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DEPARTMENT OF SCIENTIFIC AND INDUSTRIAL RESEARCH AND FIRE OFFICES' COMMITTEE
JOINT FIRE RESEARCH ORGANIZATION

COOLING CORRECTIONS FOR THE COPPER DISC RADIOMETER

by

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This apparatus consists essentially of a block of copper on which the radiation to be measured is allowed to fall. The temperature of the block is measured by thermocouples fixed in the front and back surfaces and this allows the quantity of heat received in a given time to be computed. The transient rise in temperature may be computed in the following way.

The heat received by the circular face figure 1 in any time dt is $\pi r^2 I\,dt$ where I is the incident intensity. This heat will be partly used in heating the copper disc and will partly be lost by convective and radiative cooling. This may be represented by $\pi r^2 a\rho s\,d\theta + 2\pi r^2 H\theta(1+\frac{a}{r})dt$ where ρ and s are the density and specific heat of the copper, $d\theta$ temperature rise of block in time dt, H the Newtonian cooling constant for the copper block.

Thus $$I\,dt = a\rho s\,d\theta + 2H\theta\left(1+\frac{a}{r}\right)dt$$

or $$\frac{d\theta}{I - 2H\theta(1+\frac{a}{r})} = \frac{dt}{a\rho s}$$

Thus $$I - 2H\theta\left(1+\frac{a}{r}\right) = A\,e^{-\frac{2H(1+\frac{a}{r})t}{a\rho s}}$$

When $t = 0$ $\theta = 0$ whence $A = I$

and the temperature θ is given by

$$\theta = \frac{I}{2H(1+\frac{a}{r})}\left[1 - e^{-\frac{2H(1+\frac{a}{r})t}{a\rho s}}\right]$$

$$= \frac{I}{\beta}\left[1 - e^{-\frac{\beta t}{\gamma}}\right] \quad \text{---(1)}$$

where $\beta = 2H(1+\frac{a}{r})$
$\gamma = a\rho s$

Particular values are

β very small $\quad \theta = \frac{It}{\gamma}$

$t \to \infty \quad \theta_{equlib} = \frac{I}{\beta}$

If at any time T the radiation is switched off the subsequent temperature will be

$$\theta = \frac{I}{\beta}\,e^{-\frac{t\beta}{\gamma}}\left[e^{\frac{T\beta}{\gamma}} - 1\right] \quad \text{...... (2)}$$

Approximate method of correcting final temperature

The true final temperature in the absence of cooling would be $\frac{IT}{\gamma}$. It is required to find this from a knowledge of the heating and cooling curves.

The slope of the cooling curve at the instant the radiation is switched off is

$$\frac{d\theta_c}{dt} = -\frac{I}{\gamma} e^{-\frac{T\beta}{\gamma}}\left[e^{\frac{T\beta}{\gamma}} - 1\right]$$

$$= -\frac{I}{\gamma}\left[1 - e^{-\frac{T\beta}{\gamma}}\right] \qquad \ldots\ldots (3)$$

If the cooling curve is produced backwards to cut the temperature axis at θ_1 (figure 2) the intercept will be $\theta_1 = \theta_T + T\frac{d\theta_c}{dt}$ which from 1 and 2 is

$$\theta_1 = \frac{I}{\beta}\left[1 - e^{-\frac{\beta T}{\gamma}}\right] + \frac{IT}{\gamma}\left[1 - e^{-\frac{\beta T}{\gamma}}\right]$$

and the mean value of temperature between the maximum observed temperature θ_1 & θ_T is

$$\frac{\theta_1 + \theta_T}{2} = I\left[\frac{1}{\beta} + \frac{T}{2\gamma}\right]\left[1 - e^{-\frac{\beta T}{\gamma}}\right]$$

Now if $\frac{\beta T}{\gamma}$ is small

$$\frac{\theta_1 + \theta_T}{2} = \frac{IT\beta}{\gamma}\left[\frac{1}{\beta} + \frac{T}{2\gamma}\right]\left[1 - \frac{T\beta}{\gamma}\right]$$

$$= \frac{IT\beta}{\gamma}\left[\frac{1}{\beta} - \frac{T^2\beta}{4\gamma^2}\right]$$

$$= \frac{IT}{\gamma}\left[1 - \frac{T^2\beta^2}{4\gamma^2}\right]$$

which is a very good approximation to $\frac{IT}{\gamma}$

Theoretically accurate methods

Method 1

From 1 and 3 we have the slopes at the instant the heating is switched off.

Slope of heating curve $\frac{d\theta_H}{dt}$ $(t = T)$

$$\left.\frac{d\theta_H}{dt}\right|_{t=T} = \frac{I}{\gamma} e^{-\frac{\beta T}{\gamma}} \qquad \text{...... (4)}$$

Slope of cooling curve $\frac{d\theta_c}{dt}$ $(t = T)$

$$d\theta_c/dt = -\frac{I}{\gamma}\left[1 - e^{-\beta T/\gamma}\right] \qquad \text{...... (5)}$$

Thus from 4 and 5

$$I = \gamma\left[d\theta_H/dt - d\theta_c/dt\right]$$

Although the result is theoretically accurate errors will occur in practice owing to the difficulty of correctly determining the slopes of the heating and cooling curves.

Method 2

The temperature at $t = T$ is given by

$$\theta_T = I/\beta\left[1 - e^{-\frac{T\beta}{\gamma}}\right] \qquad \text{...... (6)}$$

Similarly

$$\theta_{2T} = \frac{I}{\beta} e^{-\frac{T\beta}{\gamma}}\left[1 - e^{-\frac{T\beta}{\gamma}}\right]$$

$$\therefore \quad \frac{\theta_{2T}}{\theta_T} = e^{-\frac{T\beta}{\gamma}}$$

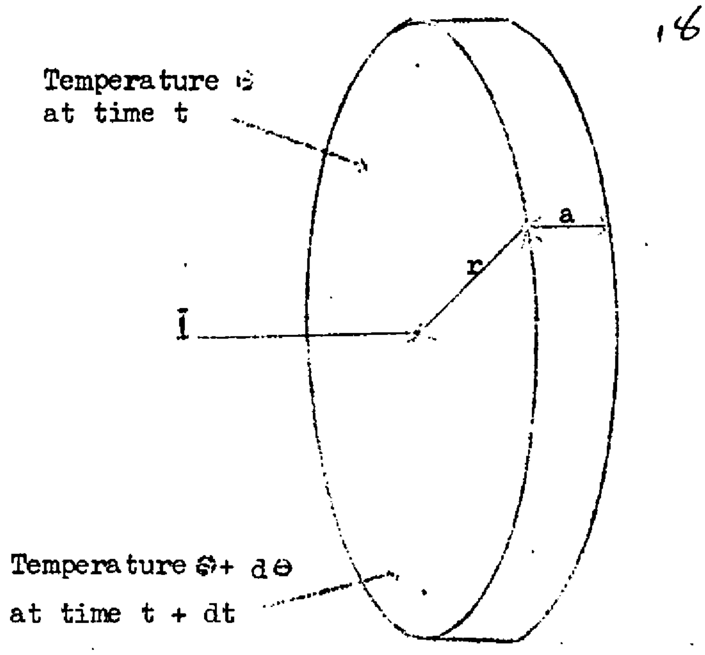
Thus from 4

$$\frac{d\theta_H}{dt} = \frac{I}{\gamma}\frac{\theta_{2T}}{\theta_T}$$

or

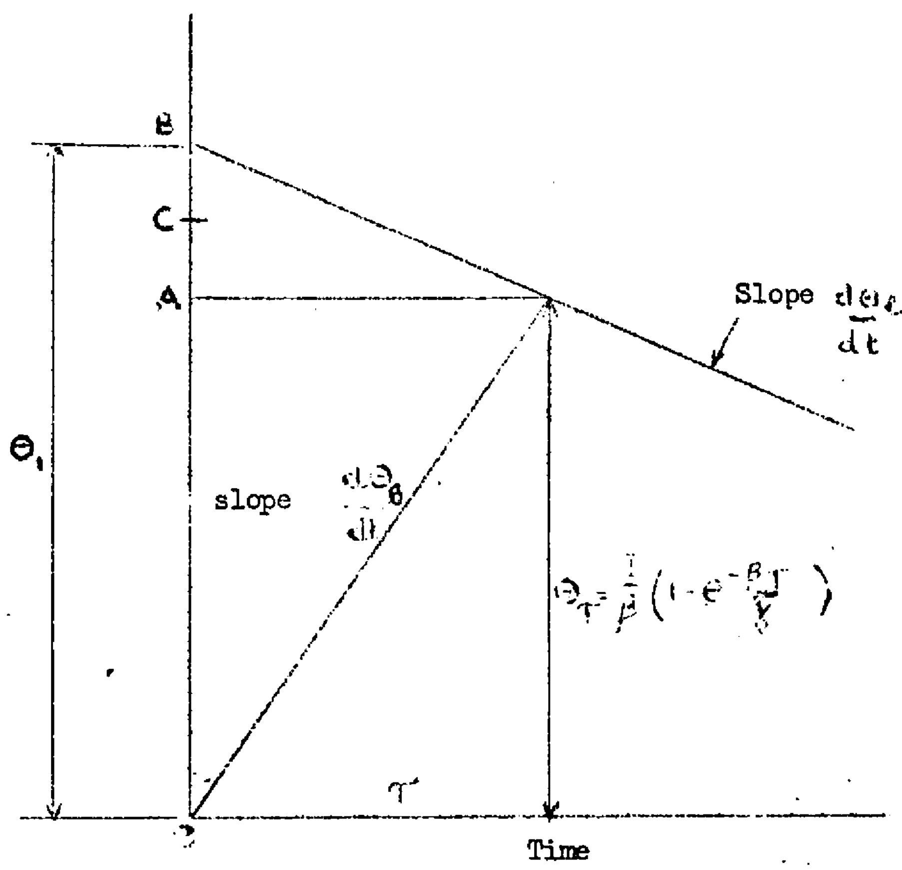
$$I = \frac{\theta_T}{\theta_{2T}}\gamma\left(d\theta_H/dt\right)_{t=T}$$

This again is open to the objection that the result depends on the slope of the heating curve.



Temperature θ
at time t
a
r
I
Temperature θ + dθ
at time t + dt

FIG. 1.



B
C
A
Slope dθ/dt
θ₁
slope dθ/dt
τ
Time

FIG. 2.