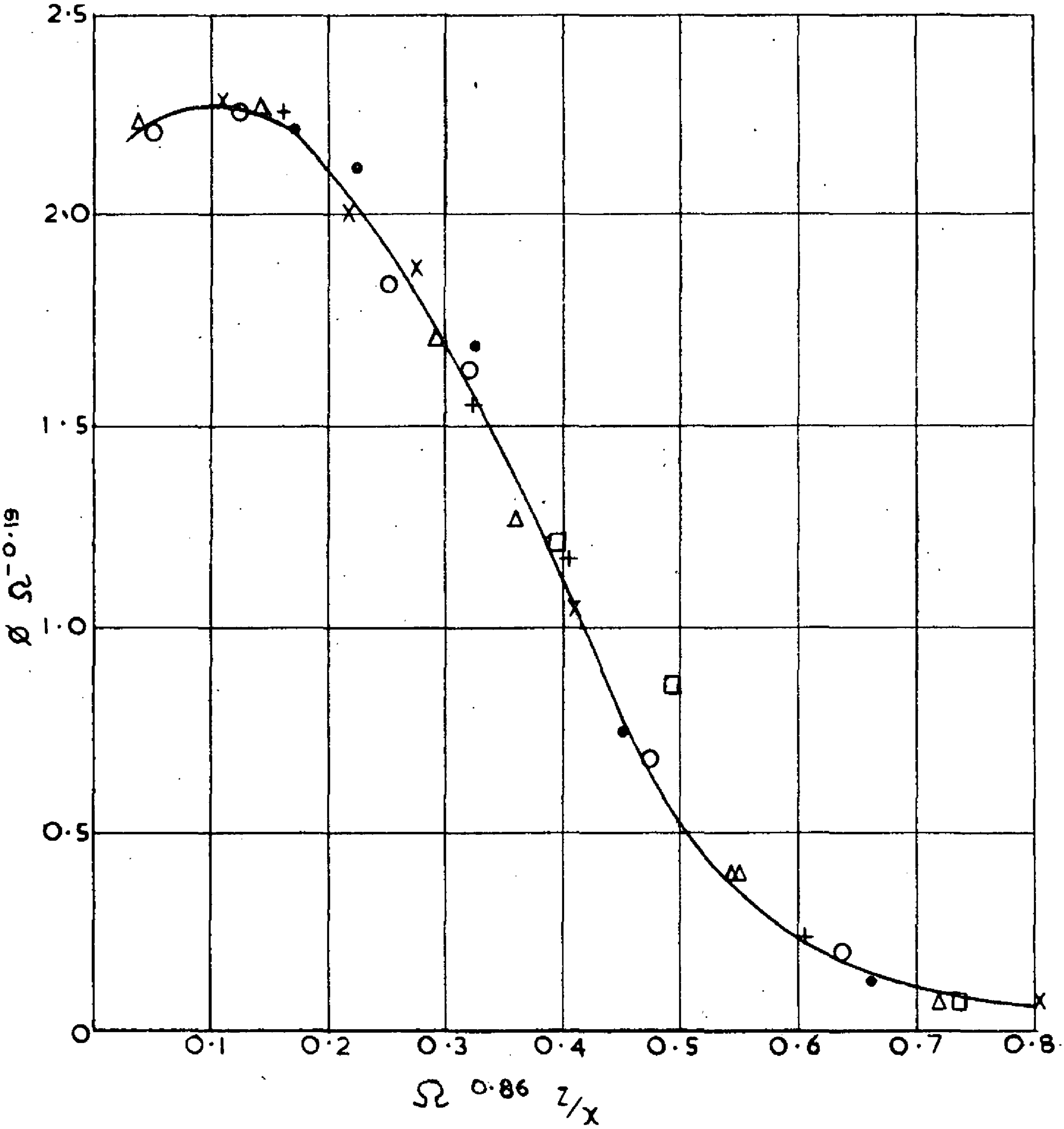
The plotted points are those of fig 5

Symbol	Ω	Symbol	Ω
•	1·0	X	1·25
○	1·5	Δ	1·75
+	2·0	□	2·5

x- - -x- - -x Theoretical distribution, equation (4A)

o- - -o- - -o Theoretical distribution, equation (4B)

FIG. 10. DIMENSIONLESS TEMPERATURE PROFILES



The points plotted are those of fig. 5.

FIG. 11. EMPIRICAL DIMENSIONLESS TEMPERATURE PROFILE