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# **Fire Research Note**

## **No.699**

**ON THE DEVELOPMENT OF URBAN FIRES  
FROM MULTIPLE IGNITIONS**

by

**P. H. THOMAS**

February, 1968

# **FIRE RESEARCH STATION**

F.R. No. 699

"The theory and that from which it is derived (Albini and Rand) is not strictly correct. The term  $p$  in  $Nqpa$  in equations 4A and 4B should be  $p'$  because given an unburnt cell (probability  $a$ ) the chance that a neighbour is alight will be less than  $p$ . Expressions for the damage will therefore be overestimates (as if  $q$  were far larger than it is). The error would be expected to be least for an initially random distribution of many fires. Putting  $N = 4$  also produces an overestimate. For all fires other than the initial fires  $N$  would be 3 on a square lattice".

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SUMMARY

Stanbury has argued that in those fires sometimes referred to as firestorms in Hamburg, Darmstadt, Kassel and Wuppertal during World War II, the number of buildings initially set alight in the fire area was nearly half the total, whereas for other fires in Germany it was 1 in 8 or less.

It is shown here, on the basis of values derived for the likelihood of spread and burn out, that if half the buildings are initially set alight, flames from separate fires might merge over large areas whereas they could never merge generally at any time during a fire starting from a much smaller fraction of ignited buildings.

A simplified theoretical model of fire spreading in an urban area from a statistically uniform distribution of ignition centres is described. It is mathematically the same as the well-known epidemic model of Kermack and McKendrik and is basically similar to that given by Albin and Rand, producing similar estimates of the probable damage, but is, however, somewhat more flexible analytically. The paper discusses the relative importance of the initial conditions and the spread parameters on the development of fire and the resulting damage.

An approximate modification of the model to describe a steadily advancing fire front is outlined, and used to estimate parameters of fire spread from published correlations of data.

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1. Introduction

In this paper we shall use the term multiple fire to mean several separate fires which may be sufficiently far apart to be burning independently of each other, close enough together to be affecting each other without being in any way physically merged, or closer still so that the flame zones from the separate fires are merged together even though the burning areas on the ground may still be separate. The use of the term multiple fire does not imply that any one form of multiple fire is or is not a mass fire or a firestorm.

Baldwin, Thomas and Wraight<sup>1</sup> have shown that if two or four square fires each of the side  $D$  are equally spaced by a distance  $S$  from each other, there is a critical value for  $S$ , viz  $S_c$ , when the flames from each of the fires join near the tips. This condition is referred to as the onset of merging. If the burning rates of each individual fire are not affected by its proximity to the other fires, the lengths of the flames, which for isolated flames are equal to their height, are relatively insensitive to deflection. If this length is denoted as  $L$  it is possible to derive theoretically a condition relating  $S_c$  to  $L$  and  $D$ . This condition is given<sup>1</sup> as:

$$\left(\frac{S}{D}\right)^3 = 1.16 \times 10^{-2} \left(\frac{L}{D}\right)^2 \quad (1)$$

Baldwin<sup>2</sup> has shown that this condition can be employed independently of the number of separate fires. For the purpose of this paper we shall need to assign a particular value to the ratio of  $L/D$  and for simplicity this is taken as 1 for large fires. Thus, in Project Flambeau, the piles of wildland

fuel were 47 ft square and flames were observed 30 - 70 ft high. Continuous bodies of flame of over 100 ft are rarely observed in building fires so a value of L/D equal to unity is of the appropriate order, though in the laboratory, small-scale fires frequently attain higher values of this ratio. Inserting L as D in equation (1) gives the condition for merging as:

$$S_c \doteq D/4 \quad (2)$$

It therefore follows that, for the regularly spaced square fires, the fraction of the ground area which must be covered by fire at the onset of merging is:

$$\frac{f_c}{c} \doteq \left(\frac{D}{D+S}\right)^2 \doteq 0.64 \quad (3)$$

If  $\frac{L}{D}$  is 2 the value of  $\frac{f_c}{c}$  is reduced to 0.55

These are high values: there are many areas outside the centres of cities where the fraction of land which is built-up is considerably less so that on this view there are few areas where separated fires can merge.

Although merged fires are not necessarily firestorms, they are severe and it is perhaps relevant that Stanbury<sup>3</sup> has asserted that, in the firestorms\* in Germany in World War II, about 1 in 2 buildings were initially set alight compared with 1 in 8 or less in the fires for which the term firestorm is not conventionally used. It is doubtful if one could argue from these data that 1 in 2 fires alight is a minimum condition for a firestorm even if the use of the word firestorm were generally agreed. However, we note that according to equations (1) and (3) the condition of 1 in 2 buildings alight is the order of the requirement for flame merging in densely built-up areas. In the next section we shall discuss a mathematical model of growth and see whether the condition for flame merging can be reached as a result of fires spreading from

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\*The term was used by Stanbury for Hamburg, Darmstadt, Kassel and Wuppertal.

a much smaller number of initial ignitions. Another observation by Stanbury which will be relevant later in this paper is that in all the mass raids the overall ratio of the number of buildings destroyed by fire to the number first ignited by incendiary bombs was about 2.

Theories of fire spread, by and large, lack confirmation and it is difficult to produce simple physical theories governing the rate of spread in this situation. There are a number of statistical formulations and they require the insertion of numerical values into their parameters. It is instructive to consider an admittedly highly idealized version of one of these and, amongst other things relate its predictions to the condition given in equation (3).

## 2. A Model of fire spread for a multiple fire

If we consider a large area with fires distributed over it in a statistically uniform manner, the probability that any part of this area is burning is the fraction of the total area which is burning and similarly the probability that a particular area has not yet been lit is the fraction of the whole area which has not yet become involved in fire.

We consider a large area sub-divided into unit blocks (or buildings), separated by streets etc. We then write the fraction of the total number of these blocks (or buildings) which are alight at any time as  $p$  which is the probability that any one is alight. We consider that fire can only spread from one block (which henceforth will include the meaning of buildings) to its immediate neighbours. If  $q$  is the probability that fire can spread from a burning block to an unburnt block in unit time\* then the increase in probability from  $t$  to  $t + \Delta t$  that a particular block is burning or the increase in the fraction of all blocks which are burning is  $Nqpa\Delta t$ , where  $a$  is the probability that the block is as yet not burning and  $N$  is the mean

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\* Albini and Rand's  $q$  is  $\frac{Q_i}{V_i}$  in our notation.

number of immediate neighbours. We shall further assume that the chance of a burning block becoming burnt out in the time  $\delta t$  is  $\mu \delta t$ . We therefore obtain the following two equations:

$$\frac{dp}{dt} = Nqpa - \mu p \quad (4A)$$

$$\frac{da}{dt} = -Nqpa \quad (4B)$$

$\frac{1}{\mu}$  may be expressed as the mean lifetime of a fully-developed fire.

Blocks are not capable of spreading fire immediately after ignition but we shall provisionally neglect this effect and treat the period between ignition and flash-over as dead time (see Appendix I).  $\frac{Nq}{\mu}$  is the mean number of buildings to which a building is capable of spreading fire in its lifetime.

This model is essentially that proposed by Albini and Rand<sup>4</sup> except that here it is assumed that fire can spread during any small time interval  $\delta t$  whereas in their model all fires that were alight at any one time burnt out at the same time and are followed by a new generation of fires which all start together just prior to the burn-out of the previous generation. The Albini and Rand model is obviously more suited to dealing with the early stages of a fire when the number of generations is small, whereas the continuous spread model is more suitable when there have been several generations of fire and there is no clearly defined generation. One could always choose to evaluate the behaviour during the first or, say, the first two generations of burning using Albini and Rand's method and obtain values of  $p$  and  $a$  to use as initial conditions for the application of the differential model during the subsequent burning. However, although many fires would be initiated at the same time in a nuclear attack they would not all develop at the same rate and spread fire to other buildings at the same time. Calculations of the final damage based on the two models give very similar results and it is mathematically more convenient to deal with equations (4A) and (4B) than with those given by

Albini and Rand.

It is assumed that at time zero  $p$  has some initial value  $p_0$  which might represent the initial density of blocks which are alight. It follows that  $a_0$  is equal to  $1-p_0$ . If the differential model is only used after one or more generations of fire have elapsed then  $a_0 + p_0 \neq 1$  but we shall not deal further with the minor modification then necessary.

The value of  $p$  increases provided:

$$\frac{Nq}{\mu} (1 - p_0) > 1 \quad (5)$$

and reaches a maximum when  $a$  reaches the value  $a_c$  given by

$$a = a_c = \mu / Nq \quad (6)$$

after which  $p$  falls eventually to zero.

Thus when the fire is at its height i.e. the chance of a building being on fire is at its greatest, the chance that a building has not yet caught alight is  $\mu / Nq$ .

Kermak and McKendrick<sup>5</sup> derived these equations in connection with a deterministic model of the spread of an epidemic and equation (5) is known as their Threshold Theorem. Following them we eliminate 't' and obtain

$$\frac{d(p + a)}{da} = \frac{\mu}{Nq a} \quad (7)$$

which can be integrated to give:

$$1 - (p + a) + \frac{\mu}{Nq} \log_e \frac{a}{1-p_0} = 0 \quad (8)$$

The final value of  $a$ , that is the probable area undamaged at the end of the fire, is obtained from:

$$1 - a_{\infty} + \frac{\mu}{Nq} \log_e \frac{a_{\infty}}{1-p_0} = 0 \quad (9)$$

$(1 - a_{\infty}) / p_0$  may be called a spread factor, the ratio of final to initial damage.



If  $1 - a_{\infty} \ll 1$  and  $p_0 \ll 1$  ( $\frac{Nq}{\mu} > 1$ )

$$\frac{1 - a_{\infty}}{p_0} = \frac{1}{1 - Nq/\mu} \quad (10)$$

The likely maximum size of the fire at any one time during the course of this burning is obtained by inserting into equation (8) the value of  $a_c$  given by equation (6). The maximum value of  $p$  is therefore

$$p_{\max} = 1 - \frac{\mu}{Nq} - \frac{\mu}{Nq} \log_e \frac{Nq}{\mu} (1 - p_0)$$

Results in Figs 1 and 2 show that this model gives very similar values for  $a_{\infty}$  to the Albini-Rand model for low and moderate damage. There are differences when  $\frac{Nq}{\mu}$  is high ( $> 2$ ) but these are likely to be outside the practical range of interest and the damage is anyway very high.

If we put  $p_{\max}$  as 0.64 or 0.55 we obtain relationships between  $p_0$  and  $\frac{Nq}{\mu}$  necessary for  $p$  to reach the critical values causing merging. These are shown in Fig.3. Figures 4 and 5 show values of  $p_{\max}$ .  $p_{\max}$  has a significance apart from merging and Fig.3 may be regarded as showing the shape of the loci of fires of equal peak severity (in terms of fraction alight).

### 3. Estimating $\frac{Nq}{\mu}$

#### 3.1. Direct methods

Albini and Rand made surveys of American cities and obtained some direct values of  $\frac{Nq}{\mu}$  on the assumption that fires spread by radiation between buildings.  $\frac{q}{\mu}$  was obtained from the proportion of buildings whose height exceeded the separation from the building across the street by more than a factor of 1.24. These values of  $\frac{Nq}{\mu}$  are given in Table 1.

Table 1

City	$\frac{Nq}{u}$
Washington D.C.	0.25
Baltimore	0.85
Philadelphia	1.5
Cleveland	1.7
San Francisco	1.9

It is possible to derive an overall value of  $\frac{Nq}{u}$  for the German cities subjected to the war raids. Stanbury's spread factor of 2 in our terms means

$$\frac{1 - a_{\infty}}{p_0} = 2$$

and inserting this into equation (10) with  $p_0$  as  $\frac{1}{8}$  gives an average value of  $\frac{Nq}{u}$  as:

$$\frac{Nq}{u} = 0.6$$

### 3.2 Indirect methods

Indirect estimates of  $\frac{Nq}{u}$  can also be derived from data for the steady rates of spread of wide fire fronts in large fires. Thus, if equations (4A) and (4B) are modified to allow for a systematic directional variation of  $p$  one can obtain (see Appendix II) a relation between a steady mean rate of spread  $R$  and the parameters  $q$  and  $\mu$ . This is:

$$\left. \begin{aligned} R &= 4q \left( 1 - \frac{\mu}{4q} \right)^{\frac{1}{2}} \\ \text{or } \frac{R}{\mu} &= \frac{4q}{\mu} \left( 1 - \frac{\mu}{4q} \right)^{\frac{1}{2}} \end{aligned} \right\} \quad (11)$$

$N$  is here taken as 4 and  $l$  is the distance between the blocks (or buildings) which are regarded as the unit fires. Phung and Willoughby<sup>6</sup> give a mean value of  $R$  for light stone and concrete

structures (40 per cent ground coverage) as 45 m/h in the absence of wind, rising linearly to 130 m/h with a wind of 20 m/s. Comparable values of R for Japanese conflagrations<sup>7</sup> are about 60 m/h in the absence of wind, rising non-linearly to 700-900 m/h in a wind of 20 m/s. If one takes  $\mu$  as  $2/3 \text{ h}^{-1}$  (a mean duration of  $1\frac{1}{2}$  hours) and  $l = 100 \text{ m}$  for U.S. blocks one gets  $\frac{4q}{\mu} = 1.4$ , while, if for Japanese cities we take  $\mu = 3 \text{ h}^{-1}$  and  $l = 10 \text{ m}$ , we get  $\frac{4q}{\mu} = 2.5$ . Table 2 summarises these estimates of  $\frac{Nq}{\mu}$ .

Table 2.

Estimates of  $Nq/\mu$  - the average no. of blocks (or buildings) to which one fire in a block (or building) spreads in its lifetime.

Source of estimate	Value of $Nq/\mu$
Albini and Rand's direct survey of parts of 5 U.S. cities.	0.25 — 1.9
Stanbury's 'spread factor' for German cities in World War II	0.6
From a theoretical model of spreading fire fronts ( $N=4$ ) and historical U.S. data correlated by Phung and Willoughby <sup>6</sup>	c 1.4
Ditto for Japanese data	c 2.5

#### 4. The effect of wind

The effect of wind on rates of spread in Japan is greater than in western cities, but since the values of  $\mu$  are altered one cannot immediately translate the known variation of R into a variation of  $\frac{Nq}{\mu}$ .  $\frac{R}{\mu l}$  is essentially the number of rows of unit fires burning simultaneously in the fire front, so if this increases with wind speed  $\frac{Nq}{\mu}$  does too.

In any real situation the value of the wind speed is not uniform over the fire area; if the unit fires are close enough the wind speed  $V$  at the periphery of the fire edge is proportional to the cube root of the rate of heat release.

If  $F$  is the total amount of fuel in the fire area, the rate of fuel consumption, if it all burnt simultaneously, would be  $\dot{m}F$  so that instantaneously it is  $p \dot{m} F$ .

and 
$$V \propto (p \dot{m} F)^{\frac{1}{3}}$$

Using data which relates  $R$  to  $V$  and some relationship between  $\dot{m}$  and  $V$  which is probably weak for western cities, one can derive as a first approximation a relationship between  $q$  and  $p$  as follows:

Equation (11) gives  $\frac{q}{\dot{m}}$  as a function of  $\frac{R}{\dot{m}}$ , and hence of  $V$  and hence of  $(p \dot{m})^{\frac{1}{3}}$ . Such a relationship does not suggest a strong relationship between  $q$  and  $p$  for western cities. Lomasson<sup>8</sup> has used a procedure on these lines and calculated the peripheral velocity to estimate the positive feedback effect of the induced wind. Such a positive feedback does not in itself guarantee that  $p$  reaches unity even if  $\frac{Nq}{\dot{m}} (1 - p_0) > 1$  because of the presence of the negative term  $\dot{m}$  in equation (4A).

### 5. Discussion

For our purposes it is sufficient to note the order of the magnitude of the estimates of  $\frac{Nq}{\dot{m}}$  in relation to Fig. 3. Such values of  $\frac{Nq}{\dot{m}}$  would presumably be typical of modern constructions in other cities since the values of  $\dot{m}$  are primarily dependent on the fuel loading of buildings while the value of  $q$  is primarily dependent on the size of streets and building heights. It is seen from Fig.3 that except for what are probably unrealistically much higher values of the ratio  $\frac{Nq}{\dot{m}}$ , the value of  $p_0$  must approach or exceed 0.5 for merging to occur at any time. In other words, except when the fires spread at a very much faster rate than is likely to take place, it is unlikely that a high value of  $p_{max}$  approaching 0.5, say,

can be reached except when the initial value is made almost equal to it.

Figure 4 shows that  $p_c$  is small,  $p_{\max}$  is  $< 0.15$  if  $\frac{Nq}{\mu} < 2.0$

Estimates of the number of primary fires in Hiroshima<sup>9</sup> give 4 000 fires in about 36 000 buildings in the central 3 square miles i.e. a mean  $p_0$  of 0.11. Between 10,000 and 33,000 ft from ground zero  $p_0$  has a maximum of 0.21. This is relatively low compared with the Hamburg fire. Despite the differences between Japanese and western cities it is doubtful if the parameter  $\frac{Nq}{\mu}$  in Japanese cities is so much larger than in western cities that it offsets the effect of the difference in  $p_0$  between 0.2 and 0.5 (see Fig.3).

None of the above discussion involves the time scale of the burning and to obtain the variation of  $p$  or  $a$  with time one must numerically integrate equations (4A) and (4B). Some results of such integration are shown in Figs 6, 7, 8 and 9. The time  $t_{\max}$  to the maximum value of  $p$  can be obtained from equations (4B), (6) and (8) as

$$t_{\max} = \frac{1}{Nq} \int_{Nq/\mu}^{1-p_0} \frac{da}{a \left[ 1 - a + \frac{\mu}{Nq} \log_e \frac{a}{1-p_0} \right]}$$

In Fig.5 it can be seen that with a low  $p_0$  the maximum  $p$  is reached sooner, the higher the value of  $\frac{Nq}{\mu}$ . If  $p_0$  is high the reverse is true: the maximum is reached later, the higher the value of  $\frac{Nq}{\mu}$ . The reverse in the variation of the time of the maximum is shown in Fig.9 for  $p_0 = 0.1$ . The values of  $t_{\max}$  attributed to Albin and Rand were obtained from their graphs of the variation of  $1 - a$  with the number of time steps. For purposes of comparison  $\mu$  was taken as unity for one time step and  $t_{\max}$  was taken as the time when  $a$  was equal to  $\frac{\mu}{Nq}$ . The figures beside the curves in Fig.3 are approximate values of  $\mu t$  for the onset of merging. Except when  $p_0 \rightarrow 0$  they are less than unity, indicating that merging occurs while many of the first generation of fires are still alight. If, in

equation (9), we put  $a_{\infty}$  equal to 0.01 and 0.1 corresponding respectively to 99 per cent and 90 per cent of buildings damaged we obtain relations between  $p_0$  and  $\frac{Nq}{u}$  and these have been shown in Fig.3.

The shape of the loci and of  $a_{\infty}$  and  $p_{\max}$  differ and equally disastrous damage can clearly result from fires having different values of  $p_{\max}$ . Indeed the maximum value of  $p$  can remain relatively small in a fire where the final damage is high. Thus if  $\frac{Nq}{u}$  is 2, equations (6) (8) and (9) show that 80 per cent of the buildings can be expected to be damaged with  $p$  initially very small having a maximum of 0.15. Quite apart from other considerations it is clear that some studies of plumes and firestorms which implicitly assume that all the fuel can be burning at once may considerably overestimate the energy release rate.

## 6. Conclusions

A simplified model of fire spread related to that described by Albini and Rand and two methods of estimating the principal parameter from fire experience have been described. One of these is based, rather tentatively, on a model of the spread of a fire front and gives values for U.S. cities somewhat higher than an estimate based on German experience in World War II but comparable to direct estimates by Albini and Rand for U.S. cities. Neglecting the effect of the induced wind on spread, the model suggests that unless the initial number of fully-developed fires is high, approaching 1 in 2, the fire at its point of maximum severity is unlikely to involve a high enough proportion of buildings simultaneously at their peak burning rates, for the fires to merge over extensive areas - that is these multiple fires would remain "group" fires.

APPENDIX I

The effect of allowing for delay between the ignition of a fire and its becoming able to ignite a neighbour

Baldwin has shown<sup>10</sup> that if the probability that a building has been ignited but is not yet capable of spreading fire is  $s$  and  $\nu$  is the probability per unit time that such a fire becomes capable of spreading fire then equations (4A) and (4B) are replaced by

$$\frac{ds}{dt} = Nq'pa - \nu s \quad 1.1$$

$$\frac{da}{dt} = -Nq'pa \quad 1.2$$

$$\frac{dp}{dt} = \nu s - \mu p \quad 1.3$$

so that  $d(p + s + a) = \frac{\nu}{Nq'} \frac{da}{a}$  1.4

Equation (8) remains valid if  $p$  in equation (8) is replaced by  $s + p$ ,  $p_0$  is replaced by  $s_0 + p_0$ ; and  $q$  by  $q'$ .  $s + p$  is a maximum when  $a$  is  $\nu/Nq'$ . This change does not otherwise alter equation (9). Since the chance of a fire spreading during the lifetime of a fire is  $q/\mu$  or  $q'/\mu'$  in both versions of the model, there is no difference in the estimate of damage - only in the time scale for the growth of the fire. This is slower if allowance is made for a finite period between ignition and flash-over.

$q/\mu$  or  $q'/\mu'$  are both the probability that fire spreads during the period when the burning fire is capable of spreading and these are, by this definition, the same - viz the fraction of buildings near enough to a neighbour to allow spread.

Another approach is to replace  $pa$  in the first term on the right-hand side of equation (4A) by its value at a time  $t_0$  before, where  $t_0$  is taken as a constant delay between ignition and the onset of a fully-developed fire.

An approximation is thus:

$$\frac{dp}{dt} = Nq \left( pa - t_0 \frac{d}{dt} (pa) \right) - p$$

$$\frac{da}{dt} = -Nq \left( pa - t_0 \frac{d(pa)}{dt} \right)$$

from which, on eliminating  $t$ , we obtain

$$\left( \frac{a_{\infty}^{k+1} - a_{\infty}^{k+1}}{1 + 2\mu t_0} \right) = \frac{Nq}{Nq(\mu t_0)} (a_{\infty}^k - a_{\infty}^k)$$

where

$$k = \frac{\mu t_0}{1 + \mu t_0}$$

$\mu t_0$  is unlikely to be more than  $\frac{1}{3}$ . In the limiting case  $p_0 = 1 - a_0 \rightarrow 0$ .

$$\frac{1 - a_{\infty}^{k+1}}{1 + 2\mu t_0} = \frac{(1 - a_{\infty}^k)}{Nq(\mu t_0)}$$

For  $\mu t_0 \sim \frac{1}{3}$  and  $a_{\infty} \sim \frac{1}{2}$  there is barely a 1 per cent change in the value 1.38 of  $Nq/\mu$  and the effect on the damage  $1 - a_{\infty}$  of this modification is negligible in the region of immediate interest.

Eliminating  $s$  from equations (1.1), (1.2) and (1.3) gives:

$$\frac{Nq \mu d^2 p}{(\mu + \nu)^2 d \left( \frac{\mu t}{\mu + \nu} \right)^2} + d \left( \frac{-\mu t}{\mu + \nu} \right) = \frac{Nq pa}{\mu} - p$$

If either  $\mu/\nu \ll 1$  or  $\gg 1$  the first term is of little consequence and the equation is identical with the original model except that the real time scale is altered to  $t \times (1 + \mu/\nu)$  which is roughly the real time scale with the mean time delays of fire growth removed.



APPENDIX 2

If  $\beta$  and  $a$  vary with position, one must alter the "Nq pa" term in equations (4) to allow for the difference in the probability,  $\beta$  with direction, and so re-write them, to a first approximation, as

$$-\frac{\partial a}{\partial t} = \frac{\partial \beta}{\partial t} + \mu \beta = 4q a \left\{ \beta + \frac{d^2}{4} \frac{\partial \beta}{\partial x^2} \right\}$$

where  $N$  is taken here as 4.

This result is derived using the arguments of Albin and Rand and putting  $q$  in their expression for the probability that a given block catches fire in a given time as  $q/\mu \cdot \delta t$  and neglecting terms in  $(\delta t)^2$

One way of deriving the effective steady rate of spread

$$R = - \left( \frac{\partial \beta}{\partial t} \right) / \left( \frac{\partial \beta}{\partial x} \right)_c$$

is to consider the solution in the region where  $a \rightarrow 1$

where it can be shown that there is a solution

$$\beta = e^{(4q - \mu)t} \cdot \phi$$

where  $\phi$  is a solution of the basic diffusion equation

$$\frac{\partial \phi}{\partial t} = q e^2 \frac{\partial^2 \phi}{\partial x^2}$$

of which the most powerful term in  $\phi$  is

$$e^{-x^2/4e^2qt}$$

If the combined index of the exponential is kept constant at zero so that  $\beta$  is only a weak function of  $x$  and  $t$

one gets

$$(4q - \mu)t = \frac{x^2}{4e^2qt}$$

$$\text{i.e. } R = \frac{x}{t} = 4e q \left(1 - \frac{\mu}{4q}\right)^{1/2}$$

If in the original equation with  $a \neq 1$  one seeks a solution of the kind  $\beta(x - Rt)$  one can obtain, following Kendall<sup>11</sup>, a solution for  $R$

provided

$$R \geq 4e q \left(1 - \frac{\mu}{4q}\right)^{1/2}$$

Waves faster than the minimum are possible but are perhaps not stable.

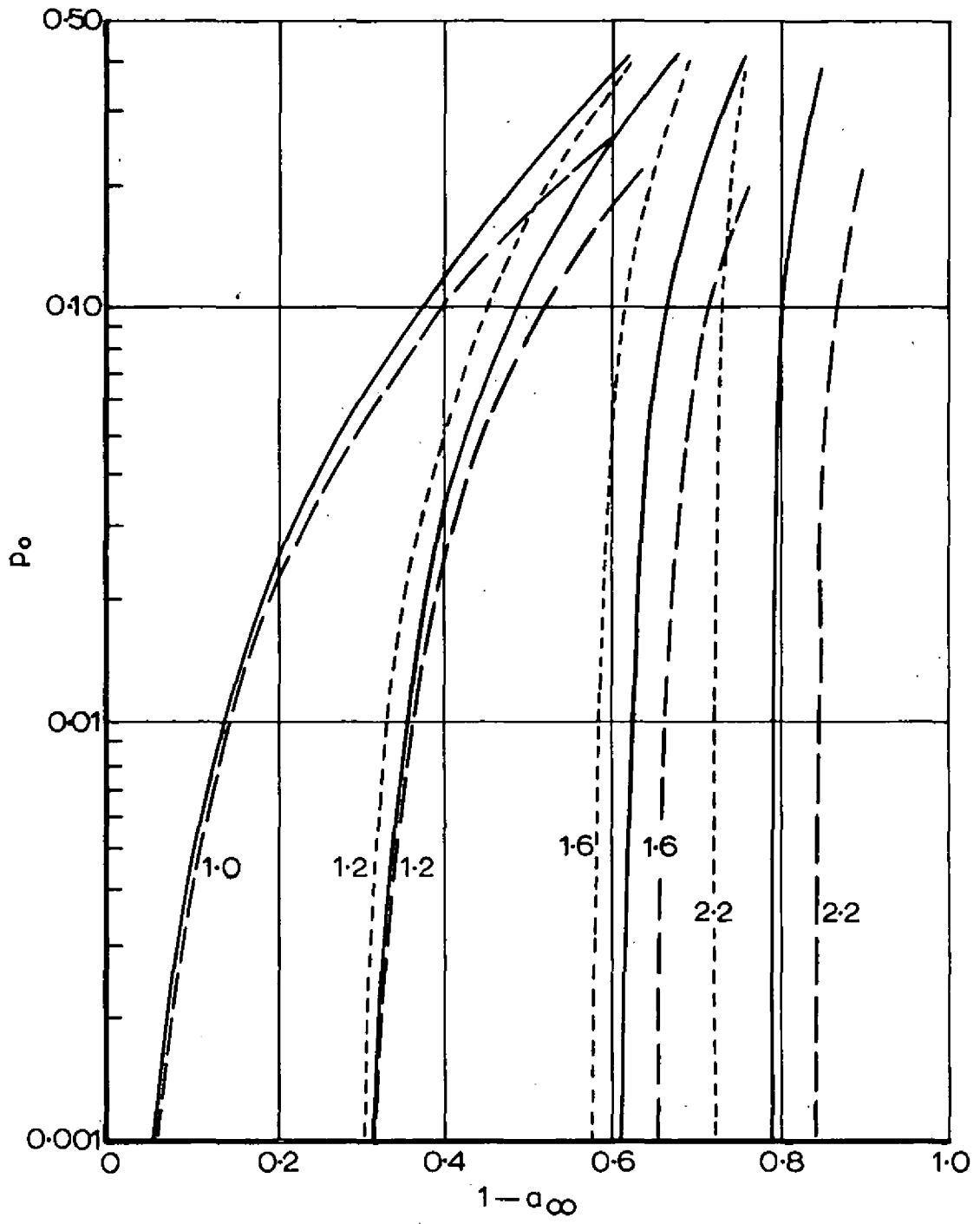
### Acknowledgment

I wish to thank M. North for computing solutions to Equations (4A) and (4B).

### REFERENCES

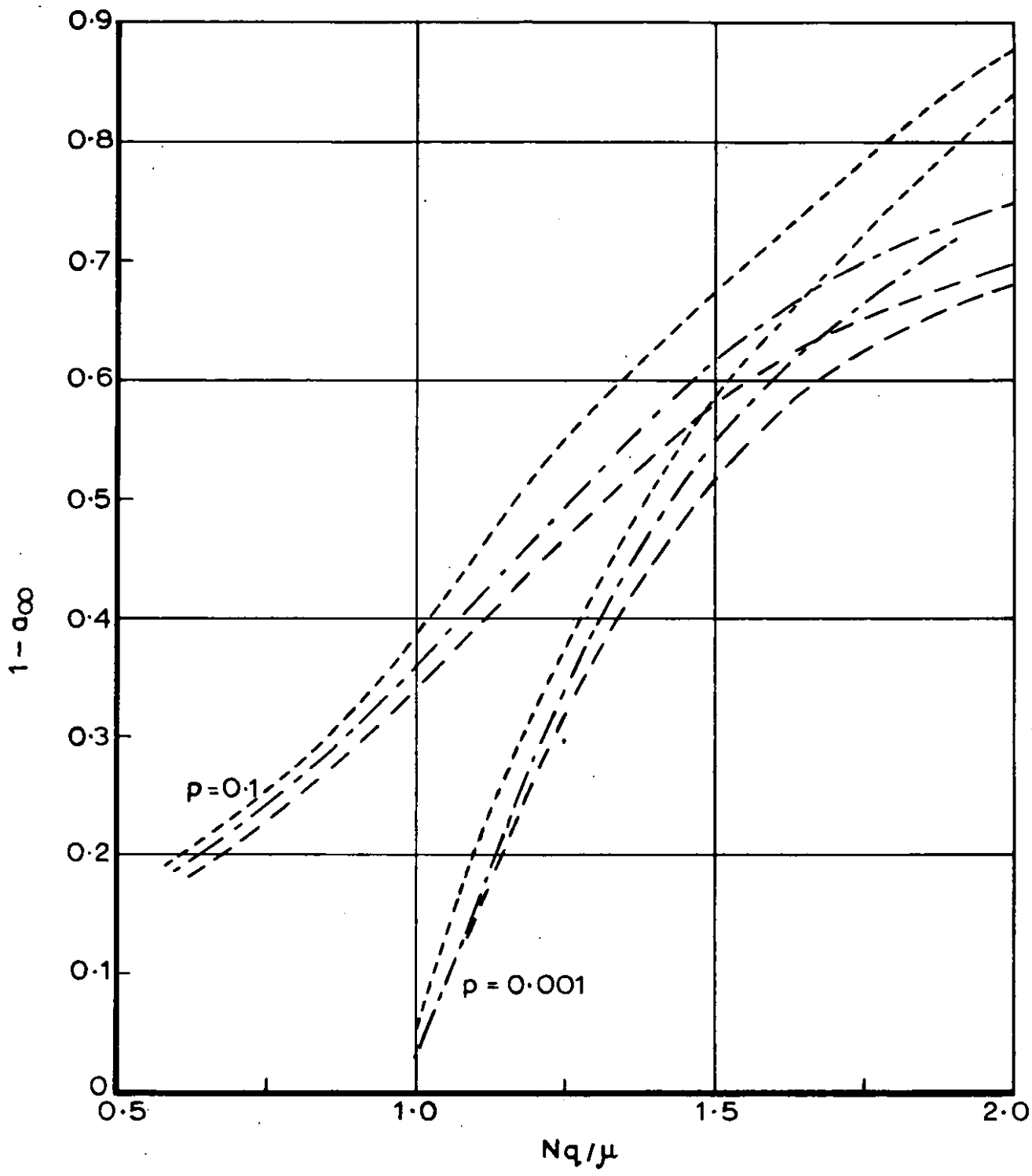
1. BALDWIN, R., THOMAS, P. H. and WRAIGHT, H. G. H. The merging of flames from separate fuel beds. Joint Fire Research Organization F. R. Note 551/1964.
2. BALDWIN, R. Some tentative calculations of flame merging in fires. Joint Fire Research Organization F.R.Note 629/1966.
3. STANBURY, G. Ignition and fire spread in urban areas following a nuclear attack. Home Office Scientific Adviser's Branch Report CD/SA 121. London, 1964.
4. ALBINI, F. A. and RAND, S. Statistical considerations on the spread of fire. Institute for Defence Analyses R & E Support Division. Washington, 1964.
5. KERMAK, W. O. and MCKENDRICK, A. G. Proc. Roy.Soc.A., 1927, 115 700 - 21.
6. PHUNG, P. D. and WILLOUGHBY, A. B. Prediction models for fire spread following nuclear attacks. United Research Services URS 641-6. Burlingame, California, 1965.
7. HISHIDA, K. Estimation of fire risk (district rate). Fire and Marine Insurance Rating Association of Japan. 1954.
8. LOMMASSON, T. E. Preliminary investigation of fire-storm criteria. Dikewood Corporation DC - TN - 1050 - 1. Albuquerque, Mexico, 1965.

9. STANBURY, G. R. The risk of fire from air attack. Home Office Scientific Adviser's Branch CD/SA 8
10. BALDWIN, R. Personal communication.
11. KENDALL, D. G. Mathematical models of the spread of infection. Proceedings of a Conference on Mathematics and Computer Science in Biology and Medicine. London, 1965. H. M. Stationery Office.



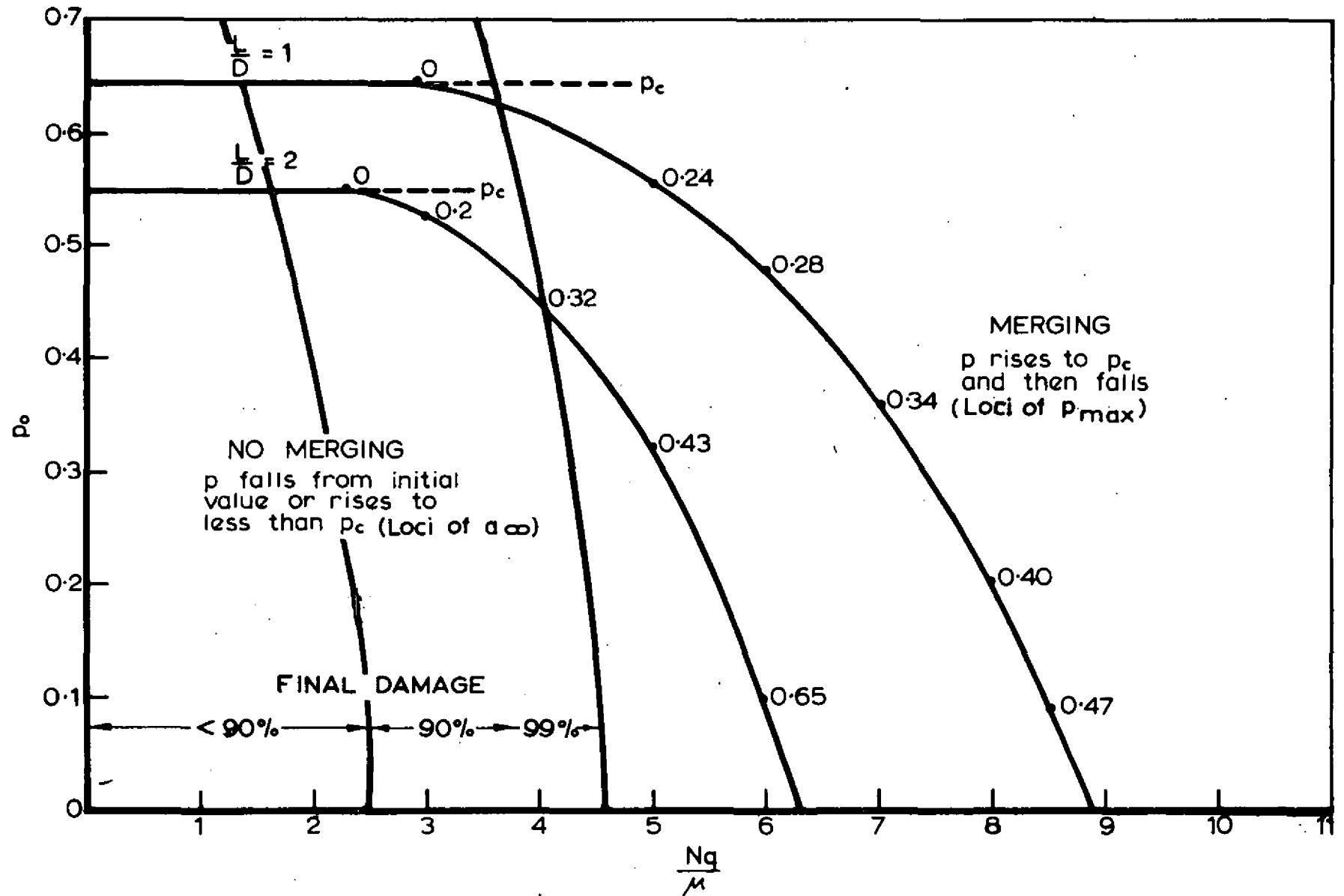
———— Upper and lower approximations  
 - - - - - used by Albini and Rand ( $\mu=1$ )  
 - - - - - Equation (9)  
 Numbers refer to value of  $Nq/\mu$

FIG.1. COMPARISON OF MODELS



----- Differential model (equation 4)  
 - - - - - Upper and lower approximations for Albini-Rand model

FIG.2. COMPARISON OF MODELS



The figures beside the curves are values of  $\mu t$  at the peak value of  $p$  viz 0.64 or 0.5

FIG.3. CONDITIONS FOR REACHING A MERGED FIRE

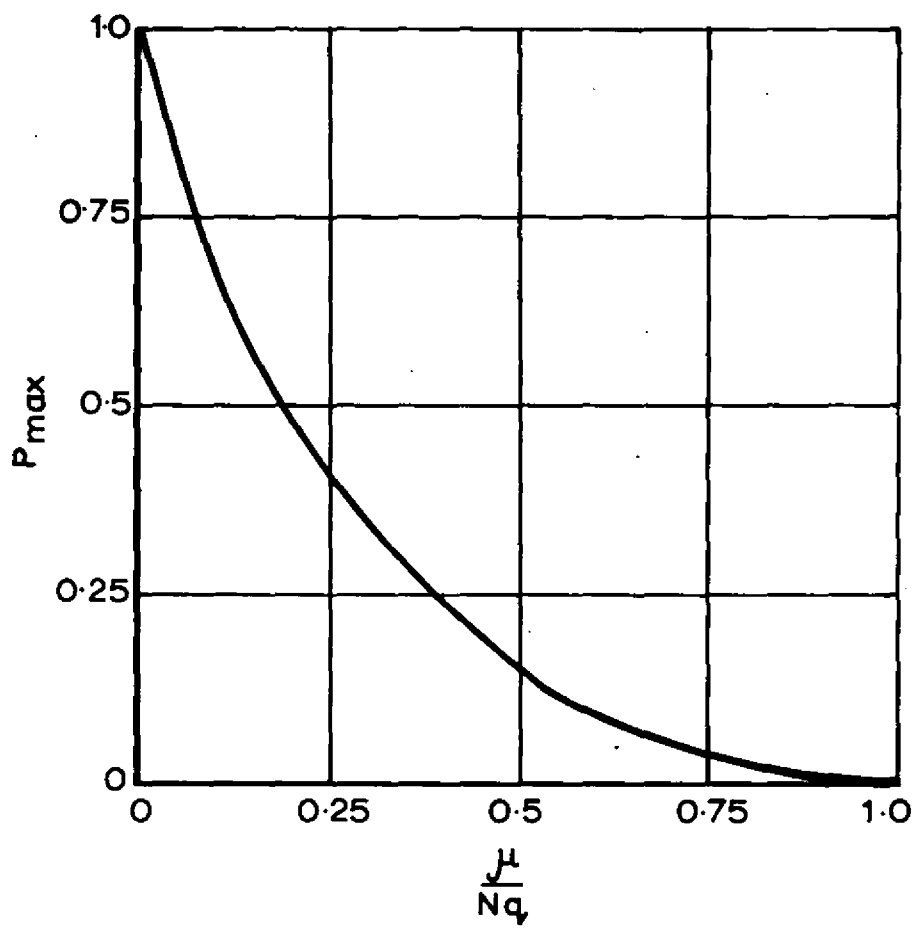


FIG-4.  $p_{max}$  FOR VERY SMALL  $p_0 (< 0.001)$

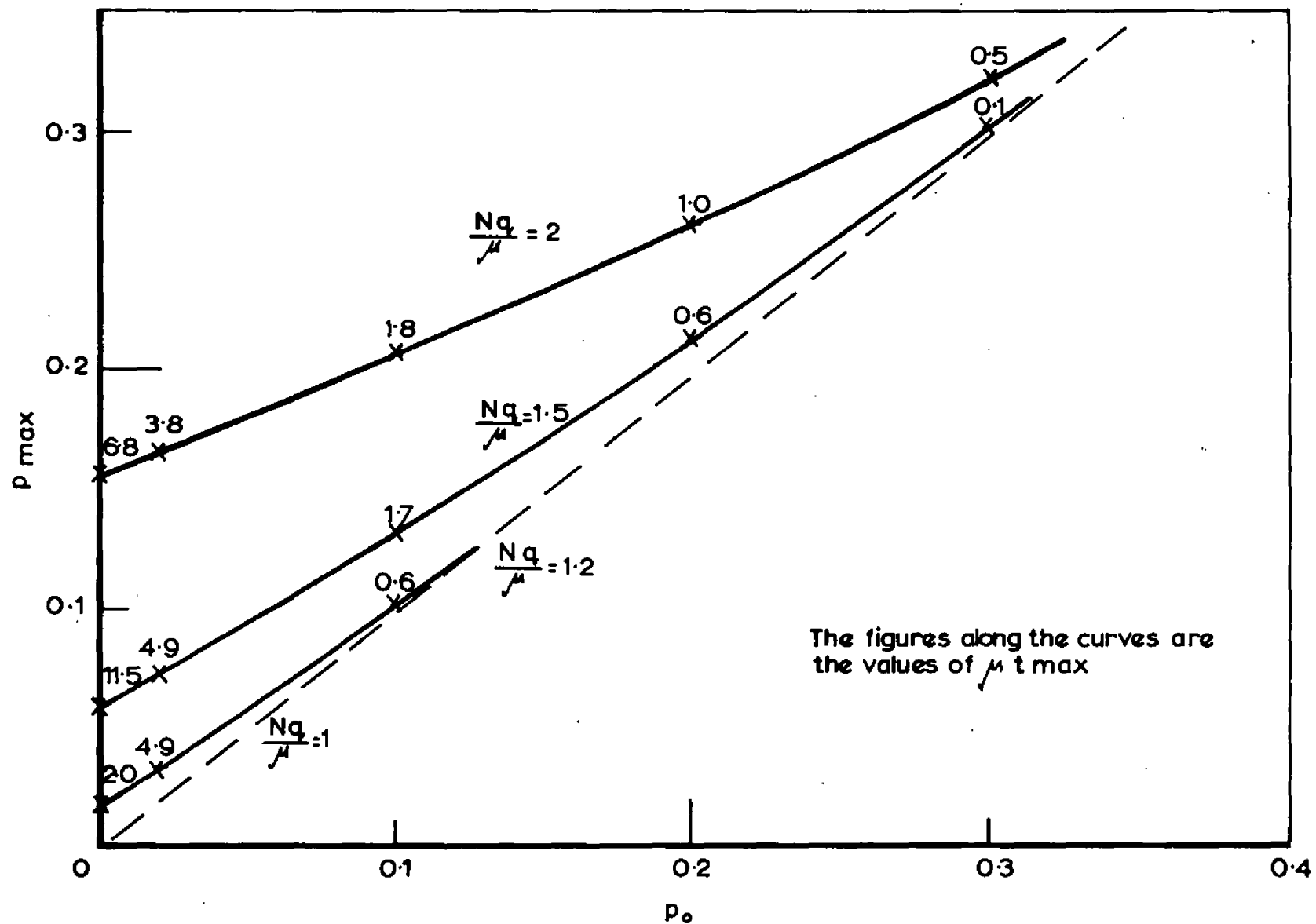


FIG.5. THE MAXIMUM p



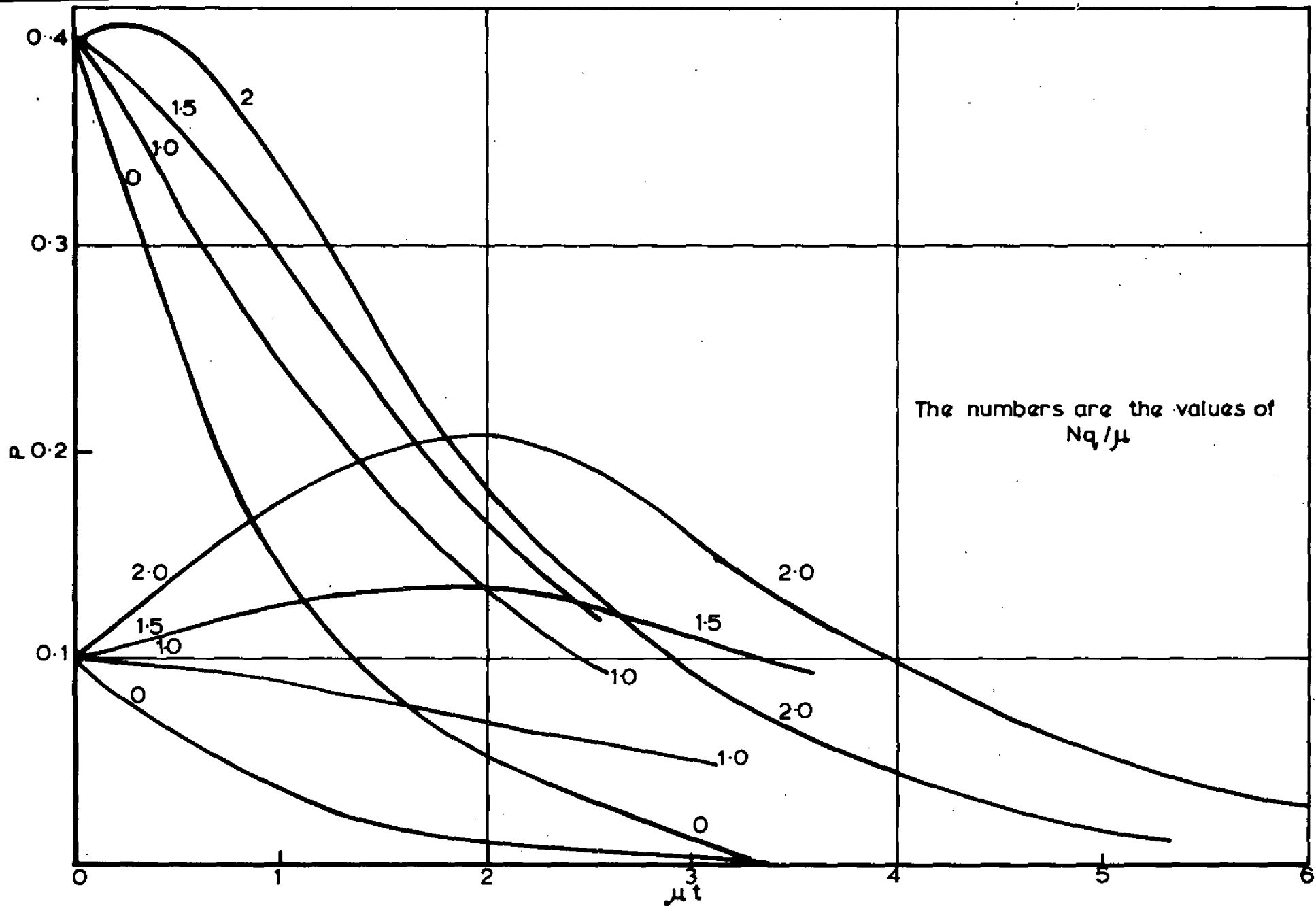
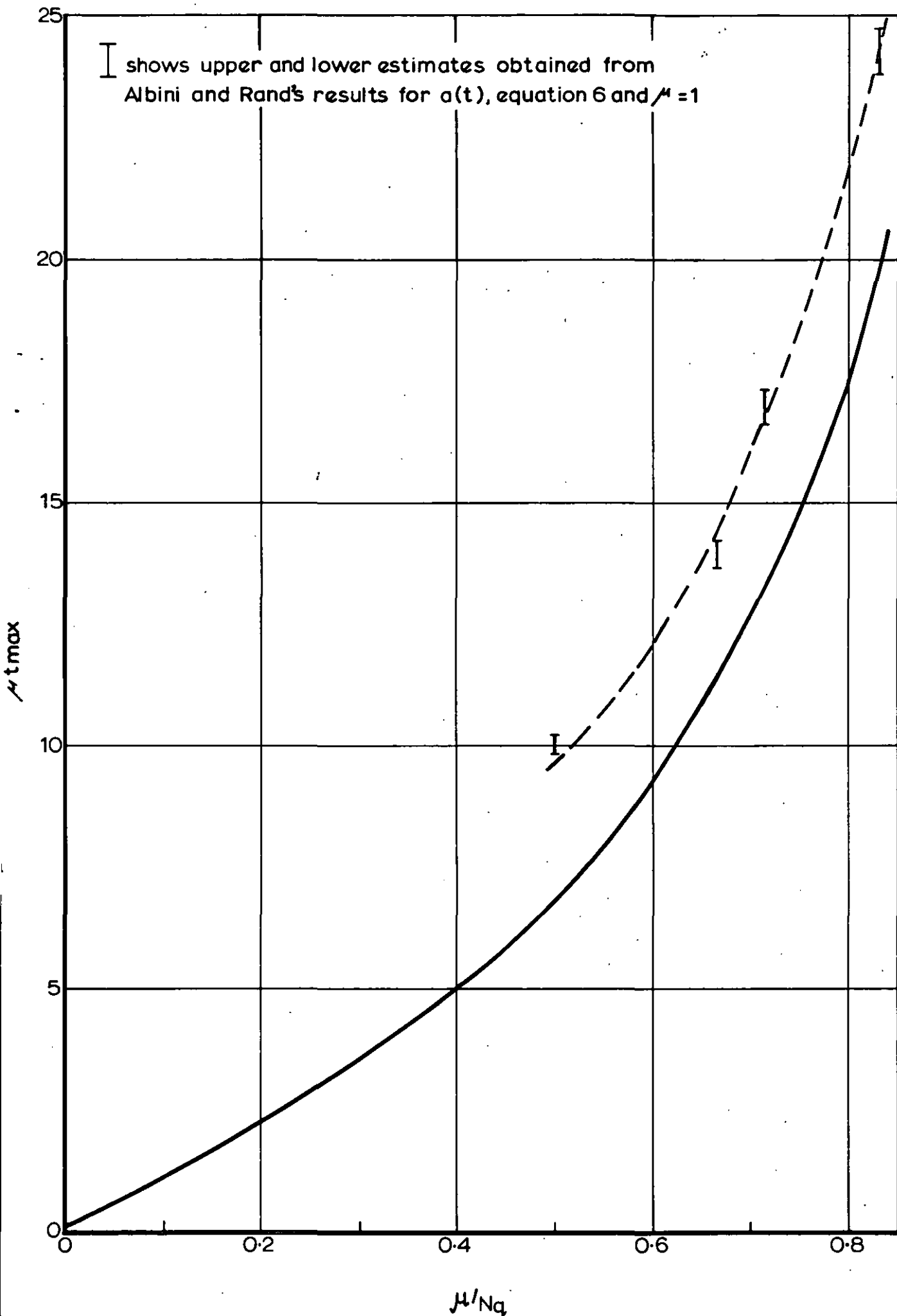


FIG.6. THE VARIATION OF  $p$  IN TIME  $p_0 = 0.1$  and  $0.4$



I shows upper and lower estimates obtained from Albini and Rand's results for  $a(t)$ , equation 6 and  $\mu = 1$

FIG 7 TIME TO MAXIMUM  $p$   
( $p_0 = 0.001$ )

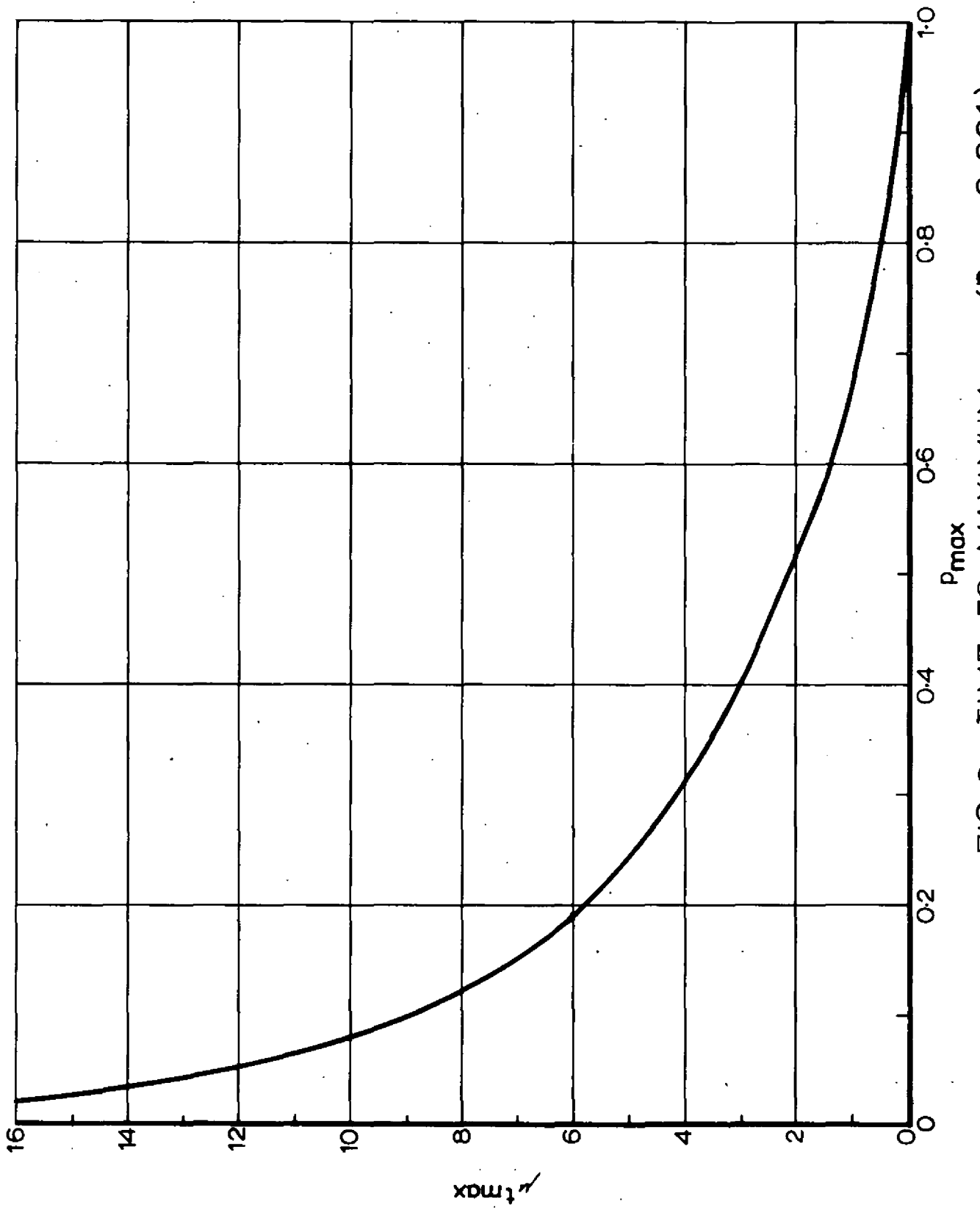


FIG. 8. TIME TO MAXIMUM  $p$  ( $p_0 = 0.001$ )

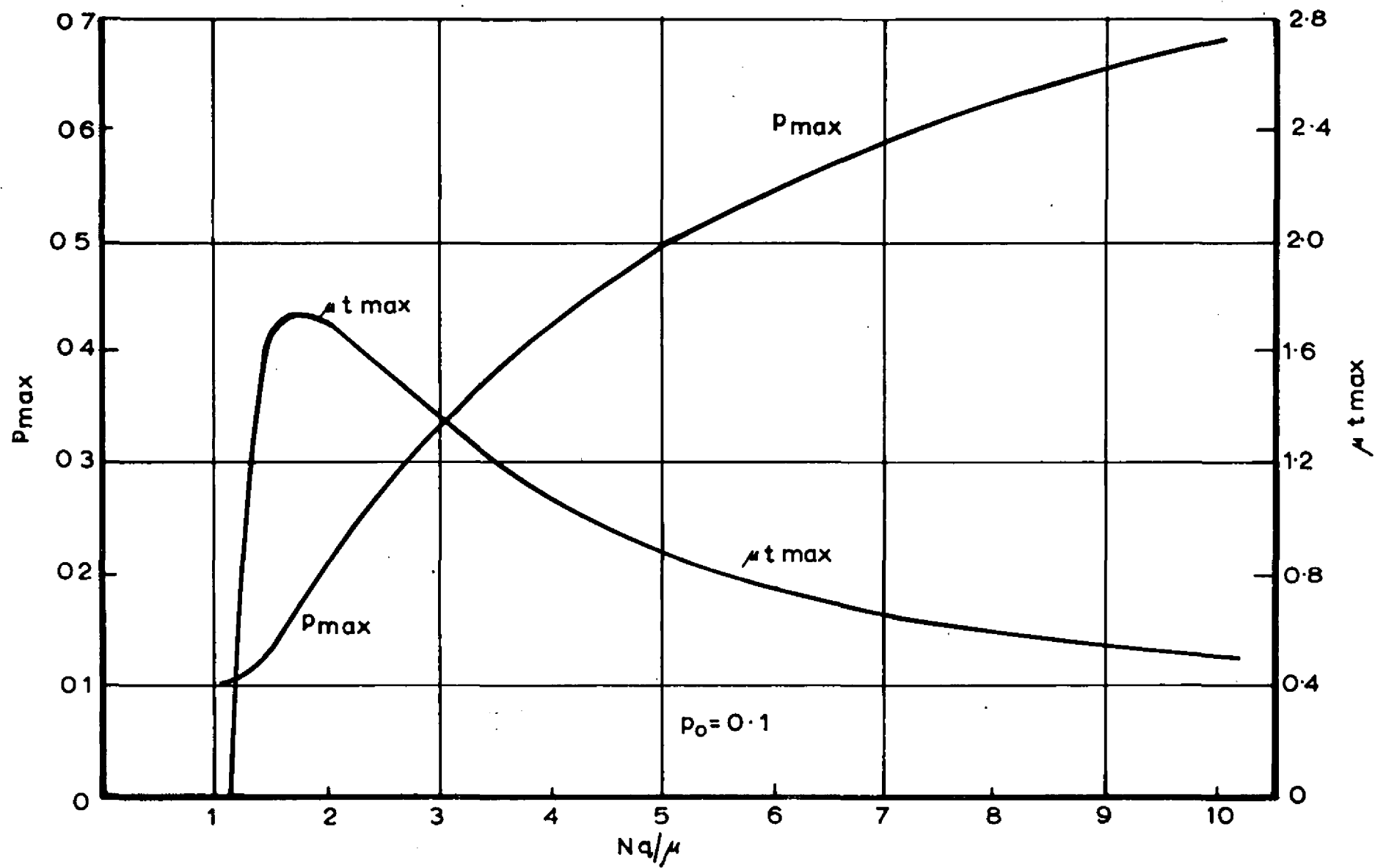


FIG.9. CONDITIONS AT MAXIMUM  $p$  ( $p_0=0.1$ )

