Fire Research Note

NO. 594

THE CONTRIBUTION OF FLAME RADIATION TO FIRE SPREAD IN FORESTS.

by

P. H. THOMAS

FIRE
RESEARCH
STATION

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SUMMARY

If the heating of unburnt fuel ahead of a steadily moving fire front is by radiation from the flames above the fuel bed and by radiation transmitted through the fuel bed there are two possible rates of spread. In one the flames above the fuel bed are essentially "optically thin" and the spread is slow - in the other they are "optically thick" and the spread is fast. Between these two states is an unstable state. Some calculations are made which suggest that only fire fronts larger than of order 10 m are capable of fast spread. There is, however, a necessary condition for fast spread involving values of flame properties. These, however, are not sufficiently well enough established quantitatively to give a clear answer as to whether fast spread controlled by flame radiation is never possible or possible sometimes.

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NOTATION

a	=	$\dfrac{\dot{i}_B L^*}{q''}$
A	-	constant
B	-	constant
c	-	effective specific heat of fuel
C	-	constant
D	-	length of burning zone
F	-	radiation exchange factor
g	-	acceleration due to gravity
h	-	height of fuel bed
H	-	vertical height of flame
i	-	intensity of radiation (black body)
k	-	attenuation coefficient of radiation in flame
L	-	flame length
$\dot{m}''$	-	mass burning rate per unit cross sectional area
$\dot{m}'$	-	mass burning rate per unit width of fire front
M''	=	$\dfrac{\rho_b h}{t_B}$ nominal mean burning rate per unit ground area
N	-	$\dfrac{q'' t_B}{L^* \rho_s h c \theta_i} = \dfrac{q''}{L^* c \theta_i M''} = \dfrac{\tau}{k a h}$
q''	-	$(\rho_a{}^2 g c^2 \theta_i{}^2 h i_B)^{\frac{1}{3}}$ which has the dimensions of heat flux.
R	-	rate of spread
t_B	-	residence time of fuel element in burning zone.
u	-	dummy variable
U	-	wind speed
ω	-	$\rho_b h$ = fuel loading per unit ground area
W	-	width of fuel bed
x, y, z	-	dummy variables for length, width and height
α	-	radiation attenuation coefficient in fuel
β	-	$\dfrac{i_F a \tau}{2 i_B}$
γ	-	$\tau/N^3 = \dfrac{k L^{*3}}{g \rho_a^2} M''^2$
δ^*	=	D/L
ϵ	-	emissivity
Δ	-	flame thickness
ξ	-	dummy variable
ρ	-	density
θ	-	temperature rise
Θ	-	dimensionless rate of spread
σ	-	specific surface of fuel
λ	-	volume of voids per unit surface of fuel
ϕ	-	inclination of flame
τ	-	$\dfrac{k i_B t_B}{\rho_b c \theta_i}$ dimensionless residence time.

Suffices

F.	flame
B.	burning zone
b	bulk fuel bed
a	air
s	solid fuel
i	ignition
c	critical
*	dimensionless defined in text.

TABLE I

Numerical values given to quantities when they are treated as constant.

i_B	intensity of radiation from burning zone	4.2 J cm^{-2}
i_F	intensity of radiation (black body) from flame	12.6 J cm^{-2} s^{-1}
g	acceleration due to gravity	981 cm s^{-2}
k	attenuation coefficient of radiation in flame	0.003 cm^{-1}
h	height of fuel bed	15 cm
$c\theta_i$	enthalpy to raise fuel to ignition	670 J/g
t_B	residence time for fuel	250 s for cribs 75 s for shavings
ρ_a	density of air	1.28×10^{-3} g cm^{-3}
ω	fuel loading	1.5 g cm^{-2} for cribs 0.15 g cm^{-2} for shavings
L^*	dimensionless flame length	47

Quantities derived from above

q''	36 J cm^{-2}
λ	5.5
M''	0.006 g cm^{-2} s^{-1} for cribs and 0.002 g cm^{-2} s^{-1} for wood shavings
γ	$= 7.2$ for cribs, 0.8 for shavings
N	$= 0.19$ and $\tau = 0.05$ for cribs
N	$= 0.57_3$ and $\tau = 0.15$ for shavings

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Introduction

The purpose of one part of fire spread theory is to explore some of the different possible types of behaviour arising from the various possible modes of heat transfer to the unburnt fuel. If a fire is able to spread at a steady speed the burning zone can be considered a stationary combustion zone into which passes a supply of fuel at a steady rate. The fire heats up the unburnt fuel. Also the loss of heat from the burning zone and the restriction on the air supply to the burning fuel may influence the burning zone sufficiently to control the rate of heating of the unburnt fuel but in this paper we shall be considering fire behaviour on the assumption that any such interaction is so weak it can be neglected.

In two previous reports [1,2] the spread of fire in still air through beds of fuel of various thicknesses and porosities including cribs has been attributed to the heating of unburnt fuel by radiation transmitted through the fuel bed. The source of this radiation is likely to be the solid surfaces of the fuel, which are heated by the flames, the flames themselves are usually optically thin, heating the fuel by convection.

The theoretical model for thin fuels was essentially a simple heat balance, which if heat loss is neglected, is described by

$$R \rho_b c \theta_i = i_B \epsilon_B \qquad (1)$$

where R = rate of spread

ρ_a = bulk density of fuel bed

c = specific heat of fuel

θ_i = the ignition temperature rise of the fuel

and $i_B \epsilon_B$ is the net flux of heat transmitted from the burning zone to the fuel ahead of it.

The above equation has to be modified when fuels thicker than about 0.5 cm are burning because then only a fraction of the total mass has to be heated before the surface is hot enough for ignition to occur and

$$R \rho_b c \theta_i > i_B \epsilon_B$$

The theoretical modification has been given elsewhere [1]. Because $i_B \epsilon_B$ is the net heat transfer various forms of the allowance for heat loss have to be discussed. Nevertheless, over a wide range of data it has been found that for these fuels

$$R \rho_b = 6 \text{ mg cm}^{-2}\text{s}^{-1} \qquad (2)$$

With c for dry wood equal to 1.4 Jg^{-1} deg C^{-1}, $\theta_i \sim 300°C$ and a nominal moisture content of 7 per cent, $c\theta_i$ is 670 J/g and equation (1) then gives a nominal value of $I_B \epsilon_B$ of about 4 W cm^{-2}

The value of ρ_b is of course governed by the porosity of the fuel bed.

It may be shown that conduction along fuel elements orientated in the direction of spread is generally negligible. In this paper no account will be taken of this and heat losses from the fuel bed will be allowed for by using a net transfer rate. The loss of accuracy in this method is not large compared with the uncertainties in some of the other parameters that will be employed.

Equation (1) may be regarded as the basic "law" for still air. Notably the height of the fuel bed is not involved in this expression when the fuel bed is described by ρ_b. However, if h is the height of a fuel bed and w is the weight of fuel per unit area

$$\rho_b = w/h \qquad (3)$$

It should also be noted that in fires not all the fuel is burnt. Here ρ_b is the bulk density of fuel heated to ignition whether eventually burnt or not. It is a consequence of this mechanism of fire spread that R is independent of the depth of fuel burnt, provided ρ_b is constant and in beds of real fuel some fuel is thin and some thick and this simple theory does not take account of the distinction.

So far too we have assumed that $I_B \epsilon_B$ is constant. This is not strictly so since the emissivity of the zone may be less than unity, and this we discuss below. The main purpose of this paper is to consider what the effect of flame radiation may be on fire spread.

The effect of the finite emissivity of the burning zone

We assume that the emissivity of the burning zone follows the well known exponential law

$$\epsilon_B = 1 - e^{-\alpha D} \qquad (4)$$

where α is an effective attenuation coefficient and D the thickness of the flaming zone in the direction of spread. It is readily shown that for a bed of randomly arranged fuel elements

$$\alpha = \frac{\sigma \rho_b}{4 \rho_s} = \frac{1}{4(\lambda + 1/\sigma)} \tag{5}$$

$$\frac{\rho_s}{\rho_b} = \frac{1}{1 + \sigma\lambda} \tag{6}$$

where λ is the volume of voids per unit surface area

σ is the surface of fuel per unit total volume

and ρ_s is the density of the solid fuel.

The use of equation (5) for regular arrangements of fuel such as cribs is only approximate but is acceptable.[1]

The duration of flaming or "residence" time is

$$t_B = D/R \tag{7}$$

so that we may write

$$\epsilon_B = 1 - e^{-\left(\frac{R}{4(\lambda + 1/\sigma)}\right)} \tag{8}$$

We now modify equation (1) to allow for the emissivity of the burning zone, and introduce m'' ($= R\rho_b$) for the mass rate of spread per unit cross-sectional area.

$$m'' c\,\theta_i = i_B\left(1 - e^{-\frac{m'' t_B}{4\rho_b(\lambda + 1/\sigma)}}\right) \tag{9}$$

Inserting equation (6) into equation (9) gives

$$\frac{m'' c\,\theta_i}{i_B} = 1 - e^{-\frac{m'' t_B \sigma}{4\rho_s}} \tag{10}$$

Fig. 1 shows the left and right hand sides of equation (10) plotted against

$\frac{m'' t_B \sigma}{4 \rho_s}$

No spread is possible if

$$\frac{4 \rho_s c \theta_i}{t_B \sigma i_B} > 1 \tag{11}$$

This condition has been given by Emmons [3] who has also discussed the influence of the initial conditions on the development of the fire.

It has been found that for a given thermal environment the burning time tends to increase with fuel thickness; the thick fuel tends to burn slower because a greater thickness of char is produced [4] and this is also true in cribs [5,6]. Thus $t_B \sigma$ increases with fuel thickness and m'' rises. Similarly, for thin fuels $t_B \sigma$ and m'' fall and when m'' is small, - near the extinction limit, the expansion of the exponential in equation (10) leads to

$$m'' = \frac{8 \rho_s}{t_B \sigma} \left(1 - \frac{4 \rho_s c \theta_i}{i_B t_B \sigma} \right) \tag{12}$$

as a first approximation. If the bulk density is reduced equations (1) and (10) predict an increasing rate of spread because m'' is constant and R is inversely proportional to ρ_b. As ρ_b falls the linear scale of the fuel bed increases and this produces a corresponding increase in the spread rate. However, the model at some point fails to describe a real situation. An increase in R implies an increase in D (equation (7)). Infinitely low ρ_b implies infinitely large D and whether propagation is possible depends on the initial condition in which only a finite zone is ignited. Emmons [3] has shewn that for a given linear size of this initial fire there is a lower limit to the bulk density of fuel bed through which fire can propagate. In this paper we shall not give any further consideration to this nor to the finite emissivity ϵ_B; we shall instead modify the heat balance to include the effects of radiation from the flames.

3. A theory of fire spread by radiation

3.1. The contribution of the flames

The value of $R \rho_b c \theta_i$ the rate at which heat is transferred forward per unit area must be increased from $i_B \epsilon_B$ to allow for the heat transferred from the flames.

Allowing for this, the heat balance for a fuel bed of height h and width W, see Fig (2), is, in the steady state

$$R \rho_b c \theta_i h = i_B \epsilon_B h + i_F \epsilon_F L F \tag{13}$$

where i_F is the black body intensity of radiation emitted by the flame of length L .

ϵ_F is the flame emissivity

F is the radiation exchange factor which is a function of $\frac{L}{W}$ and $\frac{D}{L}$ (see Fig. (2).)

Before discussing the implications of equation (13) we shall discuss various items in it, reviewing briefly the relevant information we shall need to exploit.

3.2. Flame emissivity

The emissivity ϵ_B depends on the flame thickness. This varies along the length of the flame but, for simplicity, a mean thickness will be considered and the approximate relation between emissivity and thickness assumed for this paper is

$$\epsilon_B = 1 - e^{-k\Delta} \qquad (14)$$

where k may be different for different fuels and conditions of combustion but is taken here in numerical examples as 0.0031 cm^{-1} .

3.3 The flame thickness

At present we have no data on which to base Δ but as a first approximation we take

$$\Delta = D \sin \phi \qquad (15)$$

where ϕ is defined in Fig. 2.
This should be reasonable for the most important section of the flame; near the flame front. In the absence of a wind we shall take $\phi = 90^{\circ}$

It may be argued that there is an inconsistency in taking the effective radiation for the flame at its centre so that F decreases as $\frac{D}{L}$ increases whilst putting Δ equal to D. However, where $\frac{D}{L}$ is significant in reducing F the value of D is generally large enough to make any error in using D for Δ in equation (14) of little consequence. For fast spreading large flames, errors in Δ may well be irrelevant since ϵ tends to be high; though this would not necessarily be so for small scale fires.

3.4. The flame length

For long flame fronts where the extensive width of the front is assumed to be irrelevant to determining the flame length it is convenient to define the dimensionless variables

$$L^* = \frac{gL}{(\dot{m}'g/\rho_a)^{2/3}} \tag{16i}$$

where ρ_a is to be taken as the density of air and g is the acceleration due to gravity, and m' the burning rate per unit width of front

$$D^* = \frac{gD}{(\dot{m}'g/\rho_a)^{2/3}} \tag{16ii}$$

and $$U^* = \frac{U}{(\dot{m}'g/\rho_a)^{1/3}} \tag{16iii}$$

where U is the wind speed.

Experiments have provided data for the following conditions $U^x = 0$ and $3 < U^x < 10$ and $7 < D^x < 25$ [7,8]. Theory suggests that when $\frac{D^x}{L^x} \to 0$, L^x should become constant and the experiments suggest this limit can be taken for a wood fire as

$$L^x = 47 \tag{17}$$

For still air [6,7] we have

$$L^x = 85\, D^{x\,-0.3} \qquad (7 < D^* < 25) \tag{18}$$

and for $3 < U^* < 10$

$$L^x = 70\, U^{*\,-0.21}\, D^{*\,-0.19} \tag{19}$$

when $D^* \to 0$ we employ

$$L^x = 55\, U^{*\,-0.21} \tag{20}$$

These formula are inconvenient for studying the transition from still air to windy conditions and for this purpose, a more general empirical equation needs to be devised, such as

$$L^x = \frac{A}{(1 + BU^x)^m (1 + CD^x)^n} \tag{21}$$

Values of A, B, C, m and n would need to be chosen to give the same degree of correlation with experimental data as the earlier analyses. The choice of the constants is not essential for the following discussion which will be qualitative at first.

3.5. The deflection of the flames

We have data [6] on L and H as defined by Fig. (2). For the purposes of this preliminary discussion we shall use the concept of a line source and write

$$\sin \phi = H/L \tag{22}$$

where $$H^x = 38\, U^{x\,-0.69} \tag{23}$$

H^x is defined in relation to H as L^x is to L.

It may be noted in passing that the coefficient 55 in equation (20) was obtained by plotting $L^x\, U^{x\,0.21}$ against D^x and extrapolating to zero D^x.

Here we use the equation only to obtain

$$\sin \phi = 0.69 \quad U^{x\,-0.48} \qquad (3 < U^x < 10) \tag{24}$$

$$= 1 \qquad (U^x \rightarrow 0)$$

4. Fire spread theory

From the above equations it may be seen that we shall sometimes need to use m' as a measure of the rate of fire spread and sometimes m", the double dash showing it is per unit area and the single dash per unit of line. m' is proportional to the heat released per unit of fire front which is often employed in describing wild fires.

We then have $$m'' = R \rho_b = m'/h \tag{25}$$

and we rewrite equation (13) as

$$m'' = \frac{i_B\, t_B}{c\,\theta_i} + \frac{i_F\,(1 - e^{-kD})\, m''^{2/3}}{(\rho_a^2\, g h)^{1/3}\, c\,\theta_i}\, L^*\, F. \tag{26}$$

where we retain L and F in their dimensionless forms and take c as an effective value allowing for moisture. For still air and light winds $L^x \sim 47$ and since it depends only weakly on D^x (and U^x) it will be considered constant. If the solution to the equation gives m'' and hence D^x (see below) such that L^x should be altered it is likely that only a few steps in iterative calculation would be necessary.

When D/L becomes large enough the flame can be considered as radiating from a plane behind the front (see Fig. 2). The exchange factor F between the shaded areas in Fig. 2. is readily shown by methods given in text books to be

$$F = \frac{1}{\pi L W}\int_0^W dy \int_0^W dy' \int_{\frac{D}{2}-L\cos\phi}^{\infty} x\,dx \int_0^H \frac{z\,dz}{(y-y')^2+x^2+z^2} \qquad (27i)$$

F is shown in Fig. 3. for $\phi = 90$ and for various values of D/L and L/W can be evaluated in terms of elementary functions, an expression which is convenient for some purposes when the flame is vertical

$$F = \frac{1}{\pi L W}\int_{D/2}^{\sqrt{L^2+D^2/4}} du \int_0^{W/u} \frac{W - zu}{(1+z^2)}\,dz \qquad (27ii)$$

For an infinitely wide vertical flame

$$F = \frac{1}{2}\left\{\sqrt{1+\frac{D^2}{4L^2}} - \frac{D}{2L}\right\} \qquad (27iii)$$

This value of F is reduced from 0.5 by 25% if D/L exceeds 0.6.

The complete evaluation of F leads to cumbersome expressions and we shall deal with the variation of F by using approximate forms. An important feature of the fire spread equation (26) will now be demonstrated and some quantitative results evaluated for special cases, in which we assume firstly that $\epsilon_\beta \sim 1$.

i.e. $$\frac{4\rho_s c\theta_i}{\rho t_s \sigma} \ll 1$$

We introduce the following notation

$$\text{(H)} = \frac{\dot{m}'' c\,\theta_i}{\dot{i}_B}$$ - a dimensionless rate of spread equal to unity for spread in thin fuels controlled by a highly emissive burning zone and weakly radiating flames. (28)

$$\delta^* = D/L$$

which from equations (7) (16i) (25) and (29) gives

$$\delta^* = \frac{D}{L} = \frac{t_B\,\dot{m}''}{\rho_b L} = \frac{t_B\,\rho_a^{2/3} g^{1/3} \dot{m}''^{1/3}}{\rho_b\,h^{2/3} L^*} = \frac{\text{(H)}^{1/3} q'' t_B}{L^*\rho_b h c \theta_i} = \text{(H)}^{1/3} N \qquad (29)$$

$$q'' = \left(\rho_a^2 g\, c^2 \theta_i^2\, h\, \dot{i}_B\right)^{1/3}$$ and has the dimensions of heat flux (30)

$$N = \frac{q'' t_B}{L^* \rho_b h c \theta_i} = \frac{q''}{L^* c\,\theta_i M''} = \frac{\tau}{k a h} \qquad (31)$$

$$M'' = \frac{\rho_b h}{t_B}$$ is, a burning rate per unit horizontal area

$$\tau = \frac{k t_B \dot{i}_B}{\rho_b c\,\theta_i}$$ - dimensionless residence time (32)

$$a = \dot{i}_B L^*/q'' \qquad (33)$$

and from these definitions

$$\frac{L}{W} = \text{(H)}^{2/3} a h/W \qquad (34)$$

so that equation (26) can be written as

$$\text{(H)} = 1 + \text{(H)}^{2/3} \frac{\dot{i}_F}{\dot{i}_B} a\left(1 - e^{-\tau\,\text{(H)}}\right) . F\left\{\text{(H)}^{2/3}\frac{ah}{W}, \text{(H)}^{1/3} N\right\} \qquad (35)$$

When the ~~terms~~ in F containing (H) become small compared with unity $F \rightarrow \frac{1}{2}$ and when they become large

$$F = \frac{1}{2\pi}\,\frac{W}{ah\,\text{(H)}^{2/3}} \ln\sqrt{1 + \frac{1}{4N\,\text{(H)}^{2/3}}} \qquad (36)$$

$$\rightarrow \frac{1}{16\pi}\,\frac{W}{Nah\,\text{(H)}^{4/3}}$$

We now consider the values of ρ_b , h , W , i_F , i_B , α , k and t_B to be fixed and examine both sides of equation (35) as functions of H only. (In the numerical examples evaluated in this paper the numerical values used are those listed in Table I). The left hand side H represents the input of heat required to maintain a steady rate of spread and is shown on the straight line at 45^o to the axes in Fig. 4. The right hand side tends, when H is small, to

$$1 + \frac{i_F \alpha \tau H^{5/3}}{2 i_B}$$

If F were effectively constant while H became larger than unity, i.e. if $\frac{a h}{W \tau^{2/3}} \ll 1$ and $\gamma^{1/3} = \frac{N}{\tau^{1/3}} \ll 1$ the right hand side would behave as $1 + \frac{H^{2/3} i_F \alpha}{2 i_B}$ until $\frac{H^{2/3} a h}{W}$ or $H^{1/3} N$ became large enough to produce a significant reduction in F .

At very large values of H the right hand side behaves as

$$1 + \frac{i_F W}{16 \pi N h H^{2/3}}$$

A typical shape of the right hand side is shown in Fig. 4. from which it is seen that there are various types of intersection between the two sides of equation (35). If $(4 \rho_s c \theta_i / t_B \sigma) > 1$, there is at least one stable equilibrium, possibly two.

The significance of the presence of two stable and one unstable equilibrium may be demonstrated by the following:

We shall refer to the three equilibria respectively as 'slow', 'unstable', and 'fast' spread. The 'slow' equilibrium is usually, though as we shall see below not exclusively, associated with small values of $H \tau$ and so can be described as "thin" flame equilibrium.

Suppose there is a fuel bed of type A with a fuel bed of a more flammable type B covering a band across it. If a fire is spreading in A towards B at a steady speed corresponding to the 'slow' equilibrium it will tend to accelerate on entering B. The effect of this increase in speed is to lengthen the burning zone because t_B will not be increased so rapidly, unless heating rates in the fuel bed are drastically increased. This increased rate may be high enough to take the behaviour into the fast regime which may then persist when the fire emerges from the band of fuel B, back into fuel A.

Clearly the reverse could be postulated for a zone B less flammable than zone A. Zone B need not be a different fuel bed but a different set of conditions, e.g. a period of higher or slower wind.

The extent to which the conditions and the spread rate have to change in passing from A to B so that the process is irreversible on return from B to A is indicated by the position of the "unstable middle equilibrium" with respect to the other two.

We have previously mentioned that if $(4\,\rho_s c\,\theta_i/t_B\,\sigma) \ngtr 1$ no spread is possible. This is perhaps not entirely true. Whilst it would not be possible to start such a fire from a small source a high speed spread due to flame radiation may be possible. The parameter $\rho_s c\theta_i/t_B\sigma$ is in theory independent of the others describing the form of the heat input curve. This will be discussed below when the conditions for a high speed spread are considered with other special cases. One of the useful approximations which is shown to be possible for certain conditions is to neglect all radiation through the fuel bed.

4.1. <u>Special cases</u>

<u>Thin wide flames</u> $\quad H\tau \ll 1, \quad H^{2/3}\dfrac{ah}{W} \ll 1, \quad H^{1/3}N \ll 1$

i.e. $\quad \epsilon_B \ll 1, \quad \dfrac{L}{W} \ll 1, \quad \dfrac{D}{L} \ll 1$

For these conditions the value of F is $\frac{1}{2}$

and $\quad 1 - e^{-kD} \sim kD \sim H\tau$

and equation (35) becomes

$$H = 1 + \beta H^{5/3} \tag{37}$$

$$\text{where } \beta = \frac{i_F\, a\tau}{2 i_B} = \frac{i_F}{2}\,\frac{L^{\prime} k t_B\, i_B}{q''\rho_b c\,\theta_i} \tag{38}$$

Equation (37) is shown in Fig. 5. from which it is seen that there are two solutions; the lower one corresponding to the lower of the two stable equilibria and the upper being the middle or unstable equilibrium.

Making the linear approximation to the emissivity will overestimate the right hand side of equation (35) and thereby provide a minimum estimate of the unstable equilibrium value of H.

The critical condition for the existence of a 'slow' equilibrium is that the heat input curve is tangential to the straight line (see Fig. 4).

This maximum value of β is readily obtained by differentiating $\dfrac{Ⓗ - 1}{Ⓗ^{5/3}}$ to find its maximum when $Ⓗ > 1$. This occurs when

$$Ⓗ = 5/2 \tag{39i}$$

and $$\beta_c = \frac{3}{2}\left(\frac{2}{5}\right)^{5/3} = 0.33 \tag{39ii}$$

the suffix c denoting a critical value. If β exceeds β_c no "thin flame" equilibrium is possible.

In the absence of any contribution by flames Ⓗ is 1 and we have therefore shown that if a 'thin flame' equilibrium is established the rate of spread is less than 2.5 times the value obtained from considering only fuel bed radiation.

The unstable equilibrium will be discussed below but since for slow equilibria $Ⓗ < 2.5$ it follows that the conditions under which the above calculations are valid can be written

$$\tau \ll 1$$
$$W \gg a\ell$$
$$N \ll 1.$$

If $\beta \ll \beta_c$ the flame contributes very little heat and the second two conditions which relate to the use of $\frac{1}{2}$ for F are not of great consequence, and the result $Ⓗ \sim 1$ is independent of them.

4.2. <u>Thin flames - the effect of finite width</u>

The value of F is reduced by reducing W and this tends to reduce the value of Ⓗ.

The following discussion is restricted to fuel beds for which the numerical value of N and hence $Ⓗ^{\frac{1}{3}} N$ is small compared with 1. A simple approximate form can then be used for F viz.

$$F = \frac{1}{2}\,\frac{1}{1 + L/W} = \frac{1}{2\left(1 + \dfrac{Ⓗ^{2/3} a\ell}{W}\right)} \tag{40}$$

This is compared with the correct expression in Fig. 6.
So long as $Ⓗ\,\tau \ll 1$ we have

$$Ⓗ = 1 + \frac{\beta\, Ⓗ^{5/3}}{1 + \dfrac{a\ell}{W}\, Ⓗ^{2/3}} \tag{41}$$

The critical values of β and Ⓗ viz β_c and $Ⓗ_c$ now depend on $\dfrac{a\ell}{W}$.

The alegbra is straightforward and leads to an equation in which β_c and Θ_c are given explicitly as a function of $\frac{ah}{W}$ from which Fig. (7i) and Fig. (7ii) have been produced. The effect of narrowing the fire front, namely of increasing $\frac{ah}{W}$, is to increase the range of β in which 'slow' or 'thin' flame spread is possible and to raise the upper level of Θ_c above 2.5. In view of the restriction imposed on $\frac{1}{2}\Theta^{1/3}N$ and $\Theta\tau$ the above graphs can only be used for suitably small values of $\frac{ah}{W}$.

They show, in effect, that when the fuel bed is not effectively infinite the narrow width can be partly compensated for by a taller flame, i.e. a higher burning rate, before a steady 'thin' flame spread becomes impossible.

4.2.1. Application to experimental work

The experimental work on cribs [9, 10, 11, 12] and on beds of pine needles [13, 14] is essentially a study of a 'thin' flame regime though in the experiments under wind conditions the convective transfer may, sometimes at least, be more important than radiation from the flame above the fuel bed. This aspect of the problem has however been omitted in this discussion. Most of the cribs used experimentally have been of order 30 cm wide and 15 cm high and for these and the values given in Table I of other quantities $^{ah}/_{W}$ is about 2-3. β is $\frac{i_F a \tau}{2 i_B}$ which is about 0.4 so that a 'thin' flame equilibrium would be possible. To prevent this mode of spread $^{aR}/_{W}$ would have to be less than 0.12 (see Fig. (7i)) and this means W would have to exceed about 600 cm even if F remained at $\frac{1}{2}$.

If the fuel bed were shallower, say 1 cm layer of dried grasses, 'a' would be about 13.5 and the critical β_c would be about 0.65 for a 30 cm wide bed but the actual β (neglecting any change in τ) would be increased to about 1 and so only a "fast" equilibrium would be possible. This demonstrates that the difference between, say, bushes and grasses, may be the difference between fuel bed controlled beating and flame heating. Certainly the effect of a wind, when measured in terms of a dimensionless wind speed $\frac{U}{\left(\frac{m' g}{\rho_a}\right)^{1/3}}$ is greater on the shallow fuel bed for which m' is smaller. If in the extreme case we consider the flame to be bent over so that $F \to 1$ equation (35) becomes

$$\Theta = 1 + \frac{i_F a}{i_B}\,\Theta^{2/3}\left(1 - e^{-\Theta\tau}\right)$$

The value of Υ is not necessarily the same as for a vertical flame but if this difference is provisionally neglected the equation exhibits the same general features as does equation (35) and reduces to equation (38) if the flames are thin, except for the factor of 2 in the definition of β. β is larger the smaller is h and so we see how when the bed is shallow and the effect of wind is relatively greater the spread ceases to be controlled by the radiation through the fuel bed but becomes primarily dependent on the flame. Convection heating will, in general, have to be included in any flame spread where the fuel bed is wide enough and the wind fast enough for the flames to be severely bent over towards the horizontal.

4.3. Thick flames

Flames are thick if $H\tau \gg 1$ and we can write

$$1 - e^{-kD} \sim 1.$$

If too $F \approx \frac{1}{2}$, the condition for which we discuss below we have

$$H = 1 + \frac{i_F}{2 i_B} a H^{2/3} \tag{42}$$

which has the asymptotic solution for $\frac{i_F a}{i_B} \gg 1$

$$H = \left(\frac{i_F a}{2 i_B}\right)^3 + 3 + 6\left(\frac{2 i_B}{i_F a}\right)^3 \ldots\ldots \tag{43}$$

The condition $H\tau \gg 1$ is thus

$$\left(\frac{i_F L^*}{2 c \theta_i}\right)^3 \frac{k r_B}{\rho_b \rho_a^2 g h} \gg 1 \tag{44}$$

Clearly if $\frac{i_F a}{i_B} \gg 1$ it is reasonable to neglect the first term on the right hand side of equation (35), i.e. to neglect the contribution of radiation through the fuel bed. This is sometimes a more useful approximation than the assumption of unit emissivity so we write

$$H^{1/3} = \frac{i_F a}{i_B}\left(1 - e^{-H\tau}\right) F\{H\} \tag{45}$$

$W \gg ah$ is a condition which is appropriate for fires spreading fast on a wide front. We then have from equation (27iii) and (29)

$$F = \frac{1}{2}\left\{\sqrt{1 + \frac{H^{2/3} N^2}{4}} - \frac{H^{1/3} N}{2}\right\} \tag{46}$$

and from equations (45) and (46)

$$\left(1 - e^{-H\tau}\right)\left(\sqrt{1 + \frac{H^{2/3} N^2}{4}} - 1\right) = \frac{4 i_B}{i_F a N} \tag{47}$$

$$= \frac{4 c \theta_i}{i_F} M''$$

It is profitable to use the variable δ^{*} instead of (H), and obtain equation (47) in the form

$$\left(1-e^{-\gamma\delta^{*3}}\right)\left(\sqrt{1+\frac{4}{\delta^{*2}}}-1\right) = \frac{4\,c\,\theta_i\,M''}{i_F} \qquad (48)$$

where
$$\gamma = \frac{T}{N^3} = \frac{kL^{*3}}{g\rho_a^2}\left(\frac{\rho_b h}{t_b}\right)^2 = \frac{kL^{*3}}{g\rho_a^2}M''^2 \qquad (49)$$

from which it is seen that γ is directly related to the burning rate M''.

The necessary condition for fast spread has been found from equation (48) and over the range $0.1 < \gamma < 10$ this condition is that the minimum $\gamma^{\frac{1}{2}}\,\frac{i_F}{4\,c\,\theta_i\,M''}$ should exceed about 1.3

$$\text{i.e.}\quad \frac{i_F}{c\theta_i}\left(\frac{kL^{*3}}{g\rho_a^2}\right)^{\frac{1}{2}} \gg 5.2 \text{ approx.} \qquad (50)$$

Inserting numerical values as listed in Table I, equation (50) gives the required minimum value of i_F as about 7. w cm^{-2}, a value which is less than, though approximately the same as, those characteristic of the "black body" radiation from flames. In view of the approximate character of the theory the difference between the actual value of i_F and 7. w cm^{-2}, is hardly large enough to guarantee the possibility of "fast spread". Over the same range of γ the value of δ^{*} at the minimum of the left hand side is closely given by $\gamma\delta^{*} \sim 1$

i.e. $\quad$ i.e. $(H) \sim 1/T$

$$D \doteq 1/k = 330 \text{ cm.}$$

of the actual value of i_F is taken as 12.6 w cm^{-2} s^{-1} the lower unstable solution to equation (48) for the above range of γ is given closely by

kD between 0.2 and 0.3

i.e. $D = 60 - 100$ cm.

This means that to obtain a fire which accelerates towards the higher equilibrium one must start with flames exceeding a certain thickness of order 100 cm thick. The actual critical value of D necessary to start a fast fire will be larger than this because the whole of the zone first ignited will subside at the same time and if the fire has not advanced a sufficient distance by this time it may eventually travel at the slow equilibrium, not the fast one. In this early stage of spread none of the unburnt material has been preheated as it has in the equilibrium condition.

The fast equilibrium speed of spread can also be obtained from equation (48). With i_F as 12 J cm^{-2} it can be shown that for $0.1 < \gamma < 10.0$, $\exp - \gamma \delta^{*3}$ is less than 0.03 so that the emissivity can be taken as 1.0. However, δ^* is not small enough for F to be taken as $\frac{1}{2}$. Equation (48) with $\tau \to \infty$ gives

$$H = \left[\frac{N^2}{4}\left\{\left(1 + \left(\frac{4 M'' c \vartheta_i}{i_R}\right)\right)^2 - 1\right]^{-3/2} \tag{51i}$$

$$= \frac{8}{N^3}\left(\left(1 + \frac{4 i_B}{i_R \lambda N}\right)^2 - 1\right)^{-3/2} \tag{51ii}$$

We have calculated from equation (48) the value of H neglecting the finite width of a fuel bed. The values of the constants used in the calculations are listed in Table I and the results are given in Table 2.

TABLE 2

Fuel Bed $W \gg D$, $W \gg L$		Type of Equilibrium	
		Unstable equilibrium.	Fast Stable equilibrium
Cribs similar to Fons	H	6.50*	130
	R	0.4 cm/sec	8 cm/sec
	D	100 cm	2000 cm
$N = 0.19$, $\gamma = 7.2$			
Wood shavings	H	1.51*	40
	R	0.95 cm/sec	25 cm/s
$N = 0.57$, $\gamma = 0.8$	D	70 cm	1900 cm

*The approximation of $H \gg 1$ in equation (45) clearly leads to some error here.

The minimum width of fire front for which the above results are numerically valid can be obtained from the condition that for equation (27iii) to be valid it is necessary that $H^{1/3} a h \ll W$

$$4\frac{ah}{N^2}\left(\left(1 + \frac{4 i_B}{i_R \lambda N}\right)^2 - 1\right) \ll W \tag{52}$$

Inserting the numerical values from Table I makes the requirement for cribs as

$$W \gg 2{,}200 \text{ cm} \quad \text{i.e. about } 22 \text{ m}$$

which clearly excludes all but very large forest fires.

For wood shavings the requirement is

$$W \gg 1{,}050 \text{ cm} \quad \text{viz. } 10 - 11 \text{ m}$$

More detailed calculations using equation (45) and the exact values of F as shewn in Fig. 3. have been made and they shew that, for beds like cribs, a width of 16 m, and for shavings a width of 9 m, would be necessary to produce this fast spread. These are minimum figures and where both slow and fast rates of spread are possible the attainment of fast spread also depends on the starting conditions. It is hoped, however, that it may be possible experimentally to demonstrate its existence.

Discussion and Conclusions

A model of fire spread based on radiation transfer is clearly limited in scope. So is one which, like this, neglects the effect of speed on the burning behaviour of the burning zone, that is, which treats the value of t_B as a constant independant of R. The reason why this may not be correct is that as R increases the size of the burning zone increases and the burning rate per unit area is not necessarily independent of this, The extent of this "coupling" differs for different types of fuel beds and is probably stronger for a tightly packed than for a porous bed. There is insufficient information to include this effect which, however, for beds of practical interest is probably not of great consequence.

The main result of this paper is that it predicts two types of spread, a 'slow' and a 'fast' spread with a condition of instability between them. The extent to which this is of practical importance and is associated with the phenomena described as "blow-up" is a matter for speculation at the moment. According to this model the unstable equilibrium is intimately associated with the variation of emissivity with flame thickness and the numerical data on which one must base discussion is scanty for wood flames. The analysis suggests important groups of terms about which information is required in assessing a field situation. For example 'fast' spread is closely associated with the term γ which consists of the more or less constant terms

$$\frac{\lambda^{*3}}{g \rho_a^2},$$

the attenuation coefficient for radiation through flames and a property of the fuel bed

$$\frac{\rho_s l}{t_B} \quad (= M'')$$

which is the effective rate of burning per unit area. On the other hand 'slow' spread depends on the bulk density ρ_b rather than the fuel load $\rho_b h$ and the condition for the existence of slow spread, viz. a low enough value of β imposes an upper limit to the allowable value of

$$\frac{t_B}{\rho_b h^{1/3}}$$

Fast spread depends markedly on the exchange factor. For example in conditions where D is small and W large compared with the flame height H depends roughly on the cube of F and F can increase from $\frac{1}{2}$ to 1, or from a lower value than $\frac{1}{2}$ to 1 if D is not small and W not large.

This means that H can be increased at least 8 times by the deflection of the flames if the fire is flame controlled. We might also expect that wind, irrespective of any convection heating or increased supply of oxygen can take a fire from the 'slow' to the 'fast' condition.

Attempts are now being made to demonstrate experimentally some of the features predicted by the theory and to interpret data on fire spread in the light of it.

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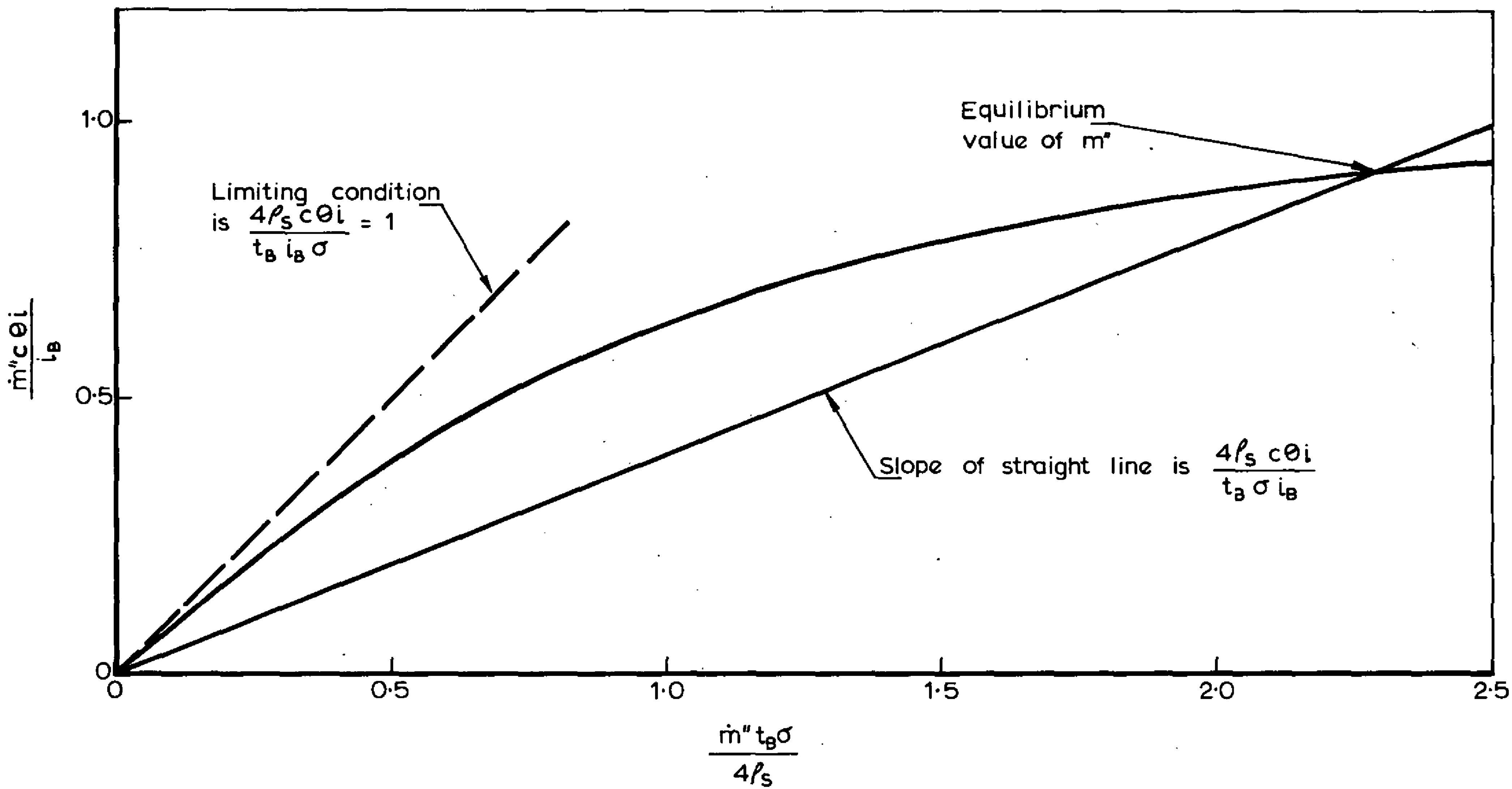

FIG. 1. THE EFFECT OF REDUCED EMISSIVITY ON FIRE SPREAD

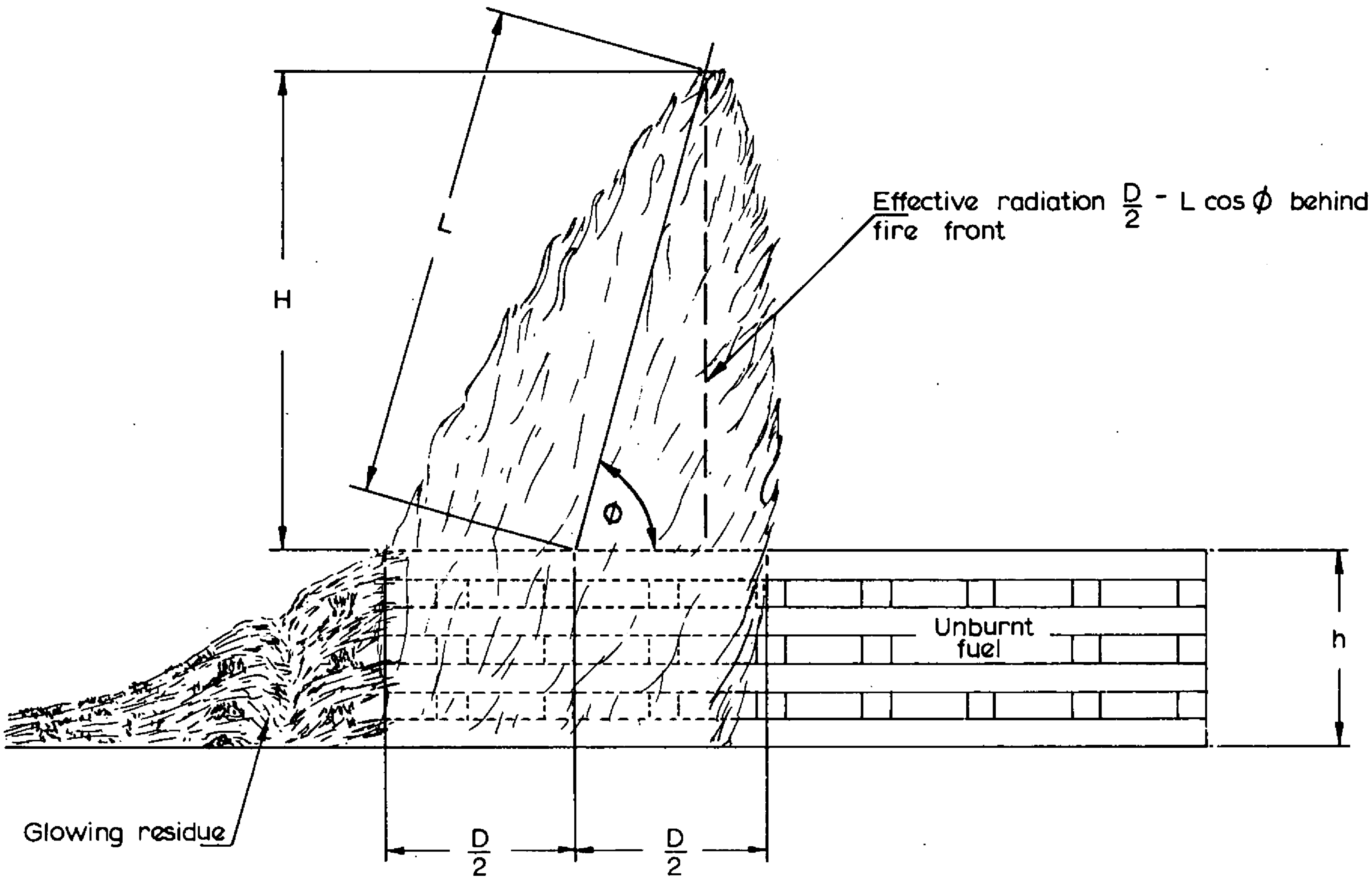

FIG.2. DIAGRAMMATIC SKETCH OF BURNING FUEL BED

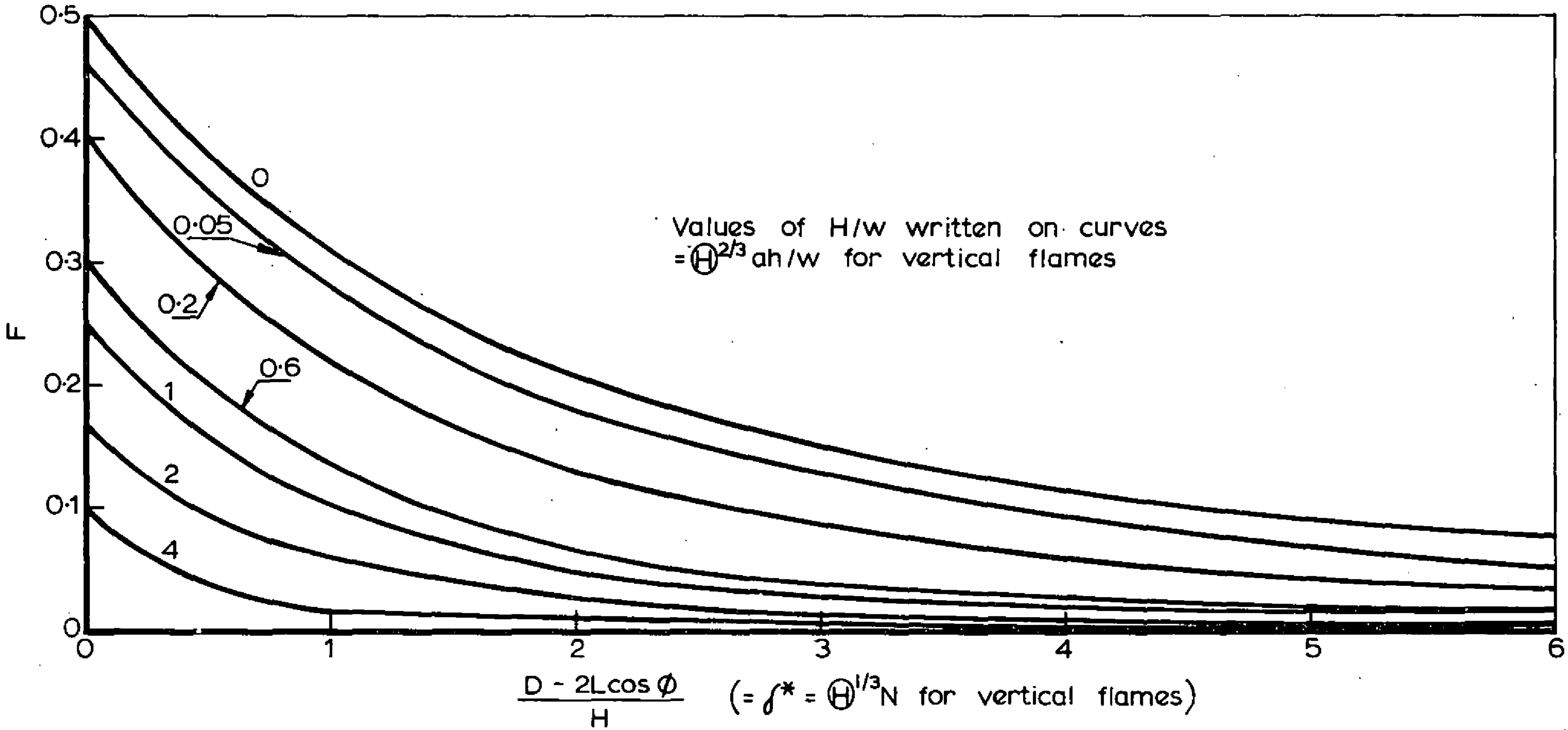

FIG. 3. EXCHANGE FACTOR F

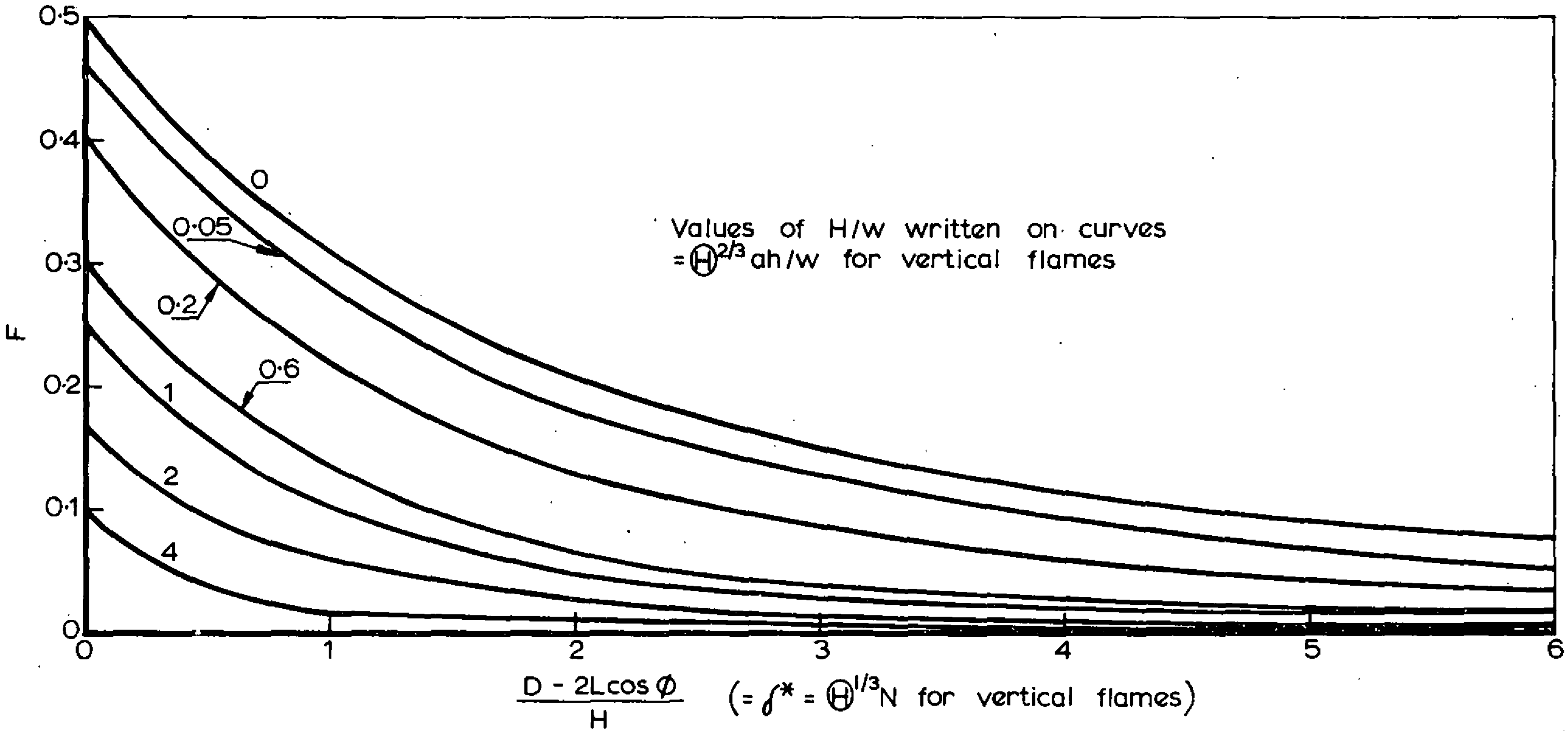

FIG. 3. EXCHANGE FACTOR F

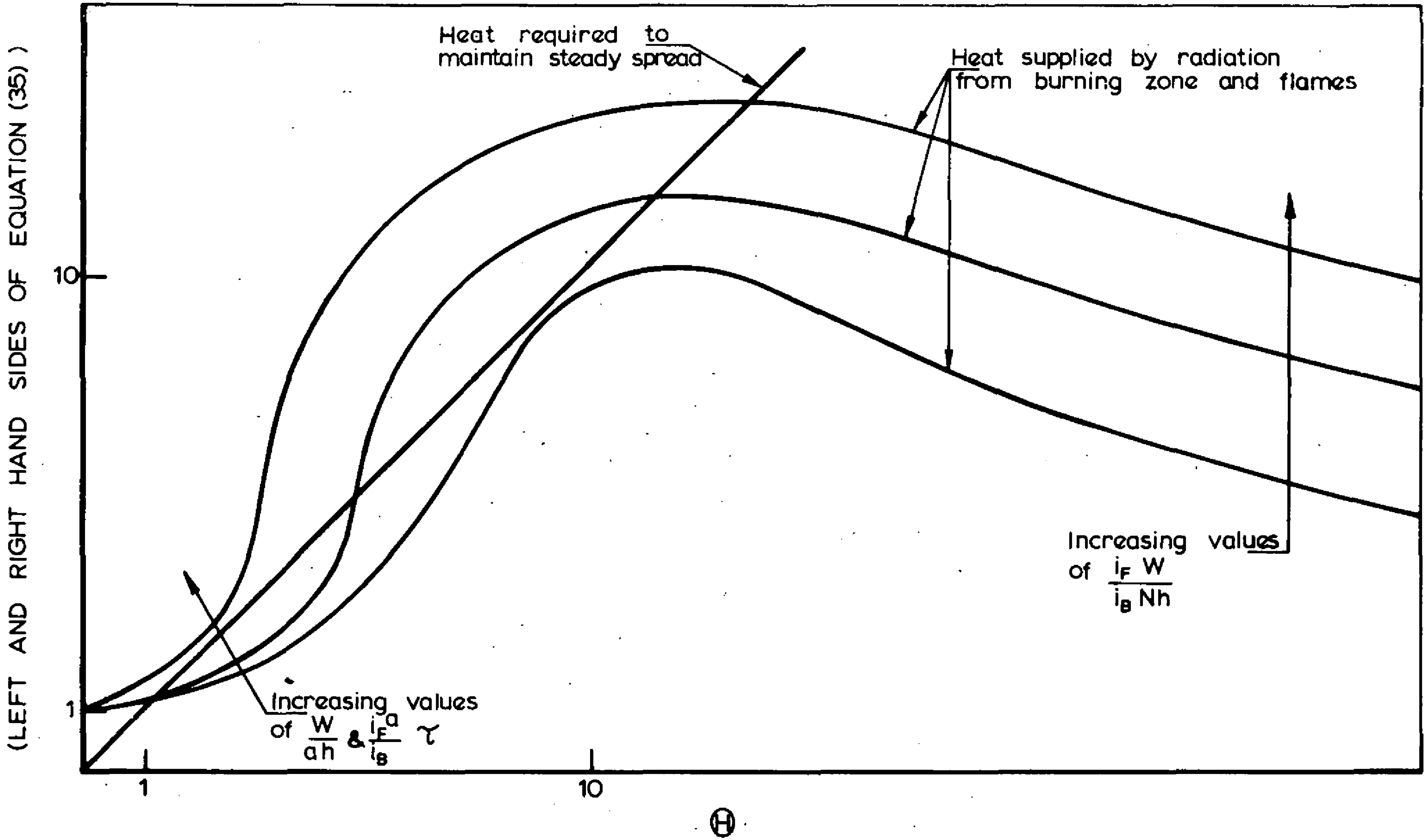

FIG. 4. REPRESENTATION OF HEAT BALANCE (DIAGRAMMATIC)

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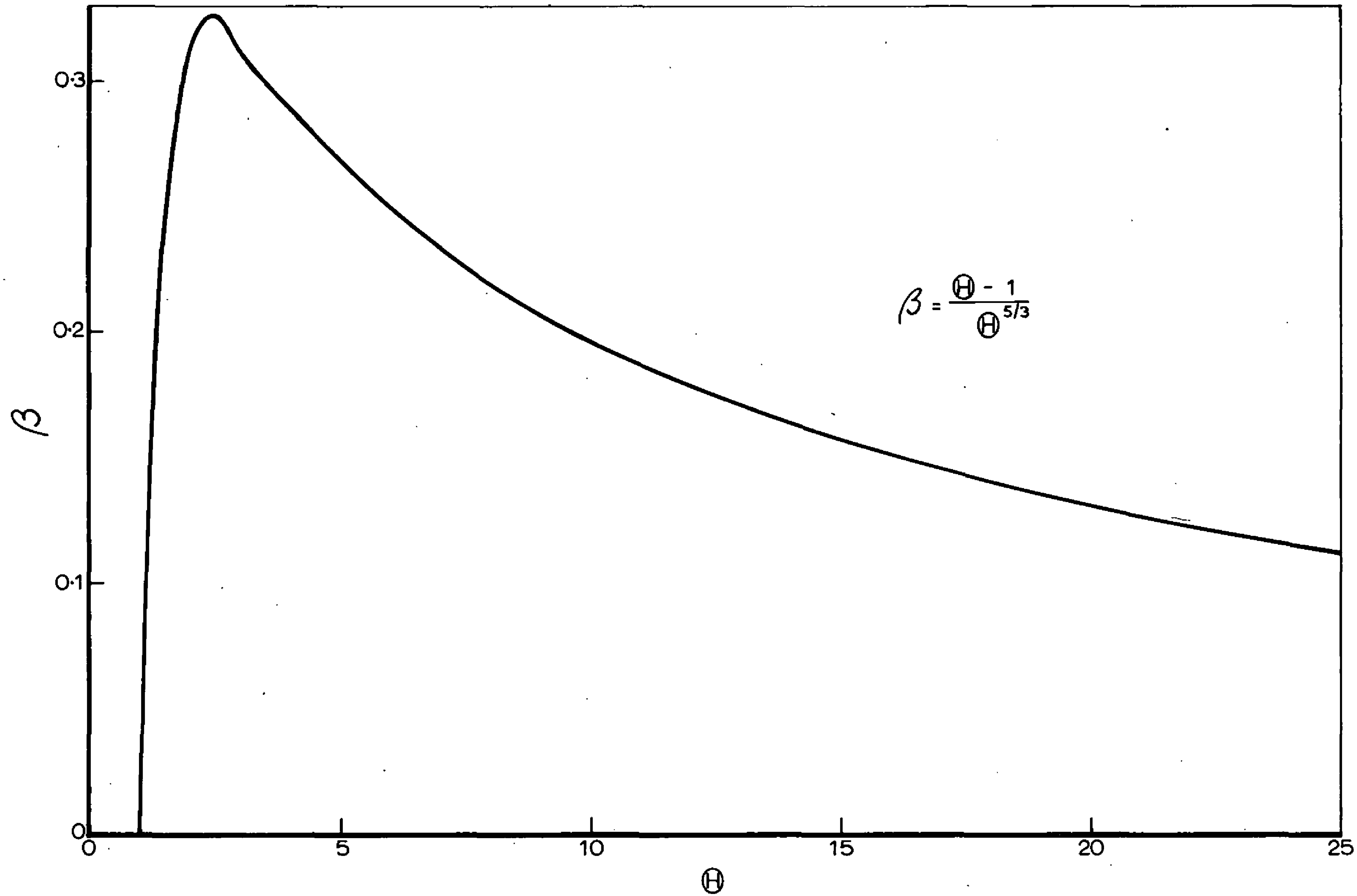
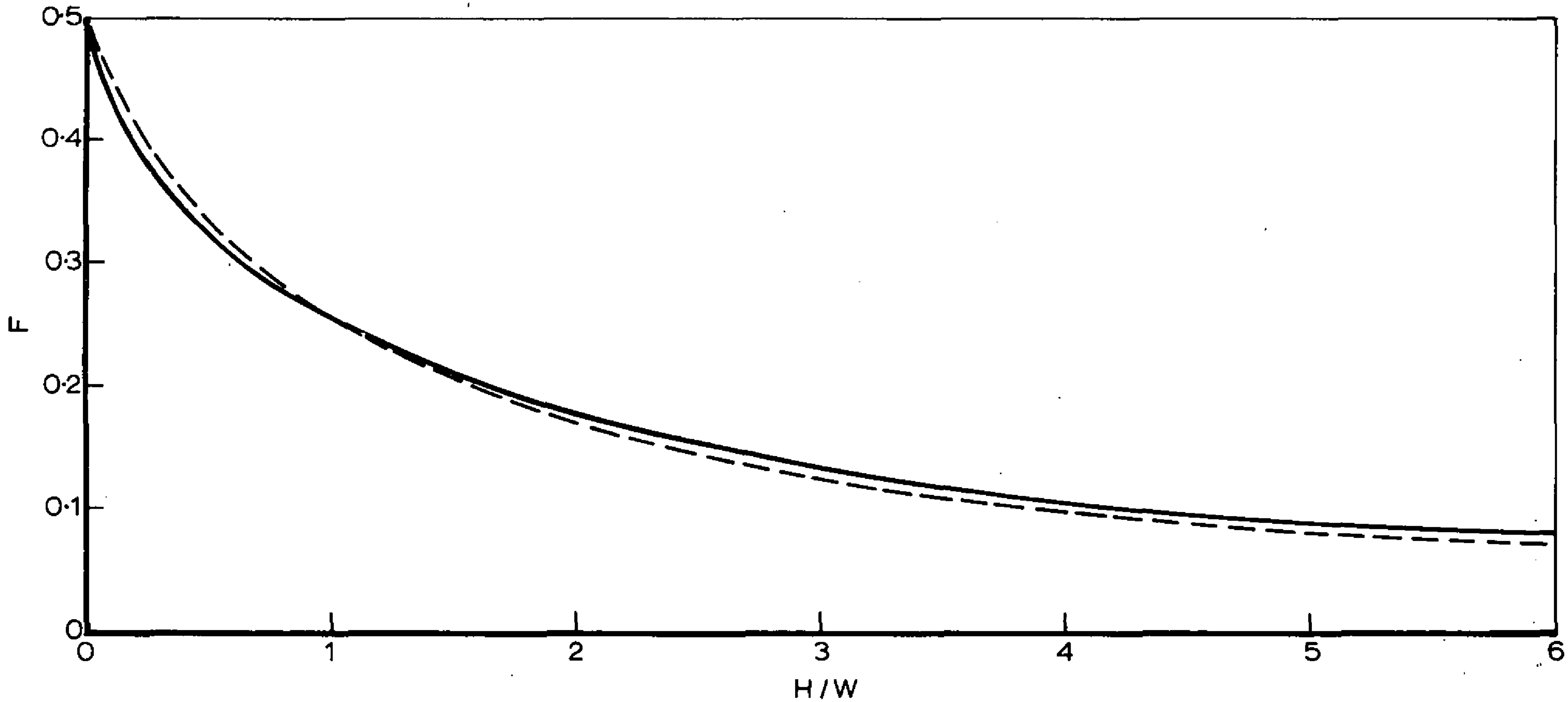


FIG. 5. EQUATION FOR "SLOW" FIRE SPREAD



$F \doteqdot \frac{1}{2}\left(\frac{1}{1+H/W}\right)\ \left(\frac{D}{H} \& \frac{D}{W} \rightarrow 0\right)$

Correct expression for F

FIG. 6. EXCHANGE FACTOR F

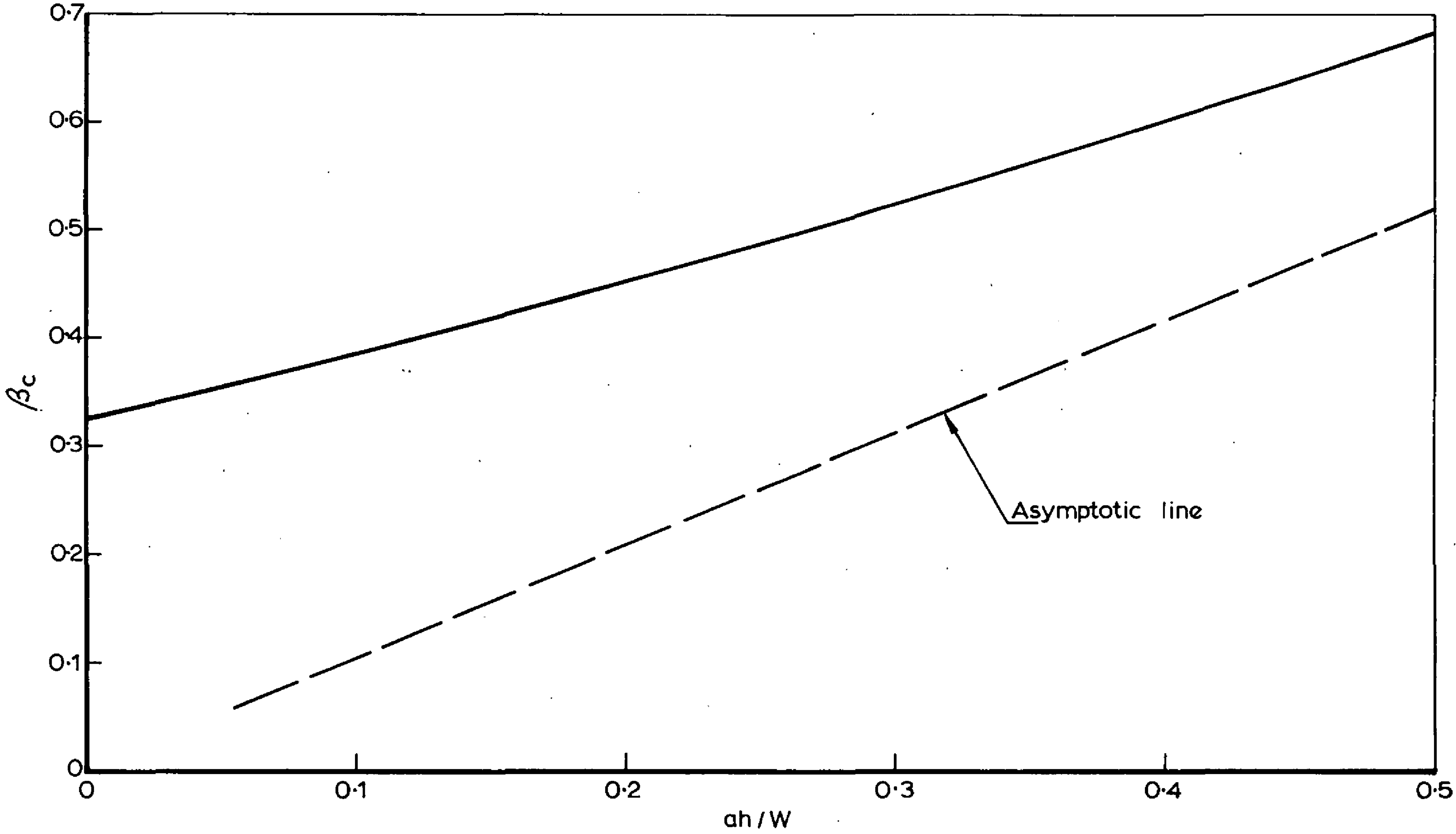


FIG. 7i. THE EFFECT OF FIRE FRONT WIDTH ON THE CRITICAL CONDITION FOR "SLOW" FIRE SPREAD

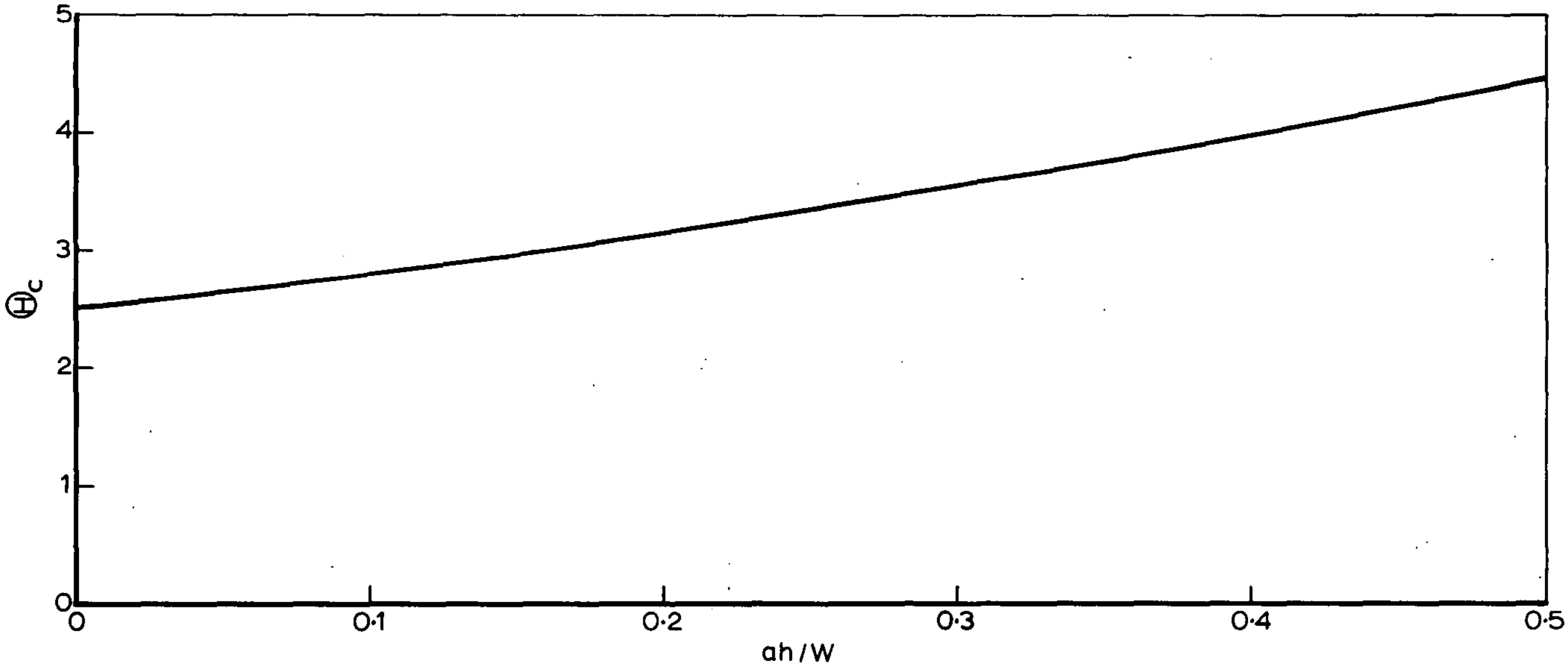


FIG. 7ii. THE EFFECT OF FIRE FRONT WIDTH ON THE CRITICAL CONDITION FOR "SLOW" FIRE SPREAD