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FREQUENCY DISTRIBUTION OF FIRE LOSS

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JOINT FIRE RESEARCH ORGANIZATION

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Introduction

Attempts are being made here and elsewhere to find an expression for the probability distribution function of direct monetary loss due to fire. The latest paper on the subject appears to be that of Mandelbrot[1]. An improved form is presented in this note supported by data on fire losses available to this Organization at present. Preliminary estimates for the values of the parameters have also been obtained.

Graphical representation

In 1965, Fire Brigades in the United Kingdom attended 83,167 fires in buildings[2]. Among these there were 777 fires each estimated to have caused material damage of £10,000 or more[3]. These 'large' fires form only 0.935 per cent of the population of fires but provide information of the situation once a fire has reached a state of 'largeness'. If $\phi(x)$ denotes the probability of loss exceeding the value x the distribution function $F(x) = 1 - \phi(x)$; it is more convenient, however, to consider the function ϕ while taking logarithms etc. than the distribution function $F(x)$. Making use of the large fire data the logarithm of $\phi(x) \cdot 10^4$ has been plotted against $\log_{10} x$ in Fig. 1. (The multiplier 10^4 is used to get the graph in the first quadrant instead of the fourth). For the range below £10,000 loss figures furnished by Surrey fire brigade in the fire reports for 1964 have been used. The range £2,000 and over is further reinforced with the information given by South Eastern Fire Brigade of Scotland in their annual reports for the period 1963 to 1966[4]. The three sets of points have fallen close to each other revealing a fairly definite pattern.

Between consecutive values of $\log x$ the curve is approximately linear. The curve has also to cut the y axis at a distance 4 from the origin if a minimum loss of £1 is assumed. These facts indicate that a curve of the second degree in $\log x$ is a better fit than a straight line. The curve appears to cut the x axis at about 5.75 meaning that the chance of loss being greater than £563,000 is one in ten thousand. The original curve is in the fourth quadrant. Hence, omitting the random error component,

$$\text{Log}\,\phi(x) = \text{Log}\,a - \alpha \log x - \beta(\log x)^2 \qquad \ldots\ldots\ (1)$$

where α and β are positive. This leads to

$$\phi(x) = a x^{-(\alpha + \beta \log x)} \qquad \ldots\ldots\ (2)$$

Since we have assumed $\phi(x) = 1$ for $x = 1$, $a = 1$. The (cumulative) frequency distribution function is given by

$$F(x) = 1 - \phi(x) \qquad \ldots\ldots\ (3)$$

and the probability density function by

$$f(x) = \frac{dF}{dx} = x^{-(\alpha + \beta \log x + 1)}(\alpha + 2\beta \log x) \qquad \ldots\ldots\ (4)$$

The probability that the damage will be equal to x has the density given by (4). If linearity is assumed expression (4) reduces to the form

$$f(x) = \alpha' x^{-\alpha' - 1}$$

or to the general form

$$f(x) = m^{\alpha'} \alpha' x^{-\alpha' - 1} \qquad \ldots\ldots\ (5)$$

with a minimum loss m as given by Mandelbrot.

Estimation of parameters.

Since a minimum loss of £1 has been assumed, fitting a straight line to log $\phi(x)$ and log x is equivalent to computing the mean $\bar{Z}$ of

$$Z = \frac{Y}{X} = \frac{\text{Log}\,\phi(x)}{\text{Log}\,x} \qquad \ldots\ldots\ (6)$$

From 40 points on the graph obtained by pooling the Survey and the large fire data, $\bar{Z}$ was estimated as

$$\bar{Z} = -0.4291 \qquad \ldots\ldots\ (7)$$

giving

$$\phi(x) = x^{-0.4291} \qquad \ldots\ldots\ (8)$$

Mandelbrot uses the law of Pareto with exponent $\frac{1}{2}$ given by

$$\phi(x) = \left(\frac{x}{m}\right)^{-0.5} \qquad \ldots\ldots\ldots (9)$$

with m as the minimum loss.

The inaccuracy in using expression (8) or (9) becomes apparent when a straight line of the following form is fitted to the data

$$Z = \alpha + \beta X \qquad \ldots\ldots\ldots (10)$$

which is the same as fitting the curve

$$Y = \alpha X + \beta X^2$$

A least square fit gave

$$\alpha = -0.0778 \text{ and}$$

$$\beta = -0.1052$$

The residual sum of squares was drastically reduced to 0.0092 from a value of 0.7412 in the first case given by $\sum(Z - \bar{Z})^2$. Hence a better expression for $\phi(x)$ would be

$$\phi(x) = x^{-0.0778 - 0.1052 \log x} \qquad \ldots\ldots\ldots (11)$$

The curve corresponding to the values given by (11) is shown in Fig. 1 and the cumulative frequency curve $F(x)$ in Fig. 2.

The values of the parameters α and β are obtained from a small sample; it is possible also that Z is non linear and of the form

$$Z = \alpha + \sum_i \beta_i x^i \qquad (i > 1) \qquad \ldots\ldots\ldots (12)$$

$\log \phi(x)$ being a polynomial of degree greater than 2 in $\log x$. When sufficient data become available, the best fit could be determined and optimum values of the parameters estimated by the methods described by Haselgrove[5] and Maddison[6].

Conclusion

From data available to this Organization at present there is reason to believe that, if $\phi(x)$ denotes the probability of loss in a fire being greater than x, $\log \phi(x)$ is a polynomial of degree two in $\log x$. A minimum loss of one pound is assumed so that the ratio $\log \phi(x)/\log x$ is approximately linear. From a least square fit the constant term to this straight line is estimated as -0.0778 and the slope as -0.1052. With more data becoming available, the goodness of a higher degree fit could be examined and optimum values of the parameters obtained.

References

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(4) "Reports of the Firemaster", South Eastern Fire Brigade, 1963 to 1966.

(5) HASELGROVE, C. B. The solution of non linear equations and of differential equations with two point boundary conditions. Computer Journal, No. 4, p 255, 1961.

(6) MADDISON, R. N. A Procedure for Non-linear Least Squares Refinement in Adverse Practical Conditions. Journal of the Association for Computing Machinery, Vol. 13, No. 1, pp 124-134, 1966.

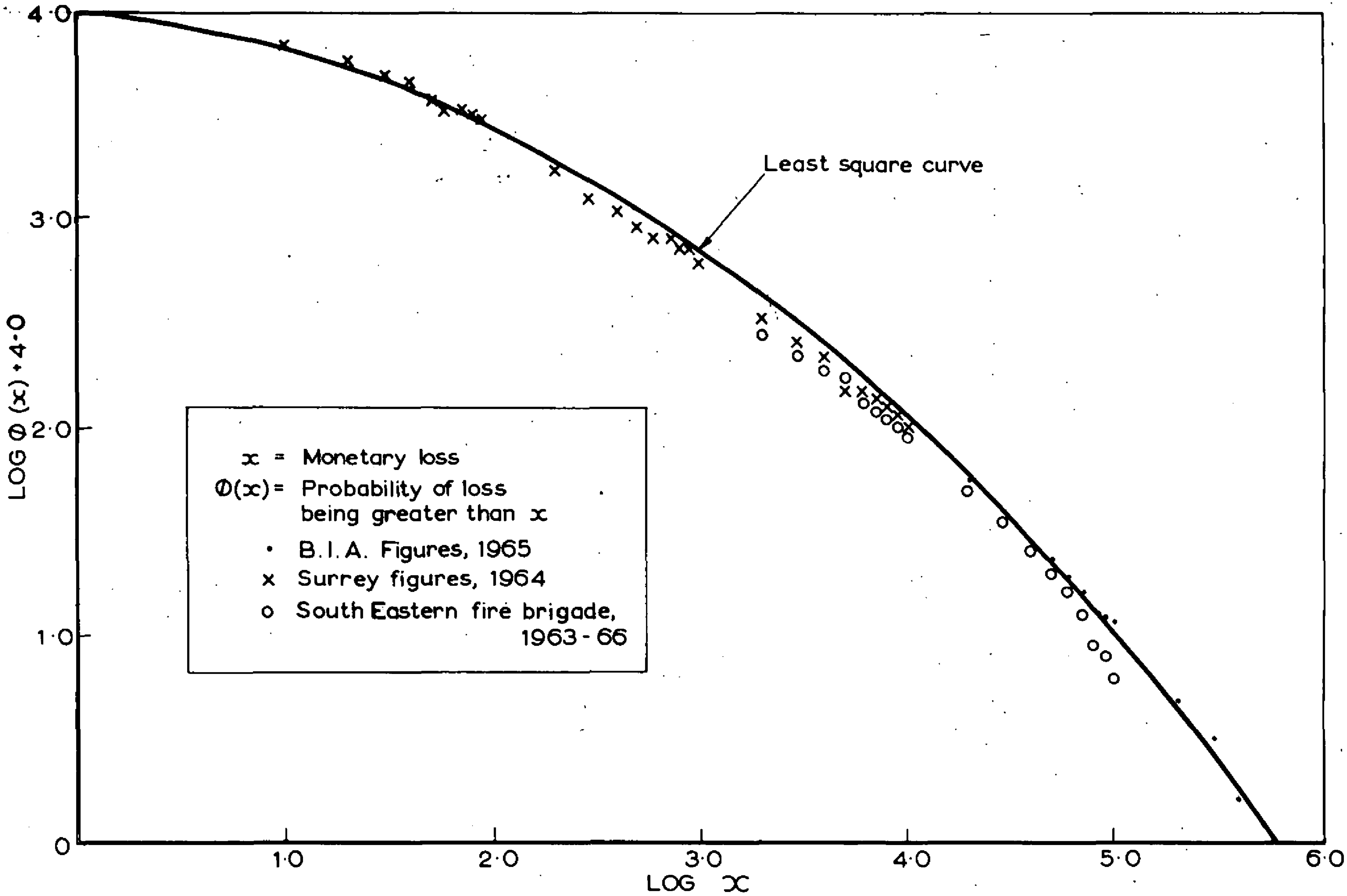
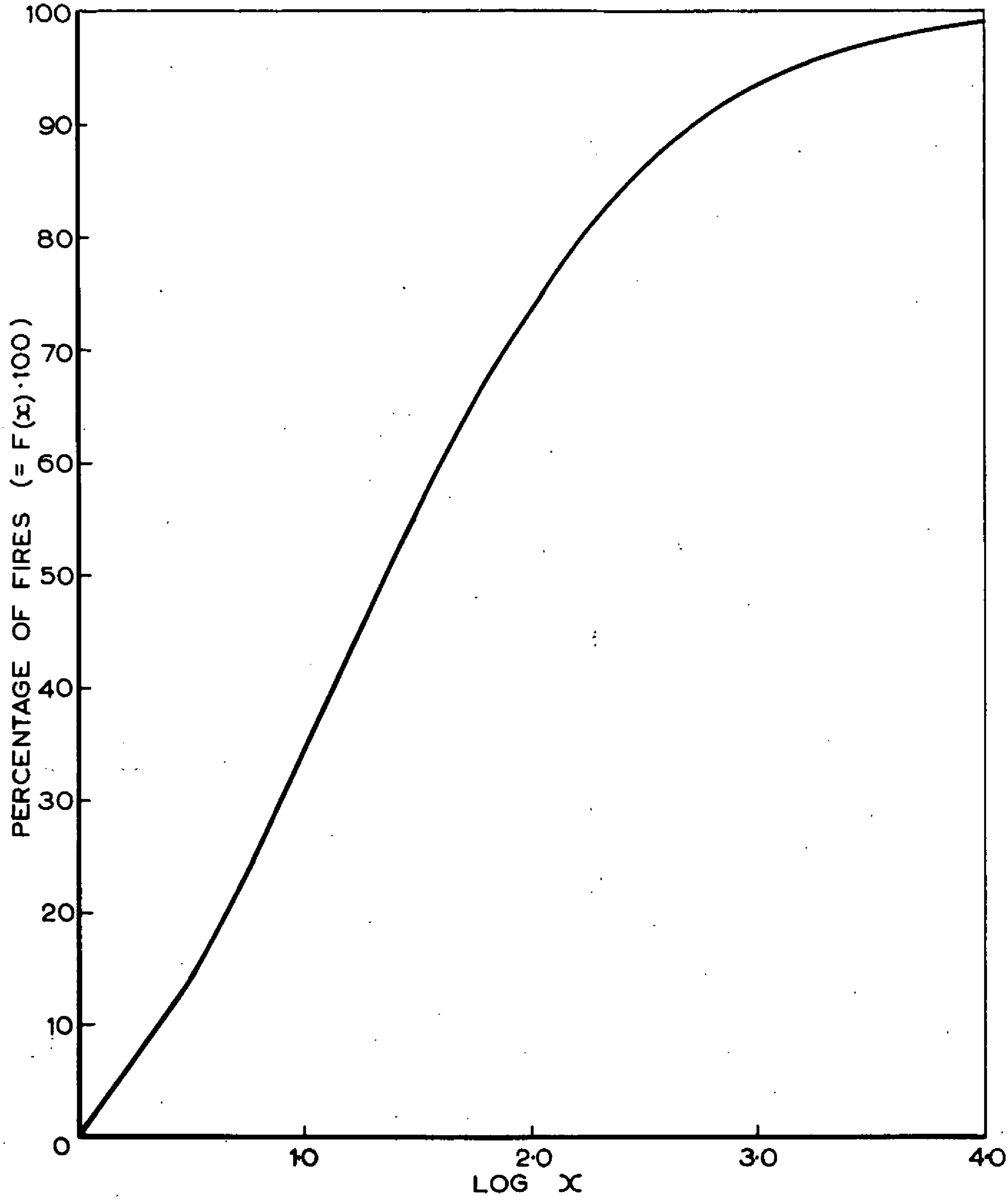


FIG. 1. DISTRIBUTION FUNCTION OF LOSS FREQUENCY FOR FIRES ATTENDED BY FIRE BRIGADES



x = Monetary loss

$F(x)$ = $1 - \Phi(x)$

= Probability of loss being less than or equal to x

FIG. 2. CUMULATIVE FREQUENCY CURVE