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A THEORETICAL MODEL OF BUOYANT DIFFUSION FLAMES

by

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SUMMARY

This report discusses some problems in predicting theoretically the height of flames from free burning fires. Some solution of the simplified equation of motion are presented and compared with other solutions and the available experimental data. It is pointed out how the basis for the conventionally made boundary layer approximation to the vertical flow breaks down near the ground for large fires.

Notation

A_1 A_2	Constants
b	Radius of flame
c	Specific heat
C	Mol fraction of nozzle fluid in the unreacted stoichiometric mixture
D	Diameter fuel bed
f	Unspecified function
E	Entrainment constant = 0.16
F	Froude Number
g	Acceleration due to gravity
H	Heat of combustion of the fuel per unit mass of oxygen
I	Rate of loss of heat by radiation
K	Constant
L	Flame height
L*	Dimensionless Flame height = $^L/_D$
m	Mass flux
m*	Dimensionless burning rate = $\ddot{m}_o / \rho (gD)^{1/2}$
M	Dimensionless mass flux
$\underline{q}$	Velocity vector
$'''Q$	Rate of heat generation per unit volume
r	Radial component of polar coordinates
S_h, S_v	Horizontal and vertical distances measured from edge of flame
T	Absolute temperature
T_F	Adiabatic flame temperature
v_e	Entrainment velocity
v	Radial component of velocity
V	Dimensionless velocity
w	Upward velocity of gases in flames
W	Molecular weight
X	Dimensionless height
z	Polar coordinate in direction of flame axis
α	Dimensionless parameter involving heat of combustion
α_T	Mols of reactants/Mols products, for stoichiometric mixture
β	Dimensionless parameter involving Froude number
γ	Dimensionless parameter involving α and β
δ	Fraction of air flowing towards flame involved in combustion
θ	Temperature rise
μ	Tangent of angle of slope of boundary of flame
ρ	Density
ϕ	Heat flux (conserved property)

<u>Subscripts</u>

fl	Flame
o	Fuel, conditions at Fuel bed
St	Stoichiometric
T	Flame tip

<u>Superscripts</u>

"	per unit area
'	fluctuating quantity

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Introduction

By a dimensional analysis Thomas[1] has related the height of flames from freely burning fuel to the rate of burning and linear dimensions of the fuel by equations of the form

$$L^* = f(m^*) \tag{1}$$

Reasonable quantitative agreement was obtained by discussing these results in terms of the entrainment of air by the turbulent flames, and defining the tip of the flame by a certain temperature rise. These concepts are employed in the present report to derive equations of the form of Eq (1) from the equations of motion of the system.

The temperature of the tip of the flame has been measured by Yokoi[2] and by Thomas, Baldwin and Heselden[3] as 300°C* for wood and 500°C for town gas and may be related to the entrainment of air by the rising gases by the following equations:

$$m_T = m_A + m_o \tag{2}$$

$$m_T \theta_T = m_o \left(\frac{H}{c_{p\ell}} + \frac{c_o \theta_o}{c_{p\ell}} \right) \tag{3}$$

where is has been assumed that c does not vary with temperature. The characteristic temperature of the tip may thus be replaced by a value of $\frac{m_A}{m_o}$, which Thomas, Baldwin and Heselden[3], by measuring the air flowing towards flames, have estimated as in the range 60–80. These values illustrate that the air flowing towards the flame is at least an order of magnitude greater than the stoichiometric requirements of the fuel, implying that only a fraction of the air flowing towards the flame takes part in combustion.

In this report the flame is represented as a rising column of hot, reacting gases; above the flame tip the flame behaves as a thermal plume. Because the vertical velocity of the gas is expected to be increasing in the region where the temperatures are high enough to keep the density difference and the buoyancy force almost constant, continuity demands an inflow of the surrounding air across some suitably defined mean outer boundary outside which will be assumed there is no upward movement. In thermal plumes, with small density differences, the assumption of local equilibrium in the turbulence leads by dimensional analysis to the result that the entrainment velocity is proportional to the axial mean upward velocity[4]. However, chemical reaction within the flames induces high temperatures and thus large density differences over the length of the flame, and the constant of proportionality must then be replaced by a function of ρ/ρ_A. Following work on flames of high initial momentum (Ricou and Spalding[5], Thring and Newby[6], Thomas[1] has assumed that the inward momentum of the entrained air is proportional to the local upward momentum. This leads to an expression

$$v_e = E\left(\rho/\rho_A\right)^{1/2} w \tag{4}$$

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* Later work indicates that this temperature when measured by more refined methods, is 500°C.

where $E = 0.16$ from the data of Rouse, Yih and Humphries[7]. Morton[8] has recently used this assumption in an analysis of plumes with high density differences.

There is as yet insufficient experimental evidence for verification of equation (4), but the general validity of the assumptions introduced above is justified by the following calculation.

Solution I

Rasbash et al[9], Heselden[10] and Thomas, Baldwin and Heselden[3] have measured the velocity of the visible flame tip during a fluctuation and their results are consistent with the velocity being proportional to the square root of the height. Equation (4), based on their measurements, then becomes:

$$v_e = 0.16 \times 0.36 \left(2gz\,\theta_{fl} / T_{fl}\right)^{1/2} \qquad (5)$$

We may now obtain an expression for flame height by finding the total quantity of air entrained by the flame from Equation (5) and defining the flame tip by its characteristic value of m_A/m_0. Suppose the effective surface over which entrainment occurs forms the frustrum of a cone, angle $\tan^{-1}\mu$. Then from geometrical considerations it may readily be shown from equation (5) that

$$\frac{2}{5}\mu\left(L^*\right)^{5/2} + \frac{1}{3}\left(L^*\right)^{3/2} = \frac{1}{8 \times 0.16 \times 0.36}\left(\frac{T_{fl}}{2\theta_{fl}}\right)^{1/2}\frac{m_A}{m_0}\,m^* \qquad (6)$$

Clearly, for large values of L^*, $L^* \propto \left(m^*\right)^{2/5}$

and for small values of L^*, $L^* \propto \left(m^*\right)^{2/3}$

and for intermediate values of L^*, $L^* \propto \left(m^*\right)^{\nu}$

where $2/5 < \nu < 2/3$

Strictly, μ is a function of L/D, but for the present we will assume μ = constant = 0.2 say. This is a reasonable first approximation if we define the flame zone as the visible flame together with the surrounding layer of hot gases. Equation (6) is plotted in Fig. 1 with L^* plotted against m^* and $m_A/m_0 = 70$ and compared with experimental data reviewed by Thomas[1]. The extent of the agreement provisionally justifies the use of the entrainment concept and the definition of the flame tip by its temperature in the analysis that follows. Equation (5) has also been employed by Thomas[1] in discussing flame length, and has been used to predict the merging properties of flames from separate fuel beds by Baldwin, Thomas and Wraight[11].

Equations of motion

The equations of continuity, momentum and energy may be written

$$\underline{\nabla}\cdot\rho q = 0 \qquad (7)$$

$$\rho\left(q\cdot\underline{\nabla}\right)w = \frac{\rho g\theta}{T_A} + \nu\nabla^2 w - \rho\underline{\nabla}\cdot\left(\overline{w'q'}\right) \qquad (8)$$

$$\rho c\left(q\cdot\underline{\nabla}\right)\theta = Q - I + k\nabla^2\theta - \rho c\underline{\nabla}\cdot\left(\overline{\theta' q'}\right) \qquad (9)$$

where w is the upward component of velocity, q is the mean velocity vector at a point, Q is the rate of heat generation per unit volume and I is the rate of loss of heat by radiation per unit volume.

Define cylindrical coordinates $\mathbf{r} = (r, z)$ for the radially symmetric flame such that the axis of the flame is the z - axis, and let the mean and fluctuating velocity vectors be $\mathbf{q} = (v, w)$ and $\mathbf{q}' = (v', w)$ respectively. Suppose the flame occupies the region $r \leq b$ where $b = b(z)$ is the mean outer boundary of the flame across which there is a mean inflow of the surrounding air.

We may simplify the equation of motion by the following approximations:

1. $I = 0$. About $\frac{1}{3}$ of the heat is lost from the visible flame by radiation and will be allowed for in calculating the effective heat of combustion of the fuel.

2. In fully turbulent flow the turbulent transport is large compared with the molecular processes and we thus ignore the terms $\nu\nabla^2 w$, $\kappa\nabla^2\theta$.

3. The longitudinal diffusion is small compared with the lateral diffusion so that we ignore the terms $\frac{\partial}{\partial z}\left(\overline{w'^2}\right)$ and $\frac{\partial}{\partial z}\left(\overline{w'\theta'}\right)$.

4. In plumes it has been shown that the distribution of velocity and temperature across the plume is Gaussian. However, it is doubtful if this property holds in flames, so as a first approximation in the absence of experimental data and for the sake of simplicity, θ, w and Q will be assumed to be constant across the flame and zero outside it. These are the so called "top-hat" profiles. At the top of the visible flame it would be appropriate to assume that the profiles are closer to those of plumes.

Integrating Equations (7), (8) and (9) with respect to r and introducing the approximations 1 - 4, we have the following conservation equations: Conservation of mass:

$$\frac{d}{dz}\left(\rho b^2 w\right) = 2E\rho_A\left(\frac{\rho}{\rho_A}\right)^{1/2} b w \tag{10}$$

Conservation of momentum:

$$\frac{d}{dz}\left(\rho b^2 w^2\right) = \frac{g\theta}{T_A}\rho b^2 \tag{11}$$

Conservation of heat:

$$c_p \frac{d}{dz}\left(\rho b^2 w\theta\right) = Q b^2 \tag{12}$$

The terms $\left(r\overline{w'^2}\right)_\infty$, $\left(r\overline{w'\theta'}\right)_\infty$ appearing in the integration vanish since the turbulent transfer at ∞ is zero.

It has been shown that the air flowing towards the flame is an order of magnitude greater than the stoichiometric requirements of the fuel, so that only a fraction of the air takes part in combustion. Suppose that at each level some fraction δ, say, of the air entering takes part in combustion. Strictly $\delta = \delta(z)$ but since the functional form is not known, we will suppose for simplicity that combustion is distributed uniformly over the length of the flame, so that δ is a constant given by

$$\left(\frac{m_A}{m_0}\right)_{st} = \delta\left(\frac{m_A}{m_0}\right)_T$$

In this way all the fuel is consumed by the reaction before it reaches the flame tip.

We may now calculate the function Q in terms of the combustion of air at each level so that Equation (12) may be written:

$$\frac{d}{dz}\left(\rho b^2 w\theta\right) = 2E\rho_A\left(\frac{\rho}{\rho_A}\right)^{1/2}_{3} \frac{\delta H}{c_p} b w \tag{13}$$

Solution II

From Equations (10 and (13)

$$\phi = \frac{mg}{T_A}\left(\frac{\delta H}{c_p} - \theta\right) = \frac{mg}{T_A}\left(\frac{\delta H}{c_p} - \theta_o\right) = \phi_o \text{ = constant}$$

where $m = \rho b^2 w$

This property has an analogy in thermal plumes where the heat flux is constant: this result may be obtained by putting $H = 0$ in the above.
Equations (11), (12) and (13) may now be written

$$\frac{dm}{dz} = 2E\rho_A^{1/2}\,(mw)^{1/2} \tag{14}$$

$$\frac{d}{dz}(mw)^2 = 2m\left(mg\frac{\delta H}{cT_A} - \phi\right) \tag{15}$$

$$\phi = \phi_o \tag{16}$$

We now define dimensionless variables V, M, X such that

$$(mw)^{1/2} = (m_o w_o)^{1/2}\,V \tag{17}$$

$$m = E^{1/2}\left(\frac{8}{5}\right)^{1/2}\rho_A^{1/4}\,\frac{(m_o w_o)^{5/4}}{\phi_o^{1/2}}\,M \tag{18}$$

$$z = \frac{1}{2E^{1/2}}\left(\frac{8}{5}\right)^{1/2}\frac{1}{\rho_A^{1/4}}\,\frac{(m_o w_o)^{3/4}}{\phi_o^{1/2}}\,X \tag{19}$$

and define

$$F = \frac{2}{3}\left(\frac{8}{5}\right)^{1/2}E^{1/2}\,\frac{\delta H}{c_p T_A}\,\rho_A^{1/4}\,g\,\frac{(m_o w_o)^{5/4}}{\phi_o^{3/2}} \tag{20}$$

where $F^{\frac{1}{2}}$ is a Froude Number,

Then Equations (14) - (16) may be written

$$\frac{dM}{dX} = V \tag{21}$$

$$\frac{dV^4}{dX} = \frac{8}{5}\,M\left(\frac{3}{2}FM - 1\right) \tag{22}$$

where $V = 1$, $M = M_o$ at $X = 0$.

Integrating Equations (21) and (22) gives

$$V^5 = 1 - (M^2 - M_o^2) + F(M^3 - M_o^3) \tag{23}$$

$$X = \int_{M_o}^{M}\left\{1 - (M^2 - M_o^2) + F(M^3 - M_o^3)\right\}^{-1/5} dM \tag{24}$$

which will be referred to as Solution II.

Evaluation of Solution II

The integral appearing in Solution II (Equation (24)) cannot be evaluated in terms of elementary functions, but in two special cases it is possible to derive approximate expressions for flame height.

First, however, we express Equation (24) in a more convenient form for finding flame heights:

$$X = \left(\frac{M_0}{1+\alpha}\right)^{3/5} \int_{1+\alpha}^{(1+\alpha)M/M_0} \left(v^3 - v^2 + \gamma\right)^{-1/5} dv \qquad (25)$$

where

$$\gamma = (1+\alpha)^2(\beta - \alpha), \quad \beta = M_0^{-2}$$

$$\alpha = \left(\frac{c\theta}{\delta H} - \frac{1}{3}\right)\left(1 - \frac{c\theta_0}{\delta H}\right)^{-1} = F M_0 - 1$$

Now $\frac{M}{M_0} = m/m_0$ takes a value of the order 40-70 at the flame tip and it can then be shown that the following approximations hold:

1. When γ is small

$$X_T = \left(\frac{M_0}{1+\alpha}\right)^{3/5} \int_{1+\alpha}^{(1+\alpha)M/M_0} v^{-2/5}(v-1)^{-1/5} dv \qquad (26)$$

This expression is exact when γ = o, i.e. when $\alpha = \beta$, but this requires $\alpha > 0$ since $\beta = M_0^{-2}$.

2. When γ is very large,

$$X_T = M_0\left(M/M_0 - 1\right)\left(1 - \alpha M_0^2\right)^{-1/5} \qquad (27)$$

Now for wood with some allowance for radiation loss the effective heat of combustion may be taken as 2500 cals/gm. c = 0.24 cal gram^{-1} degC^{-1} and $\left(\frac{m_A}{m_0}\right)_{st} = 4.5$
Therefore, $H = \frac{2500}{4.5}$ cals/gm and from Equation (2) and (3) assuming a flame tip temperature of 500°C and making allowance for the temperature profile at the tip we have $\left(\frac{m_A}{m_0}\right)_T = 34$. However, Thomas, Baldwin and Heselden[3] have measured values of $\left(\frac{m_A}{m_0}\right)_T$ of about 70, so in the following analysis both values will be considered.
θ_0 is unknown but it is small; for simplicity of calculation we will assume α = o so that θ_0 = 50-100°C.

The integral appearing in Equation (26) has been evaluated numerically, and its values are approximately

(1) 16.2 when $m_A/m_0 = 34$
(2) 23 when $m_A/m_0 = 70$

The equations expressing the dimensionless flame height L^* in terms of the dimensionless parameter m^* are then as follows

$$L^* = A_1 (m^*)^{2/5} \text{ when } (m^*)^2 << 1 \qquad (28)$$

where $A_1 = 24$, $(m_A/m_0)_T = 35$
$A_1 = 43$, $(m_A/m_0)_T = 70$

and

$$L^* = A_2 \text{ when } (m^*)^2 >> 1 \qquad (29)$$

where $A_2 = 61$, $(m_A/m_0)_T = 35$
$A_2 = 100$, $(m_A/m_0)_T = 70$

High Momentum Jets

These values of L^* may be related to previous work on high momentum jets. Hawthorne, Weddel and Hottel[13] have burned jets of different fuels and their results were in agreement with a theoretical value of L^* given by

$$L^* = \frac{5.3}{c}\left\{\frac{T_F}{\alpha_T T_0}\left(c + (1-c)\frac{W}{\omega_0}\right)\right\}^{1/2}$$

Assuming the values of heat of reaction of wood volatiles, and that the density of nozzle fluid/density of air = 1.4, and taking $\alpha_T = 1$ this expression gives

$$L^* = 75$$

which compares with the values derived above.

In some rather limited experiments Webster and Smith[12] have burned momentum jets of wood volatiles and found values of L^* lying between 30 and 60 but the rate of burning was not measured. Putnam and Speich[14] have used $\frac{3}{8}$ in jets burning city gas which contains a high proportion of methane and the value of L^* found was of the order 100 to 120.

The latter data was included amongst the flame height data reviewed by Thomas[1] and indicates a 2/5 power law for fuel jets. However, the remaining data on jets suggest that the 2/5 power law overestimates the height of the flames from jets of wood volatiles, and that the power relationship should more rapidly approach L^* = const.

Solution III

Nielsen, Tao and Wolf[15] have presented computer solutions of the conservation equations for a model having some similarities to that proposed in the derivation of Solution II. The results were given in a graphical form and the line shown in Fig. 1 is deduced by interpolation from their curves. However, the following features were incorporated in their model, which differs slightly from that developed in this paper.

1. Allowance was made in the equations of motion for loss of heat by radiation.

2. The entrainment function was assumed to be of the form

$$\rho_A v_e = E \rho \omega$$

This introduces a numerical factor into the flame height expression: the amount of air entrained is underestimated approximately by a factor of 2 compared with Equation (4).

3. The fuel and air burn to stoichiometric completion, until all the fuel has been burned, and the flame then acts as a thermal plume, cooling until $\theta = 300^{\circ}C$.

Discussion

A comparison of the three solutions presented above and the experimental data, all plotted in Fig. 1 shows that Solution I follows the experimental line best. Solution II gives reasonable estimates of L^* when $L^* > 2$, the region in which laboratory measurements have been made, but when L^* is small both Solution II and III, give a relationship of the form $L^* \propto (m^*)^{2/5}$ compared with the measured $L^* \propto (m^*)^{\frac{2}{3}}$.

Let us examine Solution I in terms of the conservation equation. Suppose $w = K z^{\frac{1}{2}}$. Then substituting in the equation of conservation of mass

$$\frac{d}{dz}\left(\rho b^2 z^{1/2}\right) = 2E\rho_A^{1/2}\left(\rho b^2 z\right)^{1/2}$$

which has the solution

$$(\rho b^2)^{1/2} = \frac{4}{5} E \rho_A^{1/2} z \qquad (30)$$

Now substitute in the momentum conservation equation

$$\frac{d}{dz}\left(K^2 z^3\right) = g\frac{\theta}{T_A} z^2$$

whence θ = constant, since by definition K = constant,

and

$$K^2 = g\frac{\theta}{3T_A}$$

$$\therefore \quad \omega = \left(\frac{\theta}{3T_A} g z\right)^{1/2} \qquad (31)$$

which compares favourably with Equation (5).

But $\rho \propto \frac{1}{T_0 + \theta}$, so ρ = constant and Equation (31) then shows that the flame is straight-sided, its bounding edges passing through the origin. The solution $w = K z^{\frac{1}{2}}$ thus corresponds to a flame from a point source in which θ = constant. The latter approximation is probably a reasonable first approximation, but the point source requirement cannot be justified for large area fires when L^* is small.

Comparison of solutions II and III and the measured values of flame heights (Fig. 1) show that for small values of L^* there is not only poor numerical agreement, but also a difference of form: measured flame heights and qualitative observation of flame shape indicate a line of slope $\frac{2}{3}$. This indicates that one or more of the postulates and basic assumptions underlying a formulation of the conservation equations is not justified when L^* is small. These will now be examined:-

1. The variation of radius with height in the model of solution 3, given in reference (15), shows that initially the flame gases flow approximately horizontally; for this region, therefore, the boundary layer approximations for the vertical flow cannot be expected to be valid. However, this initial region occupies only a very small part of the flame compared with its height, and therefore it is expected that this approximation will not affect the overall properties, such as the flame height.

This view may be justified indirectly by considering the power relationship between flame height and rate of burning: if the initial horizontal flow region is neglected, then the boundary of the flame approximates to that of a flame from a point source. The point source approximation then applies, giving a two-fifth power relationship and this agrees with the relationship found using the more complete theory.

In the model of solution 2, it is difficult to obtain readily evaluated analytic expressions for the variation of width with height, but it is evident that when m^* is small an initial zone of horizontal flow does exist. We have:

$$\frac{1}{2E\rho_A^{1/2}}\frac{d}{dz}\left(\rho^{1/2} b\right) - \frac{d}{dX}\left(\frac{n}{V}\right) = 1 - \frac{2}{5}\frac{n^2}{V^5}\left(\frac{3}{2}\frac{n}{n_0} - 1\right) \qquad (32)$$

$$\rho^{1/2}\frac{db}{dz} = \frac{d}{dz}\left(\rho^{1/2} b\right) + \frac{2E\rho_A^{1/2}}{T}\left(\frac{\delta H}{c} - \theta\right) \qquad (33)$$

Thus initially,

$$\frac{d}{dz}\left(\rho^{1/2} b\right) = 2E\rho_A^{1/2}\left(1 - \frac{n_0^2}{5}\right)$$

and since $n_0 \propto \frac{1}{m^*}$ when $m^* \ll 1$, b initially decreases rapidly with height. The extent of the horizontal flow zone is defined by a stationary value of $\frac{db}{dz}$, and for large values of M_0 it can be shown from (32) and (33) that the only stationary value with $M \gg M_0$ lies within the limits $1 \leq M/M_0 \leq 3/2$: stationary values with $M < M_0$ are inadmissible, since M is an increasing function of X. Thus, the initial zone of horizontal flow is very small compared with the height of the flame which is defined by $M/M_0 \sim 40$. In this region also, the disposition of the fuel will be important. For example, in practical situations the fire may well be from a number of individual sources, merging at some height above the fuel bed; this may greatly increase the rate of air entrainment and thus possibly eliminate the zone of horizontal flow.

2. Comparison of solution II and III indicates that the vertical distribution of heat release by combustion is of little consequence in determining the form of the function $L^* = f(m^*)$ when L^* is small. However, these two models are not strictly comparable because of the different assumptions concerning the entrainment of air and the loss of heat by radiation and the different values of the heat of combustion of this fuel. The effect of changing the vertical distribution of heat release may be studied by comparing solution II with the extreme case in which all heat is released at the fuel bed, the rising column of gases thereafter behaving as a thermal plume. Putting $\theta_0 = H/c$, $\delta = 0$ in the model of solution II leads to an expression for flame height of the same form as (28) and (29) the constants being approximately 20 per cent smaller when m^* is small, and three times smaller for fuel jets when m^* is large. Thus for large fires the variation of vertical distribution of heat release is not important in determining overall properties, such as flame height, but this is not true for fuel jets.

The rate of air entrainment by the flame is very much greater than the air requirements of the fuel and it may be, therefore, that the end of the visible flame zone cannot be identified with the end of the reaction zone, combustion taking place only in the initial region of the flame where the stoichiometric air requirements of the fuel are entrained. An alternative view is that the rates of heat release by combustion is governed by some slower diffusion process than entrainment of air. For example, there are two diffusion processes taking place – entrainment of air from outside the flame and of fuel and air within the flame zone. The latter may be the slower process of the two and this would then govern the rate of reaction of air with the fuel and it would be unrealistic to relate heat release to the rate of entrainment of air. This will be the subject of further investigation.

3. The flame tip has been defined by θ = constant, and there is no reason to suppose that in large area fires this definition is substantially inadequate.

4. There is some evidence to support the view that existing entrainment theory, as described above, is inadequate particularly for large area fires.

(a) It has been shown experimentally by Thomas, Baldwin and Heselden that the boundary of the visible flame zone may not necessarily be identified with the boundary of the column of hot rising gases as in solutions II and III. This difference may in part account for the different indices of equation (28) indicated by the above theories and by dimensional analysis. The latter is deduced by evaluating a shape factor as a function of L/D, based on qualitative observations of the approximate geometry of the visible flame zone which differs considerably from the flame geometry of solutions II and III.

(b) The pressure drop is negligible on the axis of the small fires and plumes that have been the subject of most investigation, and in this case a simple relationship between the inward momentum and a characteristic momentum of the flame follows. However, in fires of large area the tendency of the hot gases to accelerate must be associated with a large pressure drop, greatly increasing the rate of entrainment. It seems proper, therefore, in considering the equation of motion, to include the pressure gradient terms in studies of large fires.

(c) Let us follow the path of the entrained air after it has passed the flame boundary. At the same time as it is imparted an upward velocity by the hot, rising gases, it will continue to be transported towards the centre of the flame. Suppose the component of inward velocity is unchanged when across the flame boundary. Then the trajectory of the air is given by

$$S_h = \frac{v_e}{\omega} S_v$$

We deduce from using Equation (4) that if air is to reach the flame axis before the flame tip, then

$$\frac{L}{D} > \frac{1}{2 \times 0.16} \left(\frac{\rho}{\rho_A} \right)^{1/2} \sim 6.25$$

which is approximately the same point at which Solution II and III tend to diverge seriously from experiment. This simple calculation provides an additional argument in favour of a greatly increased entrainment rate in large area fires.

It is clear from this model that combustion tends to take place near the edge of the flame, rather as in the potential core of a jet, the temperature and velocity thus being greatest in this region. Air entrainment would be expected therefore to be related to flow properties at the edge of the flame rather than on the axis.

For $^L/D$ we must expect some contribution to mixing from air falling into the fire zone from above.

Conclusion

Three models for flames have been examined and compared with experiment. The best fit is obtained from a theory based on a measurement of flame tip velocity, but comparison with the conservation equation deduced shows that its use is not justified except for long flames. Equations for flame height deduced from the conservation equation give a reasonable estimate of flame height when $L^* > 2$ but outside this range the divergence between theory and experiment shows that the use of some of the basic assumptions is not justified. It is suggested that there should be a greatly increased entrainment rate in large area fires, in which velocity and temperature profiles, pressure gradients and vertical mixing should be considered.

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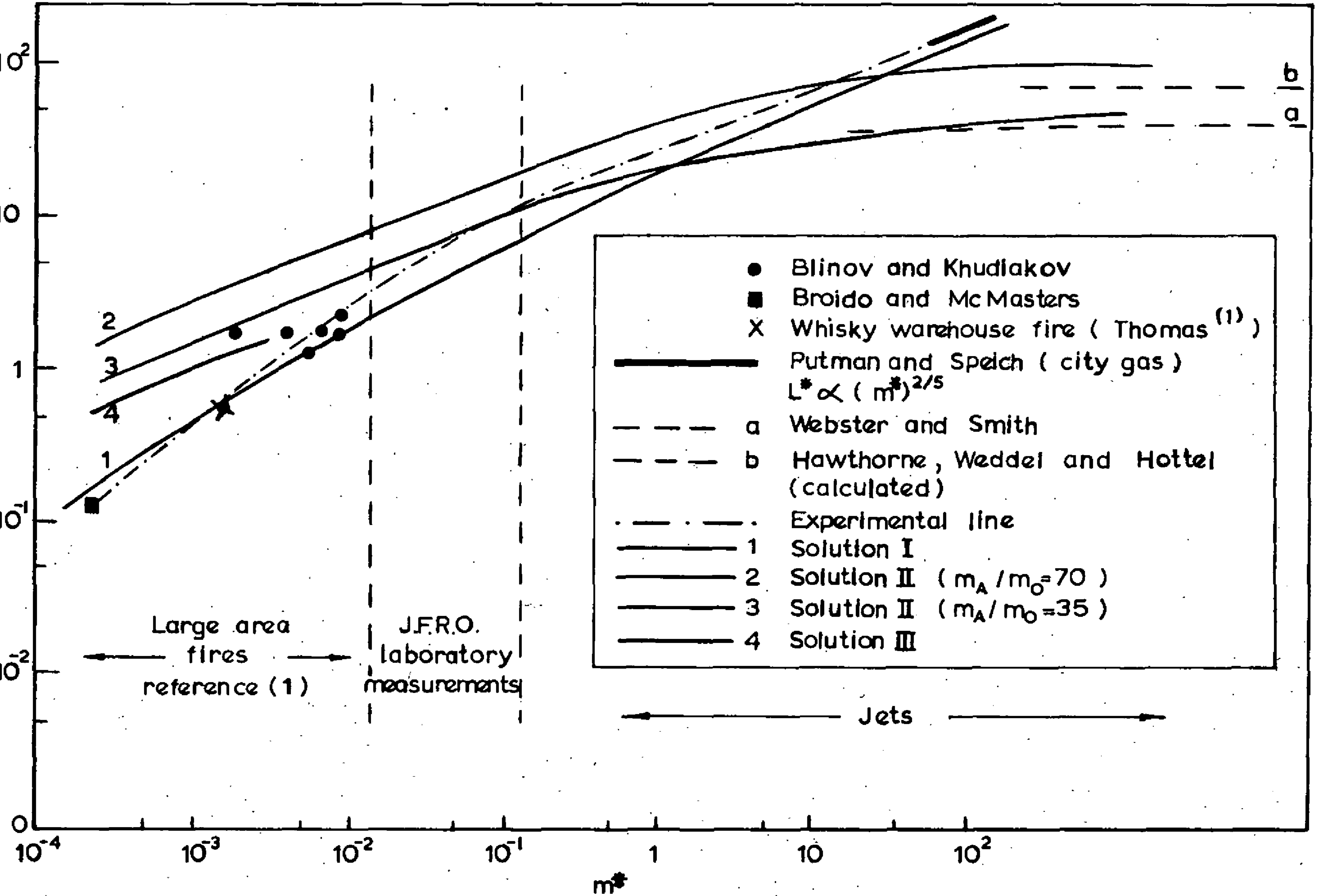

FIG.1. EXPERIMENTAL DATA AND SOLUTIONS I, II AND III