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No. 625

CONFIGURATION FACTOR FOR A TRIANGULAR RADIATOR

by

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SUMMARY

Equations for the configuration factor of an elemental receiver on a plane either parallel or perpendicular to a triangular radiating area are given. Values for isosceles triangles, useful for calculating heat transfer from flames, have been computed.

MINISTRY OF TECHNOLOGY AND FIRE OFFICES' COMMITTEE
JOINT FIRE RESEARCH ORGANIZATION

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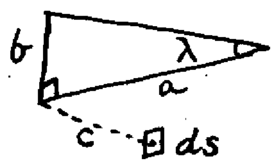
1. Introduction

This note gives equations for the configuration factor, ϕ, from a triangular radiator to an elemental area receiver, ds, on either a parallel or perpendicular plane. Values for an isosceles triangle, useful for calculating heat transfer by radiation from flames to exposed surfaces, are given in Figs 1 and 2. The theoretical basis is given elsewhere[1].

2. Receiver on parallel plane

(i) Right angled triangle



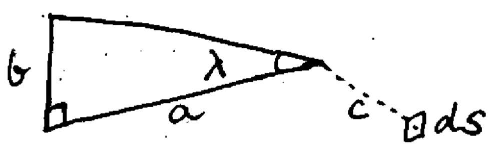
$$\frac{a}{c} = A \qquad \frac{b}{c} = B \qquad \tan\lambda = k$$

Facing right angle

$$\phi = \frac{kA}{2\pi\sqrt{1+k^2(1+A^2)}}\left[\tan^{-1}\frac{A}{\sqrt{1+k^2(1+A^2)}} + \tan^{-1}\frac{k^2 A}{\sqrt{1+k^2(1+A^2)}}\right] \quad \ldots(1)$$

or

$$\phi = \frac{AB}{2\pi\sqrt{A^2+B^2(1+A^2)}}\left[\tan^{-1}\frac{A^2}{\sqrt{A^2+B^2(1+A^2)}} + \tan^{-1}\frac{B^2}{\sqrt{A^2+B^2(1+A^2)}}\right]$$



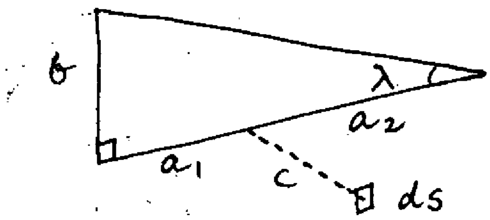
$$\frac{a}{c} = A \qquad \frac{b}{c} = B \qquad \tan\lambda = k$$

Facing acute angle

$$\phi = \frac{1}{2\pi}\left[\frac{A}{\sqrt{1+A^2}}\tan^{-1}\frac{kA}{\sqrt{1+A^2}}\right] \quad \ldots(2)$$

$$\text{or } \phi = \frac{1}{2\pi}\left[\frac{A}{\sqrt{1+A^2}}\tan^{-1}\frac{B}{\sqrt{1+A^2}}\right]$$

Right angle triangle continued



$$\frac{a_1}{c} = A_1$$

$$\frac{a_2}{c} = A_2$$

$$\frac{b}{c} = B$$

$$\tan \lambda = k$$

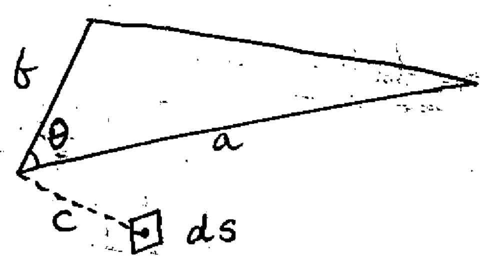
Facing base

$$\phi = \frac{1}{2\pi}\left[\frac{k\,A_2}{\sqrt{1+k^2\,(1+A_2{}^2)}}\left\{\tan^{-1}\frac{A_2}{\sqrt{1+k^2\,(1+A_2{}^2)}} + \tan^{-1}\frac{A_1+k^2\,(A_1+A_2)}{\sqrt{1+k^2\,(1+A_2{}^2)}}\right\} + \frac{A_1}{\sqrt{1+A_1{}^2}}\tan^{-1}\frac{k\,(A_1+A_2)}{\sqrt{1+A_1{}^2}}\right] \quad \ldots\ldots(3)$$

or

$$\phi = \frac{1}{2\pi}\left[\frac{B\,A_2}{\sqrt{(A_1+A_2)^2 + B^2(1+A_2{}^2)}}\left\{\tan^{-1}\frac{A_2(A_1+A_2)}{\sqrt{(A_1+A_2)^2 + B^2(1+A_2{}^2)}} + \tan^{-1}\frac{A_1(A_1+A_2) + B^2}{\sqrt{(A_1+A_2)^2 + B^2(1+A_2{}^2)}}\right\} + \frac{A_1}{\sqrt{1+A_1{}^2}}\tan^{-1}\frac{B}{\sqrt{1+A_1{}^2}}\right]$$

(ii) Acute angled triangle



$$\frac{a}{c} = A$$

$$\frac{b}{c} = B$$

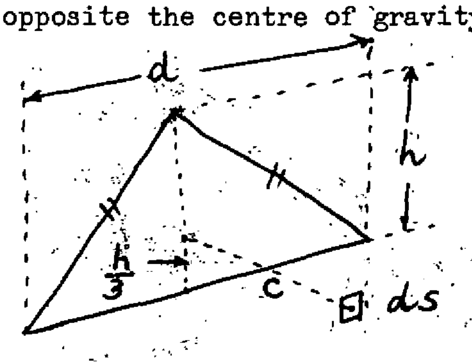
$$\tan \Theta = K$$

$$\phi = \frac{KAB}{2\pi\sqrt{(A\sqrt{1+K^2}-B)^2 + K^2B^2(1+A^2)}}\left[\tan^{-1}\frac{A(A\sqrt{1+K^2}-B)}{\sqrt{(A\sqrt{1+K^2}-B)^2 + K^2B^2(1+A^2)}} + \tan^{-1}\frac{B(B\sqrt{1+K^2}-A)}{\sqrt{(A\sqrt{1+K^2}-B)^2 + K^2B^2(1+A^2)}}\right] \quad ......(4)$$

By employing the additive property of configuration factors, equation 4 can be used for any triangle and any position of the receiver.

(iii) Isosceles triangle

The maximum configuration factor at a given distance is for a receiver opposite the centre of gravity.



$$\frac{h}{c} = H$$

$$\frac{d}{c} = D$$

$$\frac{h}{d} = f$$

From equation 3

$$\phi = \frac{1}{\pi}\left[\frac{fD}{\sqrt{f^2D^2+9}}\tan^{-1}\frac{1.5D}{\sqrt{f^2D^2+9}} + \frac{fD}{\sqrt{f^2(D^2+9)+2.25}}\left\{\tan^{-1}\frac{2f^2D}{\sqrt{f^2(D^2+9)+2.25}} + \tan^{-1}\frac{D(f^2+0.75)}{\sqrt{f^2(D^2+9)+2.25}}\right\}\right] \quad ......(5)$$

or

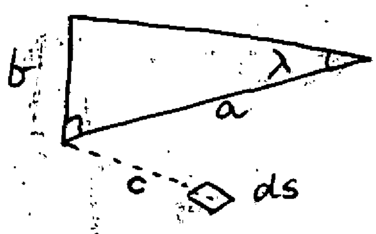
$$\phi = \frac{1}{\pi}\left[\frac{H}{\sqrt{H^2+9}}\tan^{-1}\frac{1.5D}{\sqrt{H^2+9}} + \frac{HD}{\sqrt{9H^2+H^2D^2+2.25}}\left\{\tan^{-1}\frac{2H^2}{\sqrt{9H^2+H^2D^2+2.25}} + \tan^{-1}\frac{H^2+0.75D^2}{\sqrt{9H^2+H^2D^2+2.25}}\right\}\right]$$

The solution of equation 5 is shown in Fig. 1 for different values of $f = \frac{h}{d}$

3. Receiver on perpendicular plane

Some of the solutions have already been given by Hamilton and Morgan[2]; they are reproduced here for the sake of completeness.

(i) Right angle triangle



$$\frac{a}{c} = A \qquad \frac{b}{c} = B \qquad \tan\lambda = k$$

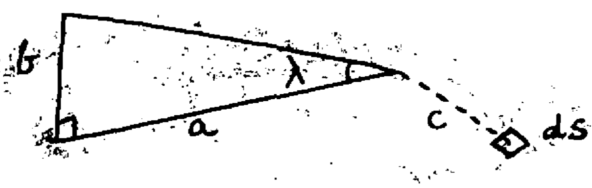
Facing right angle

$$\phi = \frac{1}{2\pi}\left[\tan^{-1}A - \frac{1}{\sqrt{1+k^2(1+A^2)}}\left\{\tan^{-1}\frac{A}{\sqrt{1+k^2(1+A^2)}} + \tan^{-1}\frac{k^2A}{\sqrt{1+k^2(1+A^2)}}\right\}\right] \quad \ldots.(6)$$

or

$$\phi = \frac{1}{2\pi}\left[\tan^{-1}A - \frac{A}{\sqrt{A^2+B^2(1+A^2)}}\left\{\tan^{-1}\frac{A^2}{\sqrt{A^2+B^2(1+A^2)}} + \tan^{-1}\frac{B^2}{\sqrt{A^2+B^2(1+A^2)}}\right\}\right]$$

Values are given by Hamilton and Morgan



$$\frac{a}{c} = A \qquad \frac{b}{c} = B \qquad \tan\lambda = k$$

Facing acute angle

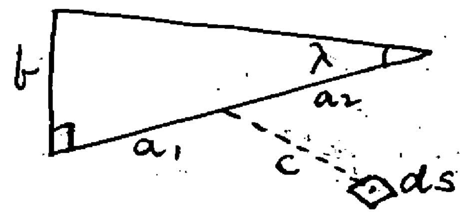
$$\phi = \frac{1}{2\pi}\left[\tan^{-1}A - \frac{1}{\sqrt{1+k^2}}\tan^{-1}A\sqrt{1+k^2}\right] \quad \ldots.(7)$$

or

$$\phi = \frac{1}{2\pi}\left[\tan^{-1}A - \frac{A}{\sqrt{A^2+B^2}}\tan^{-1}\sqrt{A^2+B^2}\right]$$

Values are given by Hamilton and Morgan

Right angle triangle continued



$$\frac{a_1}{c} = A_1$$

$$\frac{a_2}{c} = A_2$$

$$\frac{b}{c} = B$$

$$\tan\lambda = k$$

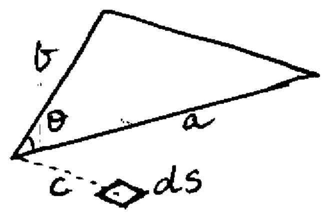
Facing base

$$\phi = \frac{1}{2\pi}\left[\tan^{-1}A_1 + \tan^{-1}A_2 - \frac{1}{\sqrt{1+k^2(1+A_2^2)}}\left\{\tan^{-1}\frac{A_2}{\sqrt{1+k^2(1+A_2^2)}} + \tan^{-1}\frac{k^2(A_1+A_2)+A_1}{\sqrt{1+k^2(1+A_2^2)}}\right\}\right] \quad ....(8)$$

or

$$\phi = \frac{1}{2\pi}\left[\tan^{-1}A_1 + \tan^{-1}A_2 - \frac{A_1+A_2}{\sqrt{(A_1+A_2)^2+B^2(1+A_2^2)}}\left\{\tan^{-1}\frac{A_2(A_1+A_2)}{\sqrt{(A_1+A_2)^2+B^2(1+A_2^2)}} + \tan^{-1}\frac{B^2+A_1(A_1+A_2)}{\sqrt{(A_1+A_2)^2+B^2(1+A_2^2)}}\right\}\right]$$

(ii) Acute angled triangle



$$\frac{a}{c} = A$$

$$\frac{b}{c} = B$$

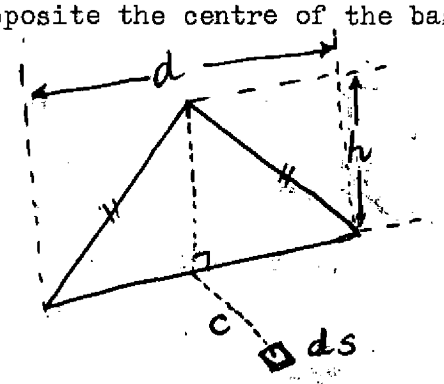
$$\tan \Theta = K$$

$$\phi = \frac{1}{2\pi}\left[\tan^{-1}A - \frac{1}{\sqrt{1+K^2}} \tan^{-1}B - \frac{Z}{\sqrt{Z^2+B^2K^2(1+A^2)}}\left\{\tan^{-1}\frac{Z}{\sqrt{Z^2+B^2K^2(1+A^2)}} + \tan^{-1}\frac{B^2K^2-BZ}{\sqrt{Z^2+B^2K^2(1+A^2)}}\right\}\right] \quad \ldots\ldots(9)$$

where $Z = A\sqrt{1+K^2}-B$

(iii) Isosceles triangle

The maximum configuration factor at a given distance is for a receiver opposite the centre of the base.



$$\frac{d}{c} = D$$

$$\frac{h}{c} = H$$

$$\frac{h}{d} = f$$

From equation (6)

$$\phi = \frac{1}{\pi}\left[\tan^{-1}\frac{D}{2} - \frac{1}{\sqrt{1+f^2(D^2+4)}}\left\{\tan^{-1}\frac{0.5D}{\sqrt{1+f^2(D^2+4)}} + \tan^{-1}\frac{2f^2D}{\sqrt{1+f^2(D^2+4)}}\right\}\right] \quad (10)$$

or

$$\phi = \frac{1}{\pi}\left[\tan^{-1}\frac{D}{2} - \frac{D}{\sqrt{D^2+H^2(D^2+4)}}\left\{\tan^{-1}\frac{0.5D^2}{\sqrt{D^2+H^2(D^2+4)}} + \tan^{-1}\frac{2H^2}{\sqrt{D^2+H^2(D^2+4)}}\right\}\right]$$

The solution of equation (10) is shown in Fig. 2 for different values of $f = \frac{h}{d}$

References

1. McGUIRE, J. H. "Heat Transfer by Radiation". Fire Research Special Report No. 2. H.M.S.O. London, 1953.

2. HAMILTON, D. C. and MORGAN, W. R. Radiant interchange configuration factors. U.S. Natural Advisory Committee for Aeronautics Technical Note 2836. Dec., 1952.

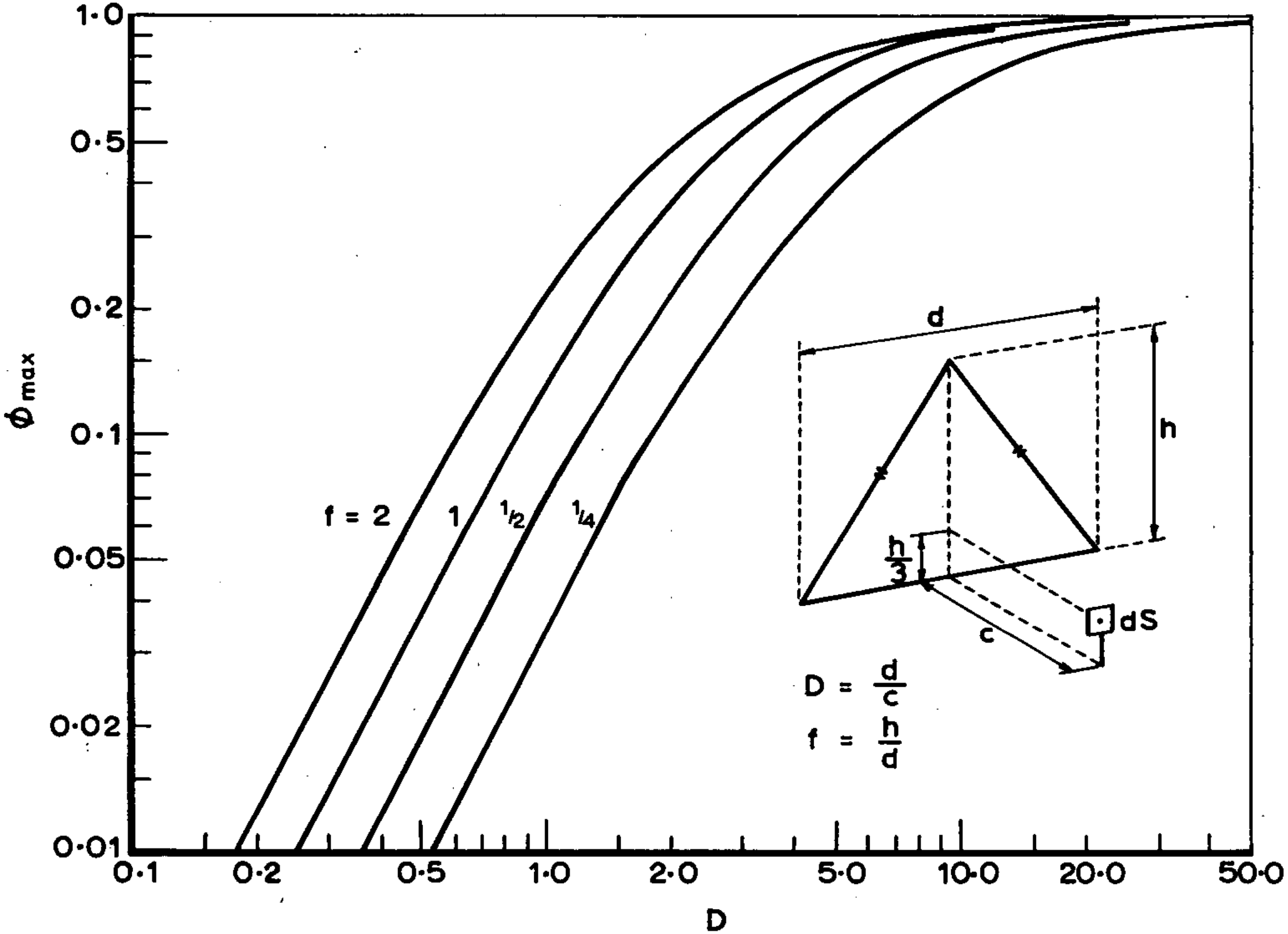


FIG.1. ISOSCELES TRIANGLE AND PARALLEL RECEIVER (EQUATION 5)

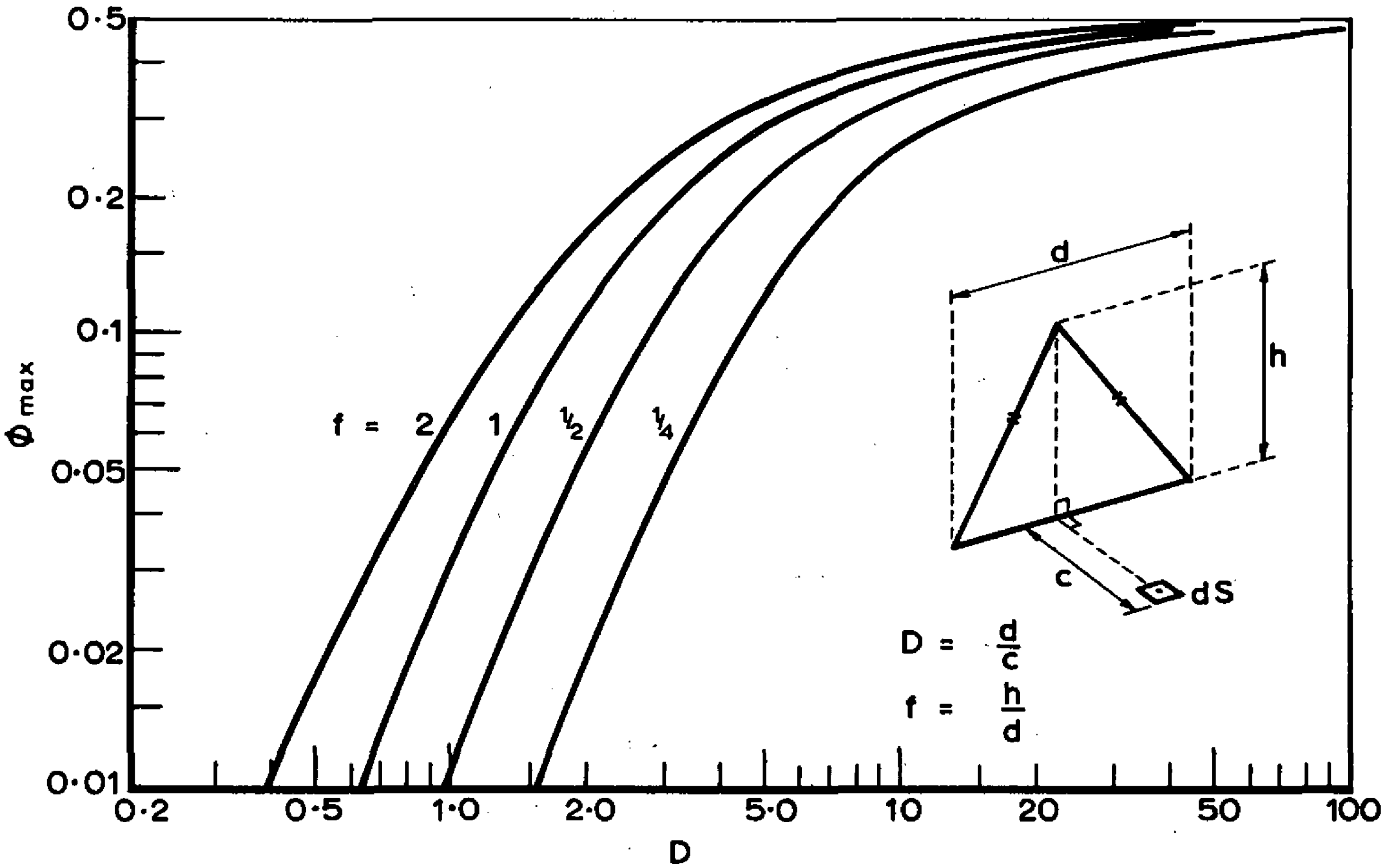


FIG. 2. ISOSCELES TRIANGLE AND PERPENDICULAR RECEIVER (EQUATION 10)