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THEORETICAL CONSIDERATIONS OF THE GROWTH TO FLASHOVER OF COMPARTMENT FIRES

by

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SUMMARY

An approximate theory of the growth to flashover of fires in compartments is described for those situations where there are no effective breaks in the continuity of the fuel. The essence of the theory is the assumption that the instantaneous rate of spread depends on the surface temperature of the fuel. It is shown that heat loss through the walls is of negligible importance on the heat balance and that the gas temperature may be regarded as responding immediately to changes in the burning rate. It is not sufficient however to regard the gases as uniformly mixed and the effect of stratification appears important. The observed differences between the behaviour of compartments fully lined with fibre insulating board and only half lined - upper walls and ceiling being brick, is consistent with the view that the fuel is heated by radiation and that because brick heats up more slowly than fibreboard it does not begin to reradiate heat to the fuel surface so soon.

The calculations demonstrate the importance of the thermal properties of the walls and ceiling etc., in determining the rate at which the fuel surface itself rises in temperature, and so controlling the fire growth.

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1. INTRODUCTION

When fire is spreading in a compartment several physical and chemical processes are occurring simultaneously and it must be expected that the relative importance of these processes in controlling the rate of spread, rate of temperature rise etc., differs at different stages of the spread. It is known, for example, that small fires in compartments, like fires in the open, burn freely, that is to say, there is no restriction on the air supply other than the exchange processes in the flame itself and the gases immediately surrounding it. On the other hand once fire has spread so that all combustible surfaces are contributing flammable volatiles and gases the fire is said to be "fully-developed" and unless the windows are large such fires are usually controlled by the rate at which air can enter the openings which become the main restrictions to the air flow.

This paper discusses a theory of fire growth up to the point at which it is "fully-developed". Customarily, this period of growth is said to culminate in "flashover" but this term is imprecise and it is possible that it is used to mean different things in different contexts. Certainly just prior to all the combustible surfaces being contributors to the fire one observes one or more of the following features:-

1. An accelerating rate of spread.
2. Rapid rises in temperature.
3. Emergence of flames from openings.
4. Fall in oxygen or rise in carbon monoxide and carbon dioxide concentrations.

These are clearly interrelated and usually occur at about the same time. In some fires however, where the windows do not fall out quickly, fire may be air limited before all the combustible surfaces are contributing to the fire and only when the windows finally fall out does the fire spread rapidly and flames emerge. On the other hand if the windows are large and open the fully developed fire is not air limited and "flashover" can only be associated with the emergence of flames or the complete involvement of all combustibles. There is thus more than one kind of "flashover". This paper discusses only one of these.

If a fire can spread, because the separate combustible materials are close enough to each other, it is capable eventually of involving all combustible surfaces. Normally, however, a fire in the open eventually reaches a relatively constant rate of spread and does not exhibit any acceleration after the very early stages. In compartments however the temperature of the gases rises and the temperature of the combustible surfaces away from the combustion zone increases. If, eventually, it rises to the ignition temperature of the combustible one may expect the fire to spread at a very high rate and, if the fully-developed fire is window restricted, it will quickly pass into this stage, and all the features listed above will appear to occur simultaneously. It is this kind of fire behaviour that appears to be best described by the word "flashover" and perhaps gave rise to it and it is this that we shall discuss.

The assumptions and approximations that will be made are admittedly drastic and oversimplify the problem, but they are thought to embody the main features of the phenomenon. The theory gives reasonable numerical results for two extreme cases and is offered as a first approximation.

2. THEORY OF "FLASHOVER"

We shall make alternative assumptions about the mixing of the gases in the compartment.

(1A) It is first assumed that the convected heat is uniformly mixed in the compartment, thus permitting the use of a mean temperature for the gases. This assumption is shown to be inadequate even by the standards of this rather approximate treatment and we have to adopt an alternative, viz., that

(1B) the convected heat is uniformly mixed in a hot layer which lies above the cold air entering the compartment. This will be seen to be better.

(2) We assume also that the whole of the surface of any one of the materials in the compartment, fuel or insulation, is at a uniform temperature which differs from one material to another but not in position(except where there is stratification and then we distinguish between those parts of a material in the hot layer and those below it). A simplifying assumption about the relation between the rate of spread and the fuel surface temperature is required and this is that

(3) the instantaneous rate of spread is given by

$$\frac{dR}{dt} = \frac{D}{1 - \psi_f/\theta_f} \qquad (1)$$

where R is the rate of heat output and is proportional to the instantaneous burning rate so that $^{dR}/dt$ is proportional to the rate of spread, ψ_f is the rise in temperature of the fuel surface at a distance from the combustion zone, D is a constant proportional to a rate of spread when $\psi_f = 0$, θ_f is the ignition temperature for the fuel surface.

This assumption embodies the concept of an infinite rate of spread when ψ_f approaches θ_f though as we shall show below this assumption has to be altered for mathematical convenience to a form which does not embody this feature as an assumption.

With the modification the rate of spread eventually increases exponentially with time and it is convenient to adopt the rate constant of this exponential increase as a measure of "flashover" time. D and θ_f may be regarded as being dependent on the flammability of the materials in the compartment and clearly this is more appropriate if only one material is involved as fuel.

2.1 The ideal case of no conduction into the walls and fuel

We shall show below that it is reasonable to neglect the rate of heat accumulation $\rho cVd\theta/dt$ compared with the heat loss $\rho c \alpha \theta$

where	ρ	is gas density
	c	is gas specific heat
	V	is compartment volume
	θ	is mean gas temperature

and α is the volume flow per sec through the window which for simplicity is taken as independent of temperature.

A simplified heat balance then gives

$$\rho c \alpha \theta = R \tag{2}$$

$$\psi_f = \theta \tag{3}$$

We note in passing that although we have treated ρ and α as independent of temperature their product is less dependent than either term.

Equations (1),(2), and (3) then give

$$\frac{dR}{dt} = \rho c \alpha \frac{d\theta}{dt} = \frac{D}{1 - \theta/\theta_f} \tag{4}$$

From equation (4) we obtain

$$\frac{\theta}{\theta_f} - \frac{\theta^2}{2\theta_f^2} = \frac{Dt}{\rho c \alpha \theta_f} \tag{5}$$

so that

$$\frac{\theta}{\theta_f} = 1 - \sqrt{1 - \frac{2Dt}{\rho c \alpha \theta_f}} . \tag{6}$$

Only the smaller root has any physical significance here, we then have

$\theta = \theta_f$ when t has its largest value with any physical meaning. This is

$$t = t_{fl} = \frac{\rho c \alpha \theta_f}{2D} \tag{7}$$

2.1.1. Alternative spread equation

For the general treatment following we have to abandon equation (1) and find some suitable linear relation between dR/dt and ψ_f. It is proposed to use

$$\frac{dR}{dt} = D\left(1 + \frac{\psi_f}{m}\right) \tag{8}$$

where m is a disposable constant of order θ_f

With ψ_f equal to θ this gives

$$\frac{\theta}{m} = e^{\frac{Dt}{\rho c \alpha m}} - 1 \tag{9}$$

and θ is θ_f when

$$t = t_{fl} = \frac{\rho c \alpha m}{D} \log_e (1 + \theta_f/m) \qquad (10)$$

m can now be chosen to give the same t_f as equation (7)

$$\therefore m \simeq 0.4\ \theta_f$$

2.1.2. The use of the Laplace Transform

We now return to the more general case where we do not omit the term $\rho c V \frac{d\theta}{dt}$. The heat balance equation then becomes

$$\rho c \alpha \theta + \rho c V \frac{d\theta}{dt} = R$$

The condition of perfect insulation gives

$$\psi_f = \theta$$

and we use the spread equation in the form

$$\frac{dR}{dt} = D\left(1 + \frac{\psi_f}{m}\right) = D\left(1 + \frac{\theta}{m}\right)$$

It will be convenient for the development of the theory to use the Laplace Transform to solve these equations. Denoting the Laplace Transform by the notation

$$\bar{\phi} = \int_0^\infty \phi . e^{-pt} . dt$$

we have, with $\theta = \psi_f = 0$ at $t = 0$

$$\bar{\psi}_f = \bar{\theta} = \frac{D}{\rho c V p \left[p\left(p + \frac{\alpha}{V}\right) - \frac{D}{\rho c V m}\right]} \qquad (11)$$

The denominator has only one positive root which we denote by p_o and from Laplace Transform theory it follows that the inversion leads to θ containing a term proportional to $e^{p_o t}$, all other terms being constant or tending to zero at long times (see Appendix I).

We have from equation (11)

$$p_0 = \sqrt{\frac{\alpha^2}{4V^2} + \frac{D}{\rho c V m}} - \frac{\alpha}{2V} \qquad (12)$$

If

$$\frac{\alpha^2}{4V^2} \gg \frac{D}{\rho c V m}$$

$$p_0 \rightarrow \frac{D}{\rho c \alpha m} \qquad (13)$$

Because the strongest term in θ for long times is $e^{p_0 t}$ we can consider $1/p_0$ as a characteristic time for the rate of rise of θ. Because $1/p_0$ at high values of $\frac{\alpha^2 \rho c m}{DV}$ tends to $\frac{\alpha \rho c m}{D}$ we shall by comparison with equation (9) define

$$t_{fl} = 1/p_0 \qquad (14)$$

The above treatment exhibits the feature that one root of the denominator in the transformed variable is positive and from this we can obtain t_{fl}. This is general in the subsequent development and is the essential feature of this theory. It is not necessary to exploit this feature in the above case because the expression for $\bar{\theta}$ is relatively simple, but as we shall see, the inclusion of more than one material and the allowance for heat transfer from one surface to another by radiation and from the gases makes the corresponding expression exceedingly cumbersome. It is a considerable simplification to use the expression only to find p_0 and hence define $1/p_0$ as t_{fl}. This procedure neglects the coefficients of the exponential term in the expressions for Ψ_f and θ when inverted. The significance of this is discussed in Appendix I.

In general it is possible to obtain the complete term containing the exponential $e^{p_0 t}$ from the calculus of residues but the remaining terms, which are not considered here in detail, but which may not always be negligible, generally involve the computation of infinite integrals.

A mathematical model based on equation (1) would give infinite rates of spread i.e. "flashover" at some finite time. Equation (8) does not exhibit this feature and the existence of a positive root p_o only means that the temperature rises exponentially with time, approaching large, but not infinite, values in a finite time. ψ_f however, does reach θ_f in a finite time and the use of $t_{f\ell} = 1/p_o$ & $m = 0.40\ \theta_f$ corresponding to an appreciable rise in the exponential, does make the two models give equal values of $t_{f\ell}$, in the one extreme case of $\psi_f = \theta$.

2.1.3. A numerical example

For a 3m cube with a window 4 m^2 the value of α is of order 4 m^3/s and $\frac{\alpha}{2V}$ is 1/13.5 s^{-1}, ρ is conveniently taken as 1.3 kg/m^3, since for most of the growth period the temperature is low rather than high, c is 10^3 J kg^{-1} $^oC^{-1}$ and θ_f is 300^oC.

$$\therefore \quad \rho c V \theta_f \simeq 1 \times 10^7 \text{ joules.}$$

An order of magnitude estimate for D can be obtained as follows.

In cubes with one side open the burning is never limited by the size of the opening and the fire spreads to involve all the fuel. When flames just emerge the burning rate can be obtained from the equation[1]

$$L = 400\ (m')^{2/3}$$

where L is the flame height in cm, here 300 cm
and m' is the rate of burning per unit width of opening in $g\ cm^{-1}\ s^{-1}$

i.e. the burning rate is 0.19 kg/s

This develops after about 10 min and if a calorific value of 16×10^3 J/kg is taken we have

$$D = 5{,}200 \text{ J/s}^2$$

$$\therefore \quad \frac{D}{\rho c V m} = 1{\cdot}25 \times 10^{-3}\ s^{-2}$$

We shall see later that a value of about twice this is necessary if numerical agreement is to be obtained in two practical extreme cases.

Clearly $$\frac{\alpha^2}{4V^2} \gg \frac{D}{\rho c V m}$$

an approximation we employed earlier and will do so subsequently. It is equivalent to

$$\left.\begin{aligned} V\frac{d\theta}{dt} &\ll \alpha\theta \\ p &\ll \alpha/V \end{aligned}\right\} \qquad (15)$$

or $$\alpha t_{f\ell} \gg V$$

The inequalities (15) mean that the gases are in quasi-steady equilibrium. Putting $p \ll \alpha/V$ means that there is only one root in the denominator of equation (11). When the surfaces are not perfect insulators p_0 decreases and the approximation is even more justified.

Even if there is no real flashover in 3 m cubes with a large window (50 per cent or over) the development of the fire is still aided by radiation from the heated walls and the flames, so we cannot use the above value of D a priori.

It is best to regard the term $\frac{\alpha \rho c m}{D}$ as a disposable constant. It may well be that in this way its value includes some allowance for the effects of flame radiation which is otherwise disregarded here. Such a procedure would in effect reduce the theory to a calculation of the effect of thermal insulation on the flashover time relative to that for a perfect insulator and does not give an absolute estimate of flashover.

From equations (13) and (14) we obtain $t_{f\ell}$ equal to about $3\frac{1}{2}$ min. The shortest flashover times observed have been in fully lined fibreboard compartments[2] and are 5-6 mins.

2.2. The effect of finite conduction transfer

2.2.1. General

We have so far discussed only perfect insulation. The effects of finite conduction loss must be considered. There are two. The first affects the relation between Ψ_f and Θ and the second affects the heat balance, i.e. the relation between R and Θ.
Let a_i be the area of internal surface having conductivity K_i density ρ_i and specific heat c_i (diffusivity $k_i = K_i/\rho_i c_i$)
As above the suffix 'f' denotes fuel.

The temperature at a depth x inside the material, which is treated as semi-infinite, is readily obtained from conventional conduction theory and its value after a Laplace Transformation is

$$\overline{\Psi_{i,x}} = A_i e^{-x\sqrt{p/k_i}} \tag{16}$$

where A_i is independent of x.

The boundary conditions where there are n different materials are each written as

$$-K_i\left(\frac{d\overline{\Psi}_i}{dx}\right)_{x=0} = H_c(\overline{\Theta}-\overline{\Psi}_i) + \sum_{j=1,2,\ldots}^{n} H_{j,i}(\overline{\Psi}_j - \overline{\Psi}_i) \tag{17}$$

where H_c is a transfer coefficient from the gases (and flame) to the i'th surface and $H_{j,i}$ is an effective radiation transfer coefficient between solid surfaces which include the effects of area and orientation.

The heat balance, neglecting $\rho V \frac{d\theta}{dt}$ c.f. $\rho c \alpha \theta$ is now written as

$$\rho c \alpha \theta = R + \sum_{i=1,2,\ldots}^{n} a_i K_i\left(\frac{d\Psi_i}{dx}\right)_{x=0} \tag{18}$$

where a_i is the area of the i'th material

It follows from equations (16) and (17) that

$$\overline{\Psi_i} = A_i = \frac{H_c\,\bar{\theta} + \sum H_{ji}.\,\overline{\Psi_j}}{K_i q_i + H_c + \sum H_{ji}} \qquad (19)$$

where for convenience we omit the suffices to $\sum$

and $$q_i = \sqrt{p/k_i}$$

We shall normalise all q's to q_f

writing $$q_i = q\sqrt{k_f/k_i}$$

omitting the suffic f to q.

Equations (18) and (19) form a set of simultaneous linear equations determining $\overline{\Psi}_i$ and $\overline{\theta}$.

2.2.2. One material only

For a single material - i.e. a fully lined compartment, we obtain from the above equations

$$\overline{R} = \frac{D}{p}\,\frac{\left(\frac{\alpha}{V}(1 + q/h_c) + \frac{a_f K_f q p}{\rho c V}\right)}{\left(\frac{\alpha}{V}p\,(1 + q/h_c) - \left(\frac{D}{\rho c V m} - \frac{a_f K_f q p}{\rho c V}\right)\right)} \qquad (20)$$

$$\overline{\Psi_f} = \frac{D}{\rho c V p}\;\frac{\quad}{\left(\frac{\alpha}{V}p(1 + q/h_c) - \left(\frac{D}{\rho c V m} - \frac{a_f K_f q p}{\rho c V}\right)\right)} \qquad (21)$$

where $h_c = H_c/K_f$

From equation (20) and (21) which have the same denominator, as also has $\bar{\theta}$, we obtain p_o by putting the denominator zero*. The terms are grouped so that it can be seen that there is only one positive

* The fact that the denominator is a polynominal in q i.e. $\sqrt{p}$ does not affect this procedure.

root p_0. When p is positive the first three terms are a function of p (or q) increasing from 0 to ∞. The last terms are a function decreasing from $\frac{D}{\rho c V m}$ to $-\infty$. There can be only one p_0 giving equality.

The value of p_0 may be derived as follows

$$p_0 = \frac{D/\rho c \alpha m - \dfrac{a_f K_f p_0^{3/2}}{\rho c \alpha \sqrt{k_f}}}{1 + \left(\dfrac{p_0}{h_c^2 k_f}\right)^{1/2}}$$

i.e. $$p_0 \left\{ 1 + \left(\frac{p_0}{h_c^2 k_f}\right)^{1/2} \left(1 + \frac{H_c a_f}{\rho c \alpha}\right)\right\} = \frac{D}{\rho c \alpha m} \qquad (22)$$

We have for fibreboard

$$k_f = 1.2 \times 10^{-7} \text{ m}^2/\text{s}$$

$$K_f = 4 \times 10^{-2} \text{ Wm}^{-1} \ {}^{\circ}\text{C}^{-1}$$

and for a 3 m cube with a 4 m^2 opening

$$a_f = 50 \text{ m}^2$$

H_c may be taken as 8 Wm^{-2} $^{\circ}C^{-1}$

so that $$\frac{H_c a_f}{\rho c \alpha} = 0.075$$

This is small and shows that whilst conductivity may be important in delaying the temperature rise of the fuel surface it is not a very significant part of the heat balance. From the above we calculate

$$t_{fe} = 1/k_0 = \quad 240\ \text{s} = 4\ \text{min}$$

which is in good agreement with experiments on the development of fire in compartments fully lined with untreated fibreboard. However as shown below, the same approach to a different compartment is less satisfactory.

2.2.3. Two conducting materials

Most compartments of practical importance contain fuel and some other material lining the walls and ceiling. Here we shall consider a brick compartment half lined with fibreboard. The two equations for this situation are obtained from equation (17) as

$$\left(K_f q + H_c + H_b\right)\overline{\Psi_f} = H_c\overline{\theta} + H_b\overline{\Psi_b} \tag{23}$$

where H_b denotes H_{ji} for the brick to fuel radiation

$$\left(K_b\sqrt{k_f/k_b}\,q + H_c + H_f\right)\overline{\Psi_b} = H_c\overline{\theta} + H_f\overline{\Psi_f} \tag{24}$$

where H_f denotes H_{ji} for the fuel to brick radiation

From equation (18) we obtain

$$\rho c \alpha\overline{\theta} = \frac{D}{p^2} + \frac{D\overline{\Psi_b}}{m p} - a_f K_f q\overline{\Psi_f} - a_b K_b\sqrt{k_f/k_b}\,q\overline{\Psi_b} \tag{25}$$

The equation to be solved for $q_0 = \sqrt{p_0/k_f}$ is

$$\left[\frac{\rho c\alpha}{H_c}K_b K_f\sqrt{k_f/k_b} + (a_f + a_b)K_b K_f\sqrt{\frac{k_f}{k_b}}\right]q_0^2 +$$

$$+\left[\frac{\rho c\alpha}{H_c}\left\{K_f(H_c + H_f) + K_b\sqrt{k_f/k_b}\,(H_c + H_b)\right\} + (H_c + H_b + H_f)\times \\ \times\left(K_f a_f + K_b a_b\sqrt{k_f/k_b}\right)\right]q_0 +$$

$$+ \rho c\alpha(H_c + H_b + H_f) = \\ = \frac{D}{m K_f q_0^2}\left(K_b\sqrt{\frac{k_f}{k_b}}\,q_0 + H_c + H_b + H_f\right) \tag{26}$$

There is only one positive root for q_0

In addition to the numerical values given previously we take

$$K_{b} = 80 \times 10^{-2} \ \mathrm{Wm}^{-1} \ {}^{\circ}\mathrm{C}^{-1}$$
$$k_{b} = 3.8 \times 10^{-7} \ \mathrm{m}^2/\mathrm{s}$$

For the radiation exchange we consider both halves of the enclosure to be approximately irradiated from a hemisphere at uniform temperature and take

$$H_{b} = H_{f} = 8 \ \mathrm{Wm}^{-2} \ {}^{\circ}\mathrm{C}^{-1}.$$

Hence we obtain q_o as 140 and this gives

$$t_{fl} = \frac{10^7}{(140)^2 \times 1.2} \doteqdot 7 \text{ min}$$

This calculated result is much less than the 20 mins or so for actual flashover in such a compartment. Adjusting the value of D to give t_{fl} as 18 min makes t_{fl} for a fibreboard compartment over 13 min, which is too long.

Altering H_{b} and H_f make little difference, the main term affecting the value to t_{fl}, other than $\frac{D}{\rho c \alpha m}$ is H_c and this suggests how the model is deficient. Cold air enters the enclosure and flows over the floor. Indeed there is little rise in temperature of the gases below the neutral pressure axis, so that convection heating of the fuel may be small though the fuel can cool to the cold air.

2.3 The effect of stratification

As an extreme case we put $\bar{\theta}$ zero in equation (17) when applied to the fuel i.e. in equation (23) but not in equation (24), to obtain a new characteristic equation (27). We are in effect assuming that all the combustible surfaces are below the neutral pressure axis where the gases are at the ambient temperature and all the non-combustible surfaces above it. Such a simplification is quite realistic for many practical problems. We then write

$$\frac{\rho c\,\alpha}{H_c}\left[\left(K_f q + H_c + H_b\right)\left(K_b\sqrt{\frac{k_f}{k_b}}\,q + H_c + H_f\right) - H_b H_f\right] \tag{27}$$
$$= \left(\frac{D}{m\,p} - a_f K_f q\right) H_b - \left(K_f q + H_c + H_b\right) a_f K_b\sqrt{\frac{k_f}{k_b}}$$

for which $q_o = 72$

$$t_{fe} = \frac{1}{p_o} = \frac{1}{k_f q_o^2} = 27 \text{ min}$$

For a compartment fully lined with fibreboard we obtain

$$t_{fe} = 11.5 \text{ min}$$

Firstly we observe that neglecting convection heating in the lower part of the compartment separates the values for t_{fe} for the two compartments rather more than before. The terms in a_f and a_b are negligible so the large values of t_{fe} are not due to any error in the formulation of the contribution of the heat loss into the surfaces to the overall heat balance, and the source of the discrepancy is presumably in the values chosen, particularly for D , m and α . It should be noted that stratification alters the way in which α is calculated though it is still to be obtained from the rate of air flow through the compartment.

2.4. Neglecting heat loss to walls and transient gas heating term (two materials)

If we omit the heat loss to the walls etc., equation (27) reduces to a simpler equation for q_o viz.

$$\left[\left(K_f q + H_c + H_b\right)\left(K_b\sqrt{\frac{k_f}{k_b}}\,q + H_c + H_f\right) - H_f H_b\right] = \frac{D H_c H_b}{\rho c\, m\, \alpha\, k_f q^2} \tag{28}$$

This gives results which are practically the same as those obtained from equation (27)

Replacing q_o by $\dfrac{1}{\sqrt{k_f t_{fe}}}$ gives an equation for t_{fe}

$$\left(\sqrt{\frac{K_f \rho_f c_f}{t_{fe}}} + H_c + H_b\right)\left(\sqrt{\frac{K_b \rho_b c_b}{t_{fe}}} + H_c + H_f\right) - H_b H_f = \frac{D H_c H_b t_{fe}}{\rho c\, m\, \alpha} \tag{29}$$

where ρ_b and c_b are the density and specific heat of the non-combustible and ρ_f and c_f are those of the fuel.
If $DH_c H_b/\rho c m \alpha$ is regarded as a disposable constant and we put t_{fl} as 6 min for a compartment fully lined with fibreboard we obtain

$$\frac{DH_c H_b}{\rho c m \alpha} = 1.2 \ j^2 m^{-4} degC^{-2} s^{-3}.$$

instead of the assumed value of 0.53

For the brick compartment the larger value of $DH_c H_b/\rho c m \alpha$ now gives $q_o = 89$ i.e. $t_{fl} = 18$ min., which is close to the typical value of 20 min.

The values of the terms in equation (28) for this situation are interesting. Firstly, $K_f q_o$ is about $\frac{1}{3}$ of (Hc + Ho) while $K_b \sqrt{k_f/k_b}\, q_o$ is nearly 3 times (Hc + Hb).

Thus conduction into the fibreboard is small compared with the surface heat loss from the fibreboard. The driving force for heating the fuel is $H_b \psi_b$ which is essentially the radiation from the brick and from equation (24) it is seen that the conduction into the brick is more important in determining ψ_b than the heat loss from its surface although the re-radiation from the brick is, of course, the only way the fuel is itself heated. Equation (29) is a quartic in $\sqrt{t_{fl}}$ and various analytic approximations are possible depending on the numerical values of some of the quantities in the equation.

However, in view of the approximations made in deriving equation (29) further progress would probably be more profitably made using a computer and the original equations.

2.5 The effect of varying $K\rho c$ on t_f

It is interesting to use equation (28) to calculate t_{fl} for different values of $K_b \rho_b c_b$. We employ the same numerical values as before for the fuel properties and for the values of H_b, H_c, and H_f and the value of $DH_c Hb/\rho c m \alpha$ is taken as 1.2. A value for t_{fl} has also been calculated for a compartment wholly lined with fuels of different values for $K\rho c$.

The results are shown in Fig. (1) which may be used to test the theory against suitably designed experiments. $t_{f\ell}$ for a compartment lined wholly with a material having the same $K\rho c$ as fibreboard is shown for various values of $DH_cH_b/\rho\, cm\, \alpha$ in Fig. (2).

Discussion

Of the two versions of the model, the latter allowing for stratification is the better since one disposable constant can be adjusted to give satisfactory agreement both for a fibreboard lined room and a room half lined with brick.

In view of the assumptions and approximations made it is not surprising that better quantitative agreement has not been obtained. What has been shown is that so far as the role of insulation is concerned one can disregard heat loss through the walls as a factor affecting the heat balance, at least for walls of conductivity as high as brick. Also one can leave out of consideration the effect of heat accumulation in the gases contained in the enclosure and treat the flow as a quasi-steady process. The analysis suggests that stratification in relation to fuel position and the influence of heat exchange between the walls by radiation are both important. Indeed the theoretical discussion here leads one to consider the heating of the roof and the conduction into it, and the subsequent radiation heating of the floor as a principle factor in fire growth. These ideas are very similar to those expressed by Waterman et al[3] who reached their conclusions as a result of an experimental approach.

If these arguments are correct one should be able to observe major differences in "flashover" time as a result of changing the roof insulation (on the inner surface particularly) or changing the pattern of window opening in relation to the fuel, e.g. putting all the fuel on the roof and cooling the floor or providing subsidiary heating to the fuel. No allowance has been made here for the effect of direct radiation from the flame which is known[4] to be dramatically altered by the deflection of the flame by the roof, though here too a large part of the radiation reaching the floor arises from the hot ceiling which has been heated by the flame.

In experimental work it would appear valuable to attempt to measure the rates of rise of temperature in the period prior to flashover. These may be more closely related to fundamental characteristics of the fire spread than is flashover time which clearly depends on the time origin and therefore on the method of ignition. An attempt to measure the value of D is clearly desirable before the theory can be regarded as a valid model.

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APPENDIX I

The complete solution for the temperature rise

Equation (11) may be written as

$$\bar{\theta} = d \frac{1}{p(p-p_0)(p+p_1)}$$

where d denotes $\frac{D}{\rho cV}$ and

$$p_0 = \sqrt{\frac{\alpha^2}{4V^2} + \frac{d}{m}} - \frac{\alpha}{2V}$$

$$p_1 = \sqrt{\frac{\alpha^2}{4V^2} + \frac{d}{m}} + \frac{\alpha}{2V}$$

both p_0 and p_1 being positive. The above equation for $\bar{\theta}$ is readily rearranged as

$$\bar{\theta} = -\frac{d}{p_0 p_1 p} + \frac{d}{(p-p_0)p_0(p_1+p_0)} \pm \frac{d}{(p+p_1)p_1(p_1+p_0)}$$

so that
$$\bar{\theta} = \frac{d\left[p_1(e^{p_0 t}-1) - p_0(1-e^{-p_1 t})\right]}{p_0 p_1 (p_0+p_1)}$$

Note as $t \to 0$ $\theta \to dt^2/2$, that is the shape of the θ - t curve is convex upwards at zero t.

When $t = 1/p_0$ the ratio of the first to the second term is

$$\frac{p_1}{p_0} \frac{(e-1)}{(1-e^{-p_1/p_0})}$$

For the numerical values used in this report this ratio is over 30 so that only the first term with the positive exponential is important.

If now we put $p \ll \alpha/V$ which is equivalent to omitting the second term above, equation (21) may be written as

$$\bar{\Psi}_f = \frac{dV}{\alpha p} \frac{1}{\left[p(1+q/k) - \left(\frac{dV}{m\alpha} - \frac{a_f k_f q p}{\rho c \alpha}\right)\right]}$$

The term in square brackets is a cubic in q and the equation may be written as

$$\bar{\Psi} = \frac{dV}{p\alpha\left(\frac{k_f}{h_c} + \frac{a_f k_f H_f}{\rho c \alpha}\right)\left(q^3 + \gamma q^2 - \beta\right)}$$

where $\gamma = \dfrac{h_c}{1 + \dfrac{H_c a_f}{\rho c \alpha}} \approx h_c$

and $\beta = \dfrac{dV}{m\alpha} \dfrac{h_c}{k_f} \dfrac{}{\left(1 + \dfrac{H a_f}{\rho c \alpha}\right)}$

For the numerical values appropriate to fibreboard $\gamma = 200\ m^{-1}$ and $\beta \approx 14 \times 10^6\ m^{-3}$ and it may be shown that the cubic has no real negative roots. The equation for $\bar{\Psi}$ can then be inverted by conventional methods to give

$$\frac{\Psi}{A} = \frac{1}{\pi}\int_0^{\infty} \frac{e^{-k_f x t} \cdot \sqrt{x}\,.dx}{(\gamma x + \beta)^2 + x^3} - \frac{1}{\beta} + \frac{2e^{p_0 t}}{q_0^2(3q_0 + 2\gamma)}$$

Where $A = \dfrac{dV h_c}{\alpha k_f \left(1 + \dfrac{H a_f}{\rho c \alpha}\right)}$

and q_o is the positive root of the cubic equation $= 190$

$\therefore \quad t_{fe} = \dfrac{1}{kq^2} = 230\ s$

The value of the $e^{p_0 t}$ term when $t = 1/p_0$ is 1.6×10^{-7} and the integral at the same time is less than $\dfrac{q_0^3}{2\sqrt{\pi}\,\beta^2}$ i.e. 0.1×10^{-7} - negligible by comparison.

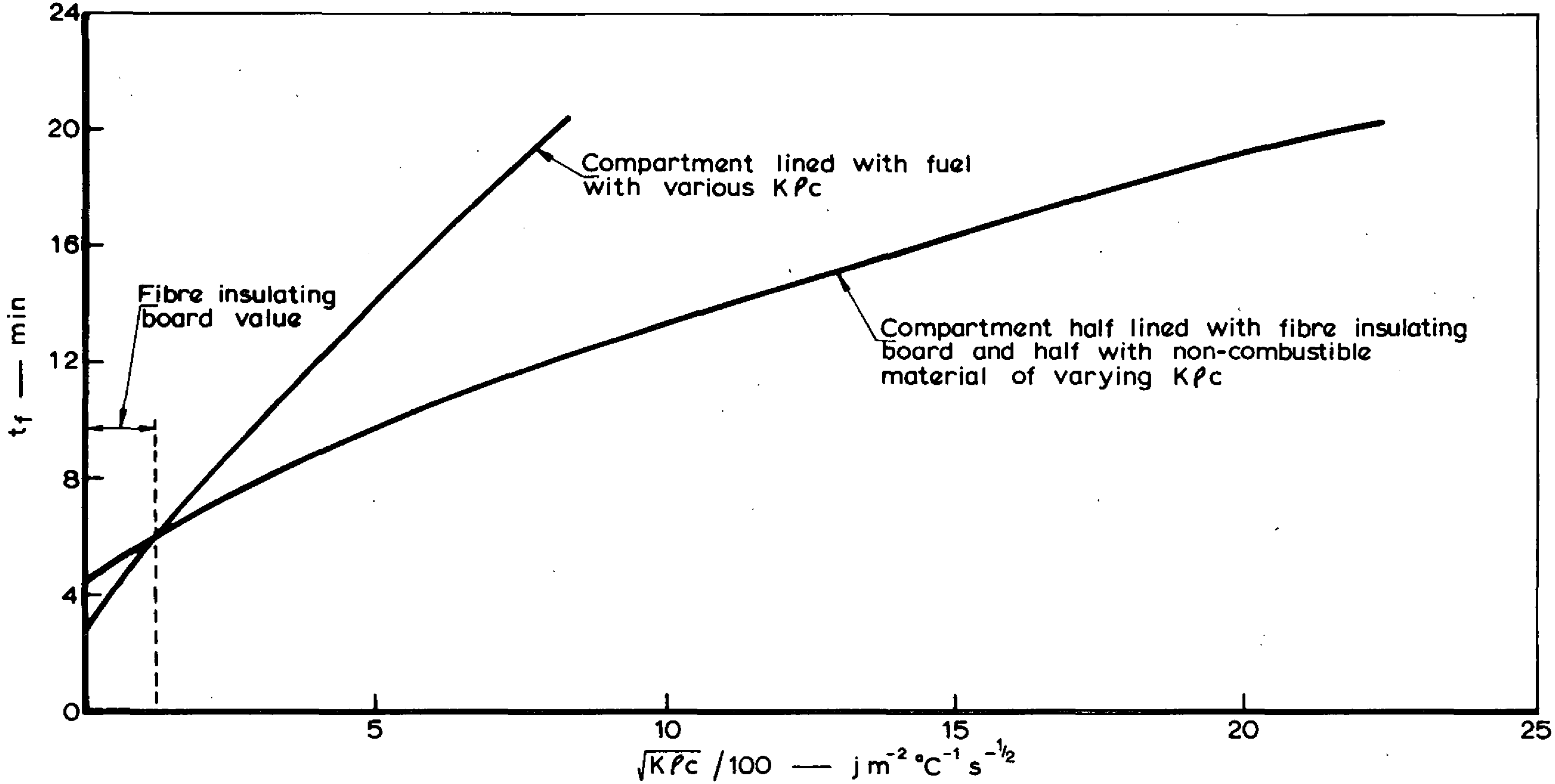

FIG. 1. THE EFFECT OF $K\rho c$ ON FLASHOVER

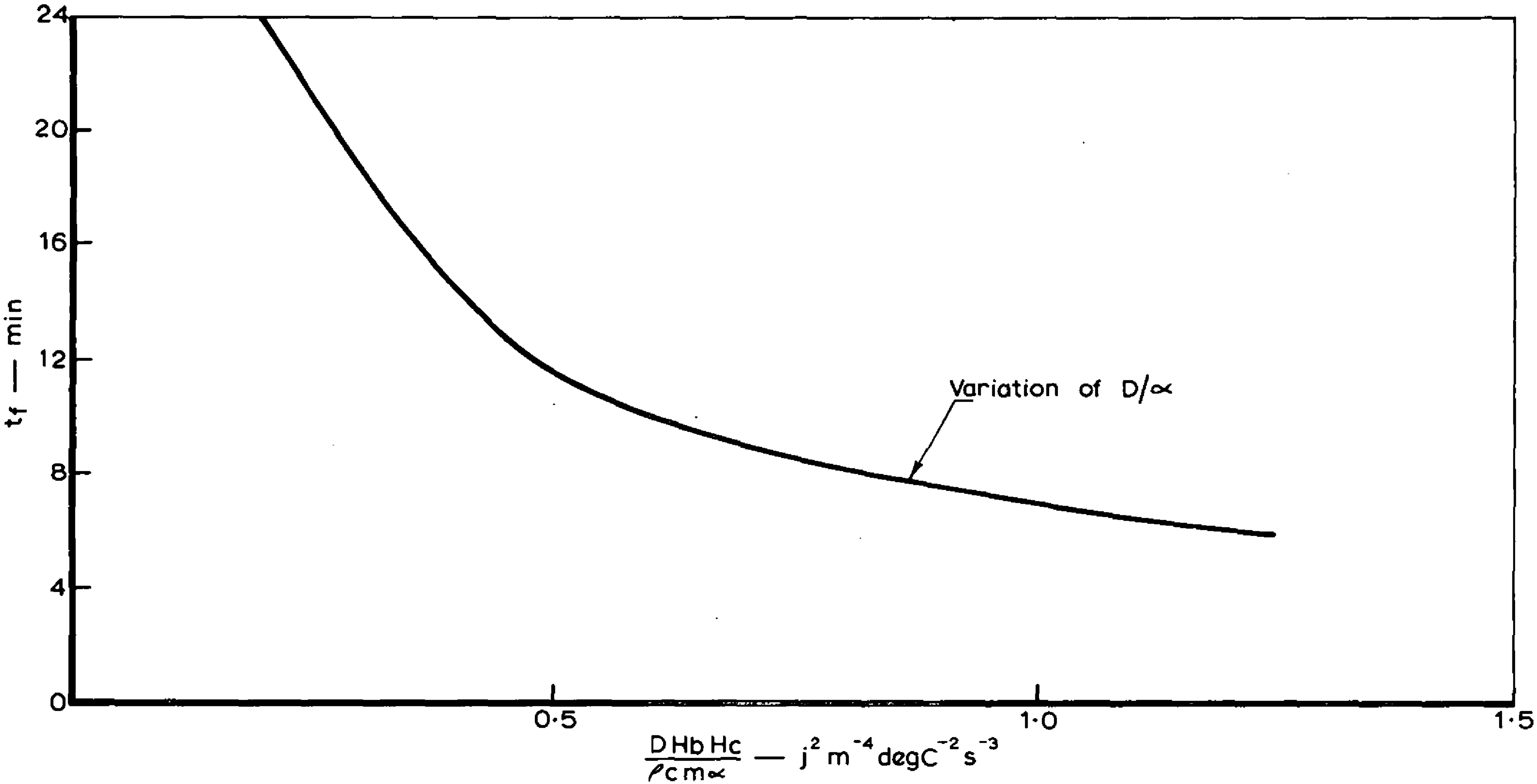

FIG. 2. THE EFFECT OF VARIATION IN D ON FLASHOVER