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THERMAL IGNITION IN THE HOLLOW CIRCULAR CYLINDER HEATED ON THE INNER FACE

by

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January, 1965

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SUMMARY

The critical conditions for thermal ignition in a reactive solid in the form of a hollow cylinder with the inner face maintained at a constant high temperature are calculated. Application to the practical case of combustible lagging on a hot pipe is illustrated briefly.

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LIST OF SYMBOLS

E = Activation energy

f = Pre-exponential factor in Arrhenius equation

F = Integration constant

G = Integration constant

h = Total heat transfer coefficient for convection and radiation

K = Thermal conductivity

Q = Heat of reaction per unit volume

R = Gas constant

r_1 = Radius of inner surface of hollow circular cylinder

r_2 = Radius of outer surface of hollow circular cylinder

T_o = Ambient temperature ^oK

T_p = Temperature of hot (inner) face of cylinder

T_s = Temperature of cool (outer) face of cylinder

r = Distance co-ordinate

z = r/r_1

z_s = r_2/r_1

α = hr_2/K

$$\delta_c = \frac{QfE\, r_1^2\, e^{-E/RT_p}}{KRT_p^2}$$

$$\delta_c^* = \delta_c \left(\frac{z_s - 1}{2} \right)^2$$

$$\theta = \frac{E}{RT_p^2} (T-T_p)$$

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INTRODUCTION

The notion of thermal ignition applied to the case of an infinite plane slab with one face maintained at a constant high temperature, while the other face loses heat to the surroundings(1), has provided a useful quantitative model for the ignition of thick layers of combustible solids in contact with hot surfaces(2).

This note gives the corresponding calculations for the case of an infinite hollow circular cylinder with the inner face maintained at a constant high temperature. The results are relevant principally to the practical problem of fires in pipe lagging.

As usual, the approach consists of a determination, in an approximate manner, of the limiting steady state for heat conduction in the body, with heat generation in the body varying with temperature in accordance with the Arrhenius equation.

THEORETICAL

The general steady-state solution of the conduction equation for an infinite hollow circular cylinder, with heat generation Qf exp (-E/RT) cal $cm^{-3}s^{-1}$, and using Frank-Kamenetskii's exponential approximation for the Arrhenius term, is(3)

$$\theta = \log_e \left\{ \frac{2F^2 G z^{F-2}}{\delta (1 + G z^F)^2} \right\} \qquad \text{........ (1)}$$

where F and G are integration constants and

$$\theta = \frac{E}{RT_p^2}(T - T_p) \qquad T \leqslant T_p \qquad \text{........ (1a)}$$

$$z = r/r_1 \qquad r_1 \leqslant r \leqslant r_2 \qquad \text{........ (1b)}$$

$$\delta = \frac{QfEr_1^2 e^{-\frac{E}{RT_p}}}{KRT_p^2} \qquad \text{........ (1c)}$$

At the inner face of the cylinder we have the boundary condition

$$\theta = 0 \qquad \text{at} \qquad z = 1 \qquad \text{........ (2)}$$

Employing the Semenov - Zeldovitch approximation, i.e. that the maximum temperature in the steady state occurs at the inner face, we have also

$$\frac{d\theta}{dz} = 0 \quad \text{at} \quad z = 1 \qquad \text{........ (3)}$$

which, as indicated elsewhere(1), is equivalent to supposing the hot face to be in contact with a perfect insulator.

With this condition, no non-critical solutions are included in equation (1) and the Frank-Kamenetskii parameter δ has its maximum value δ_c. From the more general consideration of non-stationary behaviour (e.g. ref. 4) we may expect that for $\delta > \delta_c$, the temperature in the cylinder will increase to high values associated with combustion or explosion; i.e. ignition will occur.

The most general boundary condition for the outer surface in the steady state equates the heat flux on either side of the surface, i.e.

$$-\frac{d\theta}{dz} = \alpha\left(\theta_s - \theta_o\right) \quad \text{at} \quad z = r_2/r_1 = z_s$$

where the subscripts s and o imply $T = T_s$ and $T = T_o$ respectively in θ, and $\alpha = hr_2/K$. This will lead to δ_c as a function of three variables α, θ_o and z_s for critical states. Since, for practical purposes, we are mainly interested in large values of α, i.e. in situations where the surface temperature of the cylinder is not much above ambient, we shall adopt the simpler condition

$$\theta = \theta_s \quad \text{at} \quad z = z_s \qquad \text{........ (4)}$$

and either put $\theta_s = \theta_o$ or, better, base θ_s on a surface temperature calculated on the assumption that the material of the cylinder is inert(2).

From equations (1) and (3)

$$F\,\frac{1-G}{1+G} = 2 \qquad \text{........ (5)}$$

and, further,

$$\delta_c = \frac{8G}{(1-G)^2} \qquad 0 < G < 1 \qquad \text{........ (6)}$$

From equations (1) and (4)

$$\theta_s = \log_e \left\{ \frac{2 F^2 G z_s^{F-2}}{\delta_c (1 + G z_s^F)^2} \right\} \quad \text{........ (7)}$$

These last three equations yield δ_c as a function of z_s and θ_s. However, for $G z_s^F \gg 1$, equation (7) can be simplified and combined with equations (5) and (6) to give

$$\theta_s = 2 \log_e \left(1 + \frac{1}{G}\right) - \frac{4}{1-G} \log_e z_s \quad \text{........ (8)}$$

The error in δ_c, incurred in the simplification of equation (7), is less than 5 per cent for $G z_s^F > 40$ – which proves to be a condition easily fulfilled in the range of practical interest.

Equations (6) and (8) have been used to calculate δ_c^*, in Fig. 1, where δ_c^* is the critical ignition parameter expressed conventionally in terms of the semi-thickness of the cylinder wall; i.e.

$$\delta_c^* = \delta_c \left(\frac{z_s - 1}{2}\right)^2 \quad \text{........ (9)}$$

<u>The limit of δ_c^* for $z_s \to 1$</u>

θ_s is –ve and, from equations (5), (6) and (7) we have

$$|\theta_s| = -\theta_s = \log_e \left\{ \frac{\left(1 + G z_s^{2\frac{1+G}{1-G}}\right)^2}{(1+G)^2 z_s^{\frac{4G}{1-G}}} \right\} \quad \text{........ (10)}$$

For $|\theta_s| > 0$ and $z_s \to 1$, equation (10) can be satisfied only if G is close to unity. δ_c then approaches infinity, but δ_c^* approaches a finite limit and may be evaluated as follows:

For $G \to 1$, equation (10) becomes

$$|\theta_s| \approx \log_e \left\{ \frac{\left(1 + z_s^{\frac{4}{1-G}}\right)^2}{4 z_s^{\frac{4}{1-G}}} \right\}$$

$$\approx \log_e \frac{z_s^{\frac{4}{1-G}}}{4} \qquad \text{for large } |\theta_s|$$

Putting $z_s = 1 + a$, where $a \ll 1$, and noting that

$$\delta_c \approx \frac{8}{(1-G)^2} \quad \text{for } G > 1$$

and

$$\delta_c^* = \delta_c \frac{a^2}{4}$$

we have, finally,

$$\delta_c^* \approx \frac{(|\theta_s| + 1{\cdot}4)^2}{8} \qquad \text{........ (11)}$$

This is the expression obtained earlier[1] for a plane slab with one, hot, face in contact with a perfect insulator (i.e. for the condition $d\theta/dz = 0$ at the hot face), with $\alpha \gg 1$ and for $|\theta_s| > 5$. Thus, as is to be expected, a thin-walled hollow cylinder tends to behave as a plane slab.

The error in δ_c^*

As for the case of the plane slab with a large imposed temperature gradient, the use of the Frank-Kamenetskii approximation for large values of θ requires comment. Estimation of the error in δ_c due to the extended use of the approximation showed that it was less than about 9 per cent for the plane slab[1]. For practical applications of the kind envisaged an error of this magnitude in δ_c is negligible.

Initially the use of the approximation was justified on the grounds that in the region where it ceases to hold, i.e. towards the cool face of the slab, the rate of heat generation is negligible compared with the rate in the region where the approximation is good and, hence, the error in θ near the cool face will have little effect on the ignition criterion. Since, for the hollow cylinder, the surface area increases with radius, the temperature and, hence, the rate of heat generation, will fall off more rapidly with distance from the hot face than in the case of the plane slab. It is to be expected, therefore, that the error in δ_c^* for the hollow cylinder, due to use of the Frank-Kamenetskii approximation will be no greater than for the plane slab and may even be markedly less.

APPLICATION

Experimental results are available for the ignition of plane slabs of wood fibre insulating board in contact with a hot surface[2]. With the aid of constants obtained from these results, the above analysis will be used to estimate the critical conditions for ignition of this material in the form of cylindrical lagging on pipes. Although this example is unlikely to occur in practice, it serves to illustrate the application of the analysis and, in particular, the effect of the cylindrical geometry.

From equations (1c) and (9)

$$r_2 - r_1 = 2\,T_p \sqrt{\delta_c^*} \left(\frac{KR}{QfE}\right)^{\frac{1}{2}} e^{\frac{E}{2RT_p}} \qquad \text{........ (12)}$$

where δ_c^*, corresponding to values of z_s and θ_s appropriate to a particular case, is obtained from Fig. 1 and the constants have the following values for wood fibre insulating board(2):-

$$Qf = 3.47 \times 10^8 \text{ cal cm}^{-3}\text{s}^{-1}{}^{+}$$
$$E = 25{,}300 \text{ cal mole}^{-1}$$
$$K = 1.2 \times 10^{-4} \text{ cal cm}^{-1}\text{s}^{-1}{}^{\circ}\text{C}^{-1}$$

Assuming the lagging to be inert, we have the following equation for the steady state at the cool surface:-

$$h(T_s - T_c) = \frac{K}{r_2} \cdot \frac{T_p - T_s}{\log_e z_s} \qquad \ldots\ldots\ldots (13)$$

where the left hand side represents the heat loss to the surroundings by convection and radiation and is calculable by standard methods(5). Here we shall consider horizontal pipes in still air for which the heat loss is

$$9.9 \times 10^{-5} \frac{(T_s - T_0)^{1.25}}{(2r_2)^{0.25}} + 1.37 \times 10^{-12} \left(T_s^4 - T_0^4\right) \quad \text{cal cm}^{-2}\text{ s}^{-1}$$

Usually, T_p, T_o and r_1 will be given and r_2 and T_s must then be found to satisfy equations (12) and (13) simultaneously.

Calculated in this way, the relationship between ignition temperature and thickness is shown in Fig. 2 for wood fibre insulation on pipes of external radius 1.7 cm (nominal 1 in B.S.P.) and 16.5 cm (nominal 6 in B.S.P.) for an ambient temperature of 15°C. For comparison, the relationship for plane slabs of wood fibre insulation is also shown, and the ordinates for standard lagging thicknesses of 1in and 2in are given.

The practical implication of calculations of this kind are to be discussed more fully elsewhere, but the following point may be noted. For 2in lagging on nominal 1in pipe, z_s is 4, and this is the highest value of z_s likely to occur in practice; in this case, the cylindrical geometry results in an ignition temperature of about 30°C higher than for the 2in plane slab. Reduction in thickness from 2in to 1in increases the hot surface temperature for ignition by 25-30°C.

Usually, the constants required for the above calculations will not be known and it will be necessary to carry out experimental determinations of the minimum ignition temperature for at least two thicknesses of lagging. The constants can then be obtained from the plot of $\log_e 4\, \delta_c^* T_p^2/(r_2-r_1)^2$ vs. $1/T_p$, which is linear with slope of $-E/R$. Initially, an assumed value for E must be used in calculating θ_s for the purpose of obtaining δ_c^*; this is then corrected from the slope of the final plot. This graph provides a means of linear interpolation and extrapolation. When, as is the case for fairly thick layers,

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+Values for Qf given in Table 6 of ref. 2, are incorrect. Those for mixed hardwood sawdust must be multiplied by 6.75×10^{-2} and those for wood fibre insulating board by 7.30×10^{-2}.

the variation of θ_s is relatively small and can be ignored, the necessary calculation is simplified considerably.

Fig. 3 compares the calculated critical temperature distribution for self-heating, in 2in of wood fibre insulation on a nominal 1in pipe, with the temperature distribution in the absence of self-heating. In the neighbourhood of $z_s = 1.5$, self-heating leads to temperatures of about 30°C higher than in the absence of self heating.

The temperature gradient at the cool surface, in the presence of self-heating is given by

$$\frac{dT}{dr} = -\frac{RT_p^2}{E} \cdot \frac{4}{1-G} \cdot \frac{1}{r_2}$$

and, for this example, is -29.9°C/cm. This is 18 per cent higher than the gradient without self-heating (-25.4°C/cm) and, for the system to be stable, requires an ambient temperature of about 12°C instead of the 15°C assumed in the calculations. This adjustment is negligible and, at least in the range of practical interest, the method estimating θ_s, based on the assumption that the lagging is inert, is seen to be justifiable.

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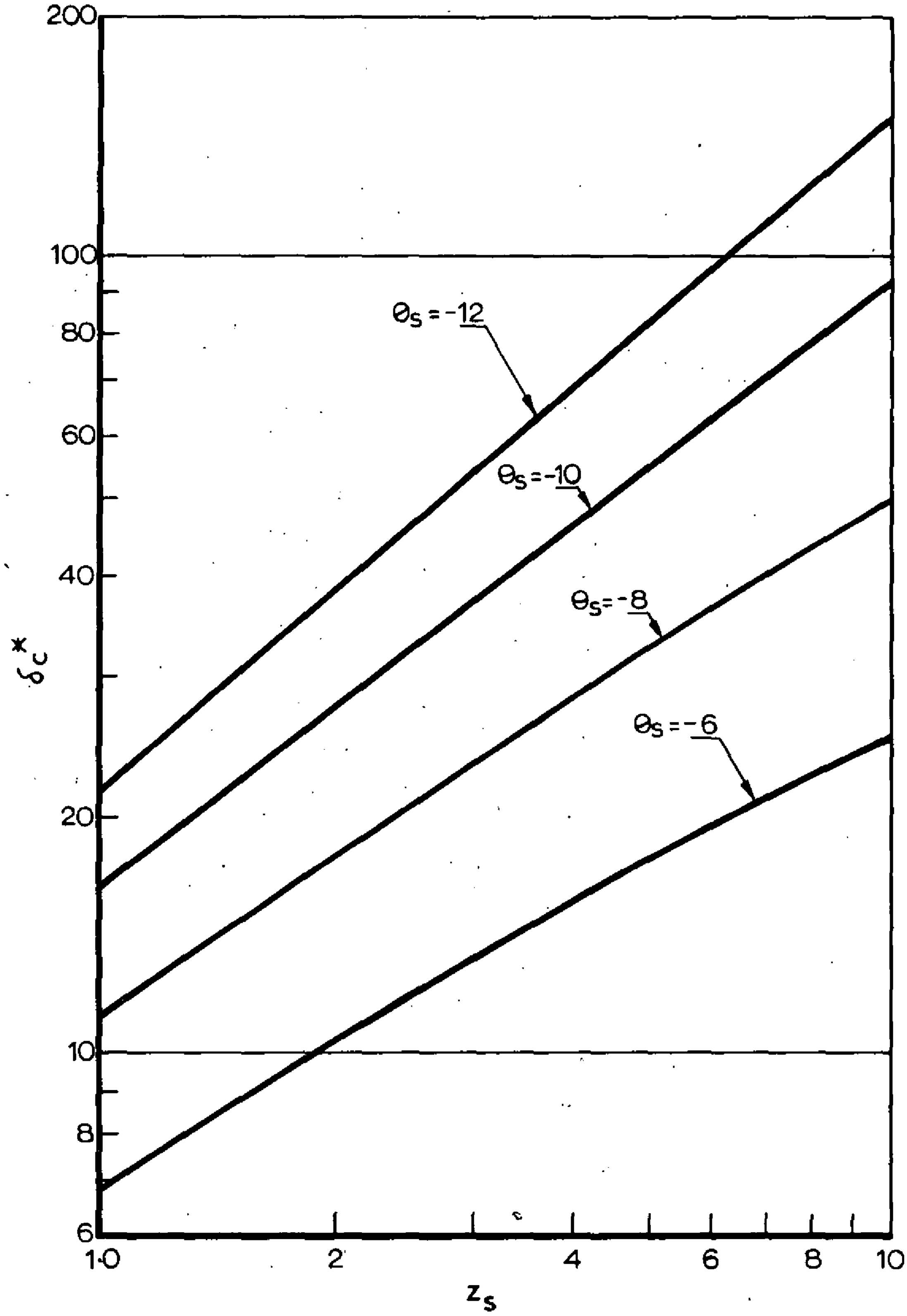

FIG. 1. IGNITION CRITERION FOR HOLLOW CYLINDER

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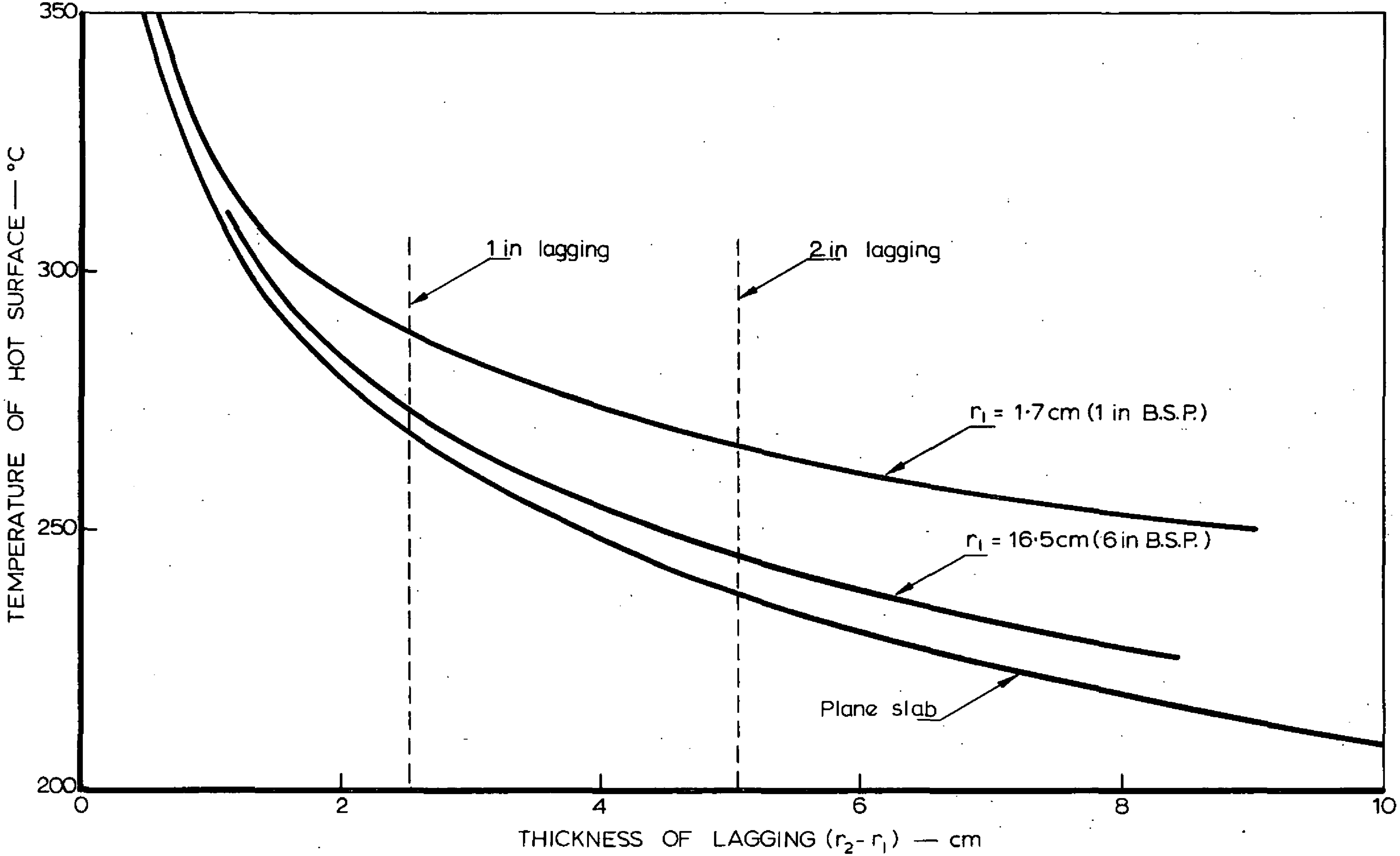

FIG. 2. TEMPERATURES FOR IGNITION OF WOOD FIBRE INSULATION

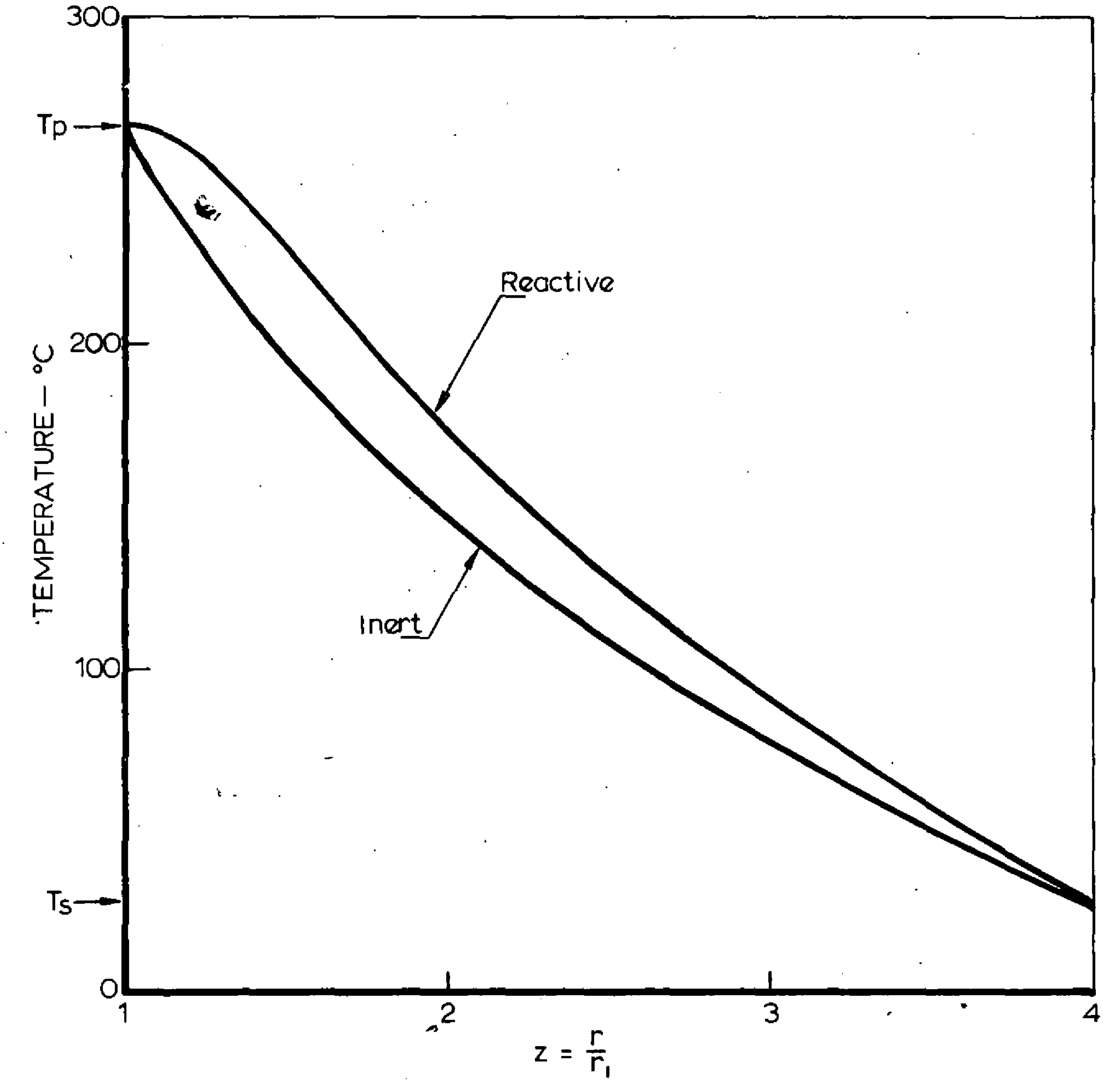

FIG. 3. CRITICAL TEMPERATURE DISTRIBUTION IN WOOD FIBRE INSULATION (2 INCH LAGGING ON NOMINAL 1 INCH B.S.P.)

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