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DEPARTMENT OF SCIENTIFIC AND INDUSTRIAL RESEARCH AND FIRE OFFICES' COMMITTEE
JOINT FIRE RESEARCH ORGANIZATION

FURTHER COOLING CORRECTIONS FOR THE COPPER DISC RADIOMETER

by

D. L. Simms and M. Law

Summary

The intensity of radiation falling on a copper block has been derived in terms of the transient temperature rise assuming:

(1) Newtonian cooling from the surfaces (1).

(2) A more exact cooling correction given by Hampton (2).

With the apparatus designed the two solutions are identical for heating periods up to two minutes.

Another method is given for calculating the intensity of radiation falling on the copper block.

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1. By assuming Newtonian cooling, Lawson [1] has derived the intensity of radiation falling on a copper block in terms of the transient temperature rise (Fig. 1). The solution obtained may be written

$$\theta = \frac{IC}{D\Psi}\left(1 - e^{-\frac{D\Psi}{Ms}t}\right) \qquad \ldots\ldots (1)$$

where M is the mass of the copper block
s is the specific heat
C is the area of the block exposed to radiation
D is the area of the block emitting heat
I is the intensity of the radiation
and Ψ is the Newtonian cooling constant.

The rise in temperature of the block during an experiment may be as high as 100°C above the ambient temperature. Newton's law of cooling is usually accepted as accurate for small temperature differences only. The error of using equation (1) may be estimated by comparing that solution with the one obtained using the more exact approximation of Hampton [2].

Assuming that the temperature of the block remains uniform throughout the experiment the transient temperature rise θ is given by

$$Ms\dot{\theta} + D\left(A\theta + B\theta^2\right) = IC \qquad \ldots\ldots (2)$$

where A and B are the constants given by Hampton.

Putting $\frac{DA}{Ms} = \alpha$, $\frac{DB}{Ms} = \beta$, $\frac{IC}{Ms} = \gamma$

and $\theta = \frac{1}{\beta u}\dot{u}$ so that $\dot{\theta} = -\frac{1}{\beta u^2}(\dot{u})^2 + \frac{1}{\beta u}\ddot{u}$

equation (2) becomes

$$\ddot{u} + \alpha\dot{u} - \beta\gamma u = 0 \qquad \ldots\ldots (3)$$

the subsidiary equation of which is

$$m^2 + \alpha m - \beta\gamma = 0$$

giving

$$m = \frac{-\alpha \pm \sqrt{\alpha^2 + 4\beta\gamma}}{2}$$

A solution to equation (3) is

$$u = Z e^{-\frac{\alpha t}{2}} \cosh\left(\frac{\sqrt{\alpha^2+4\beta\gamma}}{2}t + \phi\right)$$

Put $$n = \frac{\sqrt{\alpha^2+4\beta\gamma}}{2}$$

$$\therefore \dot{u} = Z e^{-\frac{\alpha t}{2}} n \sinh(nt+\phi) - Z e^{-\frac{\alpha t}{2}} \frac{\alpha}{2} \cosh(nt+\phi)$$

$$\therefore \theta = \frac{1}{\beta}\,\frac{Z e^{-\alpha t/2} n \sinh(nt+\phi) - Z e^{-\frac{\alpha t}{2}} \frac{\alpha}{2} \cosh(nt+\phi)}{Z e^{-\alpha t/2} \cosh(nt+\phi)}$$

$$= \frac{1}{\beta}\left[n \tanh(nt+\phi) - \frac{\alpha}{2}\right]$$

When $t = 0,\ \theta = 0, \quad \therefore \tanh\phi = \frac{\alpha}{2n}$

$$\therefore \theta = \frac{1}{\beta}\left[\frac{n \tanh nt + \frac{\alpha}{2}}{1 + \frac{\alpha}{2n}\tanh nt} - \frac{\alpha}{2}\right]$$

$$= \frac{\gamma}{n}\,\frac{\tanh nt}{1 + \frac{\alpha}{2n}\tanh nt} \qquad \ldots\ldots (4)$$

Equations (1) and (4) cannot be compared analytically. In Fig. 2, using the constants given in Appendix I, they are compared for two values of the intensity of radiation, I. The curves are identical within the limits of measurement.

2. <u>A further method of correcting for heat losses</u>

The rise in temperature of the copper block is given by equation (1).

The temperature of the block after being shielded from the radiation is given by

$$Ms\theta = D\Psi\theta$$

Substituting for Ψ in equation (1) gives

$$I = \frac{Ms}{c}\left(\frac{1}{1 - e^{+(\dot{\theta}/\theta)t}}\right) \qquad \ldots\ldots (5)$$

The mean value of Ψ can then be determined from the slopes of the tangents to the cooling curves at various temperatures. The curve given by expression (5) can also be computed and compared with the equivalent experimental one.

3. Conclusions

The transient temperature rise θ of a copper block has been calculated assuming cooling from the surface by Hampton's Law (2).

$$\theta = \frac{\gamma}{k}\left[\frac{\tanh nt}{1 + \frac{\alpha}{2n}\tanh nt}\right]$$

For periods of irradiation up to two minutes, this is identical with the transient temperature rise θ calculated assuming Newtonian cooling from the surface (1).

$$\theta = \frac{IC}{D\Psi}\left[1 - e^{-\frac{D\Psi}{MS}\cdot t}\right] \ldots\ldots (6)$$

Another method of calculating the intensity of radiation in terms of the transient temperature rise has been found. The value of , the Newtonian cooling constant, can be found from the slope of the cooling curve and by substituting in equation (6), the intensity of radiation, I may be calculated.

References

1. Lawson, D. I., Cooling corrections for the copper disc radiometer. Joint Fire Research Organization F.R. No. 69/1953.

2. Hampton, W. M., Approximate expression for loss of heat from exposed surfaces. Nature Lond. 1946, 157, P. 481.

Appendix I

Constants used in calculations

Mass of copper block		=	175 gm
Specific heat of copper block		=	0·092 cal/gm/°C
Area of block receiving radiation		=	13·8 cm^2
Total surface area of block emitting radiation		=	48·7 cm^2
Newtonian cooling constant	Ψ	=	2.8×10^{-4} cal/cm^2/sec/°C
Hampton's constants.	A	=	1.96×10^{-4} cal/cm^2/sec/°C
	B	=	1.71×10^{-6} cal/cm^2/sec/°C.

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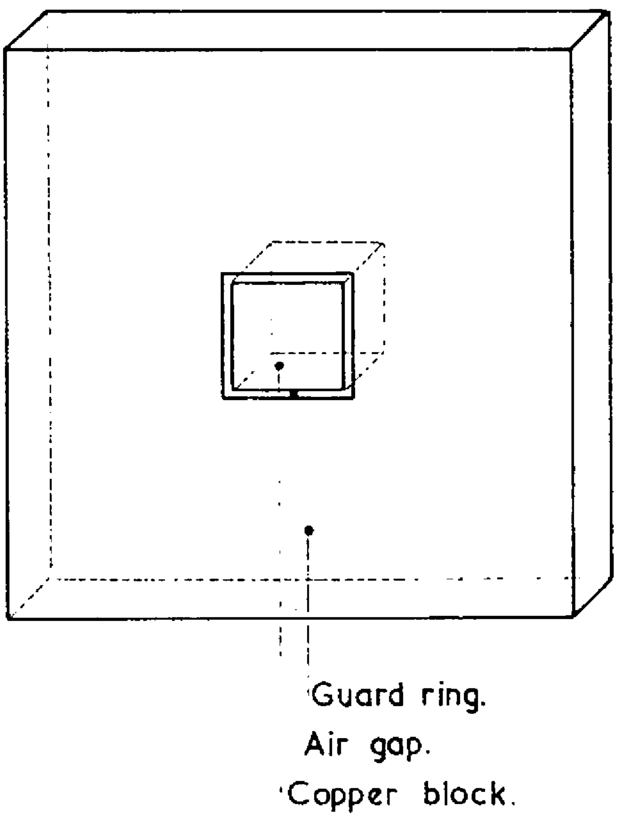

FIG 1.

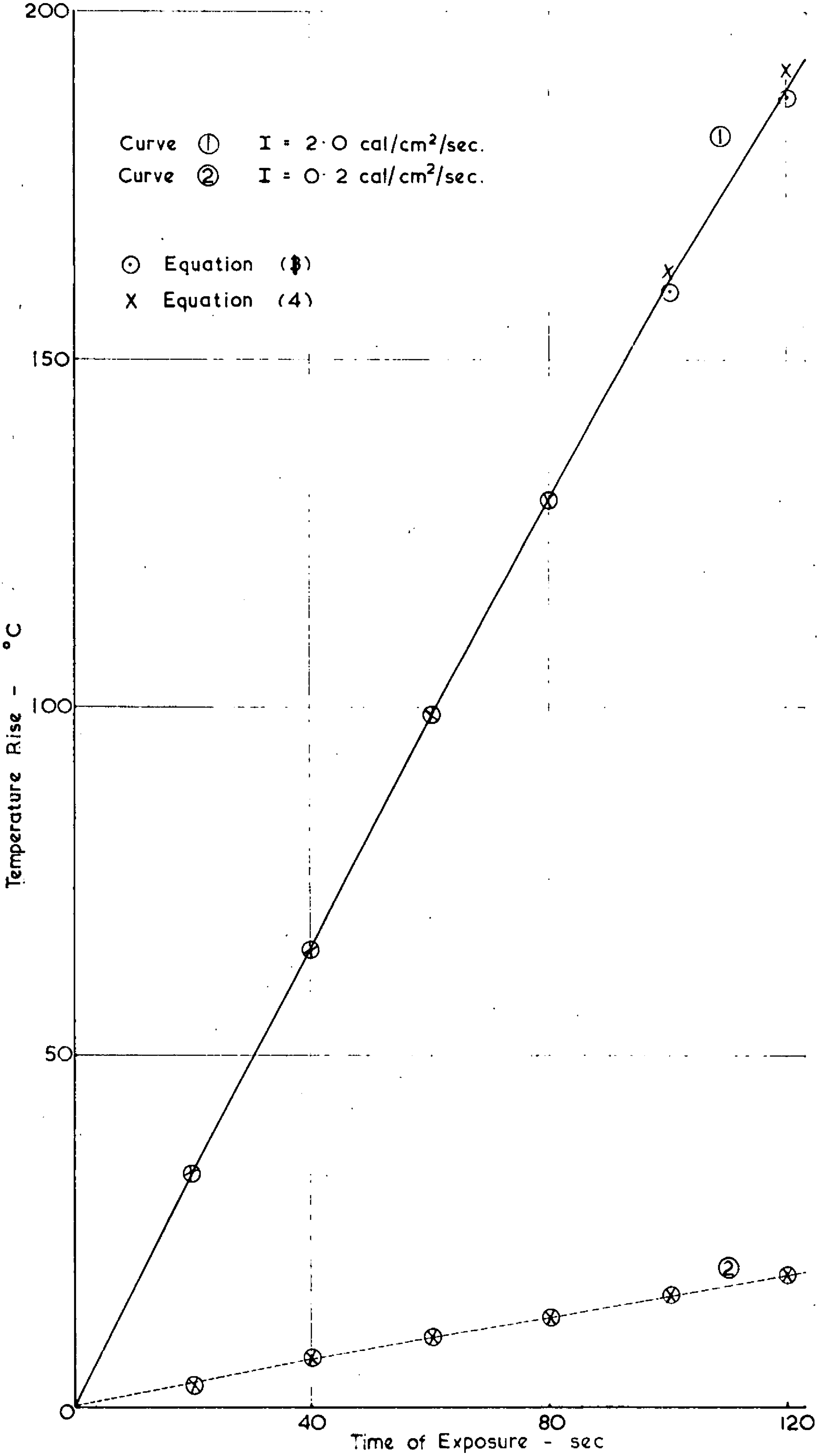

FIG. 2. COMPARISON OF EQUATION (3) AND EQUATION (4) FOR TWO INTENSITIES OF RADIATION.