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ON THERMAL THEORIES OF FIRE SPREAD ALONG THIN MATERIALS

by

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SUMMARY

Flame spread along a thin material or along the surface of a thick material is one of the features of fire spread, and, with the possible exception of the calculation of radiation transfer across gaps between combustible materials, the one which has attracted perhaps the most theoretical attention.

In such theories account must be taken of the energy balance and, although there are conditions when this is not sufficient to account for all features of the spread e.g. the generation of flammable gases can sometimes be a critical factor, the heating of the unburnt fuel ahead of the fire is frequently the controlling factor. In discussing this heating theoretical problems sometimes arise and this paper refers to some of those in the theory of smouldering of Kinbara, Endo and Sega[1].

KEY WORDS: Flame, spread, solids, theory.

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INTRODUCTION

Kinbara, Endo and Sega[1] have experimentally demonstrated certain relationships between the velocity of smouldering down strips of cardboard and have successfully correlated their data in terms of an equation based on the heating of the unburnt preheated material ahead of the fire, with the heat release treated as a disposable constant.

The theory of Kinbara et al

They wrote the heat balance equation for the material ahead of the fire front as:

$$\frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{k \partial t} - \alpha (T - Ta) \qquad (1)$$

where α is $\frac{P}{aK}\left(\frac{q}{Ti - Ta} - h\right)$, $T - Ta$ is the temperature rise above ambient, P is the perimeter of the strip, a its cross-section, x is a distance ahead of and normal to the fire front (stationary co-ordinate), K is the thermal conductivity, ρ is the density, c is the specific heat, $k = K/\rho c$, t is time, h is the surface cooling coefficient, and q is the rate of heat generation per unit peripheral surface per unit temperature rise above ambient.

The effective perimeter was defined around the boundary layer and the rate of heat generation per unit cross-sectional area was taken as $\frac{P\,q\,(T-Ta)}{a\,(Ti-Ta)}$, presupposing some heat generated at comparatively low temperatures. They did not make any assumptions about the region behind the fire front.

This feature presents some difficulties, which this paper comments on and explores.

Presuming a constant rate of spread v and writing z equal to $x - vt$, equation (1) becomes

$$\frac{\partial^2 T}{\partial z^2} + \frac{v}{k}\frac{\partial T}{\partial z} + \alpha (T - T_A) = \left(\frac{\partial T}{\partial t}\right)_z \qquad (2)$$

with z a moving co-ordinate

for which $$T - T_A = A e^{-\beta_1 z} + B e^{-\beta_2 z} \qquad (3)$$

where $$\beta_1, \beta_2 = -\frac{v}{2k} \pm \sqrt{\frac{v^2}{4k^2} - \alpha}$$

is a solution, (steady state $(\frac{\partial T}{\partial t})_z = 0$) but one which presupposes an infinite T behind the fire front. Kinbara et al then assume that there is a point ahead of the fire where the temperature just begins to rise and where $T = T_A$ and $\frac{dT}{dz} = 0$. They then argue that though equation (3) cannot be solved exactly the solution must be approximated by $\beta_1 = \beta_2$ and hence they obtained v as $2k\sqrt{\alpha}$.

With $\beta_1 = \beta_2$ the solution to equation (2) is

$$T - T_A = (C + Dx)\, e^{-\frac{v}{2k}z}$$

This certainly can give $T = T_A$ at some positive x but then $\frac{dT}{dx}$ is finite and if $T - T_A$ is zero ahead of this point there is a discontinuity in $\frac{dT}{dx}$ (a heat sink) albeit a small one.

It is clear that $2k\sqrt{\alpha}$ is a minimum value for v for a real solution but not that it is the only possible value of v.

Heat sources moving with an imposed velocity

This lack of uniqueness in v can be demonstrated by selecting various boundary conditions or various conditions along the negative x axis. Firstly, consider a disc source of heat travelling along an infinite uniform rod in which all the other conditions prescribed by Kinbara et al are satisfied. We then liberate Q units of heat per unit section per second.

Equation (2) is satisfied by

$$\theta = e^{(\alpha - \frac{v^2}{4k^2})kt}\, e^{-\frac{v}{k}z}\, w(z, t)$$

where w satisfies the equation

$$\frac{\partial^2 w}{\partial z^2} = \frac{1}{k}\frac{\partial w}{\partial t}$$

and, initially $\int_{-\infty}^{\infty} w . dz$ is $\frac{Q\delta t}{\rho c}$ at $z = 0$ and zero elsewhere.

Employing the standard solution for a plane instantaneous source and integrating over all real time we have when $t \to \infty$

$$\Theta = \frac{Q\, e^{-\frac{v}{2k}z}}{\rho c\, 2\sqrt{\pi k}} \int_0^{\infty} e^{-\left\{\left(\frac{v^2}{4k^2} - \alpha\right)ku + \frac{z^2}{4ku}\right\}} \frac{du}{u^{1/2}}$$

We employ the substitution

$$u = \frac{y^2 \, |z|}{2k\left(\frac{v^2}{4k^2} - \alpha\right)_+^{\frac{1}{2}}}$$

where the positive sign denotes that the positive real root of $\left(\frac{v^2}{4k^2} - \alpha\right)^{\frac{1}{2}}$ is taken. This leads to

$$\Theta = \frac{Q\, |z|^{1/2}\, e^{-\frac{v}{2k}z}}{K\sqrt{2\pi}\left(\frac{v^2}{4k^2} - \alpha\right)_+^{1/2}} \int_0^{\infty} e^{-N\left(y^2 + \frac{1}{y^2}\right)} dy$$

where $N = \left(\frac{v^2}{4k^2} - \alpha\right)_+^{\frac{1}{2}} \frac{|z|}{2}$

This leads to

$$\Theta = \frac{Q}{2K} \frac{e^{-\frac{v}{2k}z + \left(\frac{v^2}{4k^2} - \alpha\right)_+^{1/2}|z|}}{\left(\frac{v^2}{4k^2} - \alpha\right)_+^{1/2}}$$

i.e. $$\Theta_o = \frac{Q}{2K\left(\frac{v^2}{4k^2} - \alpha\right)^{\frac{1}{2}}}$$

where Θ_o is the temperature rise at the heat source itself

Equation (4) is of the same form as the solution by Kinbara et al except that β_1 only applies when $z > 0$ and β_2 only when $z < 0$, thereby giving infinite temperatures on the negative z axis.

If behind the fire q is zero we replace α by $-h$ in $z < 0$ and choose $Q = Q_1$ for the forward half and Q_2 for the rear half of the rod and, from the values of the fluxes at $z = 0$, obtain the values of Q_1 and Q_2 $(Q_1 + Q_2 = Q)$. This gives

$$\frac{Q}{\Theta_o} = \left(\frac{v^2}{4k^2} - \alpha\right)^{\frac{1}{2}}_{+} + \left(\frac{v^2}{4k^2} + h'\right)^{\frac{1}{2}} \qquad (5)$$

where Θ_o is the temperature rise at the heat source itself and $h' = \frac{P\,h}{a\,K}$. If Q is given, Θ_o is a function of a 'v' which can be imposed on the system.

If a rod satisfying equation (2) is driven at velocity v $(> 2k\sqrt{\alpha})$ into an infinite sink at a given temperature various steady states are possible according to the value of v.

Another model would be that the heat release is zero everywhere except at a thin zone (defining $z = 0$) in which case we have

$$\frac{Q}{K\Theta_o} = 2\left(\frac{v^2}{4k^2} + h'\right)^{\frac{1}{2}} \qquad (6)$$

Kinbara et al, by not discussing the rear of the fire neglect, in effect, the term in h' in equation (5) and if Θ_o is finite and the only heat generated is expressed in q so that Q is zero, we obtain

$$\left(\frac{v^2}{4k^2} - \alpha\right) = 0 \quad \text{which is their result.}$$

Their physical arguments leading to the choice of the heat release being proportional to T-Ta are that the heat release is controlled by the air availability and hence its velocity and this they took, for simplicity, as proportional to T-Ta. For lamina flow $(T-Ta)^{\frac{1}{4}}$ is probably more appropriate and for turbulent flow $(T-Ta)^{\frac{1}{3}}$. Either way, a heat release independent of T is as good as a linear approximation, i.e. putting the heat generation in Q - not in q, so that equation (6) is a possible alternative model. Equation (6) can be rearranged to give

$$\frac{v^2}{4k^2} = \left(\frac{Q}{2K\Theta_o}\right)^2 - h' \qquad (6A)$$

$$\Theta = \Theta_o \, e^{-\left(\frac{v}{2k} + \left(\frac{v^2}{4k^2} + h'\right)^{\frac{1}{2}}\right) z} \qquad (z > 0) \qquad (7)$$

Θ_o and Q are here treated as given parameters between which some other relationship may exist (e.g. one derivable from combustion considerations).

If the heat source is not a moving disk source but is extended over an indefinite region at the rear, equation (7) can be integrated and the result is then obtained in terms of Q_v the heat release per unit volume.

$$\Theta_o = \frac{Q}{2K} \frac{1}{\left[\frac{v}{2k} + \left(\frac{v^2}{4k^2} + h'\right)^{\frac{1}{2}}\right]\left[\frac{v^2}{4k^2} + h'\right]^{\frac{1}{2}}}$$

The equation can be rearranged to give

$$\frac{v^2}{4k^2 h'} = \frac{(1-\psi)^2}{\psi(2-\psi)} = \frac{1}{\psi(2-\psi)} - 1 \qquad (8)$$

where $\psi = \dfrac{2h' \Theta_o K}{Q} = 2h\dfrac{(T - T_A)}{q}$

in Kinbara et al's notation, where we have taken

$$\Theta_o = T_i - T_A \quad \text{and} \quad Q_v = \frac{P\,q}{a} .$$

Comparison of theories

In this notation Kinbara et al's result is

$$\frac{v^2}{4k^2 h'} = \frac{2}{\psi}\left(1 - \frac{\psi}{2}\right) = \frac{2}{\psi} - 1 \qquad (9)$$

ψ was treated as a disposable constant and q estimated as 0.11 cal $cm^{-2}\,s^{-1}$.

The range of values of Ta employed by Kinbara et al was 60°C to 150°C

Ti is given as 460°C and $^h/q$ as 1.55×10^{-3} $^oC^{-1}$ so ψ varies only from 0.96 to 1.24

Given the good agreement between their experiments and equation (9) their experimental data covered the relatively narrow range 0.78 to 1.04 in the dimensionless velocity $\frac{v}{2k\sqrt{h'}}$ and in this range it is possible to find a ψ in equation (8) which gives as good a fit, though a model with a finite zone of heating may be better

For this q needs to be taken as 0.37 cal cm^{-2} s^{-1}.

The value of q

Leaving aside the question of the use of a boundary layer correction in the effective perimeter P, some significance might be attached to q.
The value of h recorded by Kinbara et al is 1.5×10^{-4} cal cm^{-2} s^{-1} $^oC^{-1}$ and the rate of mass transfer to a surface, by the Reynolds Analogy, is thus roughly

$$m'' = \frac{h}{c} = \frac{1.5 \times 10^{-4}}{0.57} \quad gm \; cm^{-2} \; s^{-1}$$

A heat release of ΔH per unit of mass transferred gives

$$q = \frac{\Delta H \times 1.5 \times 10^{-4}}{0.57} \quad cal \; cm^{-2} \; s^{-1}$$

For $q = 0.11$ cal cm^{-2} s^{-1}, $\Delta H \sim 400$ cal/gm, and for $q = 0.37$ cal cm^{-2} s^{-1} $\Delta H \sim 1400$ cal/gm. The latter value is approaching the order of magnitude expected for the combustion of cellulosic fuels; in a comprehensive theory loss by radiation, which may double this figure, would have to be allowed for.

The relation to a thermal model for upward spread.

If the heat transfer to the unburnt fuel is high and the speed v also high ψ becomes small and

$$\frac{v^2}{4k^2 h'} \propto \frac{1}{\psi} \qquad (10)$$

This can be rewritten as

$$v = 2k \sqrt{\frac{Q_{\upsilon}}{2\theta_o K}} \qquad (11)$$

But this is in terms of θ_o at the front end of the heating zone. If heat were supplied from the sides (e.g. by convection as in spread upward along a fabric, the heat may be supplied in front of the point where the material first ignites and it is then more appropriate to consider the temperature at the rear of the heated zone. Integrating equation (4) with Q replaced by Q_v over a finite zone l and obtaining the temperature at the rear of the zone gives

$$\theta_R = \frac{Q_v}{2K} \frac{1 - e^{-\left\{\left(\frac{v^2}{4k^2} + h'\right)^{\frac{1}{2}} - \left(\frac{v}{2k}\right)\right\} l}}{\left[\left(\frac{v^2}{4k^2} + h'\right)^{\frac{1}{2}} - \left(\frac{v}{2k}\right)\right]\left[\frac{v^2}{4k^2} + h'\right]^{\frac{1}{2}}}$$

which for large values of $\frac{v^2}{4k^2 h'}$ tends to

$$\theta_R = \frac{Q_v l}{\rho c v} \left(1 - \frac{P h}{v a \rho c}\right)$$

$$\text{i.e. } \frac{\rho c v \theta_R}{Q_v l} = 1 - \frac{P h \theta_R}{Q_v} \qquad (12)$$

$$\approx 1 \text{ for large values of } \frac{a Q_v}{P h \theta_o}$$

This is the equation deducible directly from a heat balance of the material ahead of the heat source neglecting thermal conduction along the material, e.g. putting $K = 0$.

This last result is important in that for materials burning slowly controlled by conduction into the cold material equation (11).

$$v \propto \sqrt{Q_v}$$

and for materials burning quickly controlled by the capacity of the cold material equation (12)

$$v \propto Q_v$$

Since $Q_v \propto P/\lambda$, for a constant heat transfer rate to thin materials of varying thickness, the latter is in accordance with the well known result that rate of spread is inversely proportional to the weight per unit area for strips of a given material burning upwards.

Diathermancy

Cellulosic materials are not entirely opaque to radiation so allowance may have to be made for this.

If the radiation attenuation law within the solid is $e^{-\sigma z}$ the radiation transfer may be neglected if, on comparing this profile with equation (7)

$$\sigma \gg \frac{v}{2k} + \sqrt{\frac{v^2}{4k^2} + h'}$$

i.e. $$\frac{\sigma}{\sqrt{h'}} \gg \left(\frac{2-\psi}{\psi}\right)^{\frac{1}{2}}$$

$$\frac{\sigma}{\sqrt{h'}} \gg \sqrt{\frac{2\ Q_v}{\Theta_o K}}$$

Since σ is of order 10 - 100 cm^{-1} for wood, this sets a minimum thickness of order 0.1 mm below which diathermancy may have to be allowed for when the fuel is smouldering.

Conclusions

The model employed by Kinbara et al appears to be one of several possible thermal models, others of which might be expected to give as good a fit if q is left as a disposable constant, and some effort to accommodate an evaluation of q into a theory is necessary before any one model can be said to be fully satisfactory.

REFERENCE

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