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THE RELATIONSHIP BETWEEN THE CHANCE OF A FIRE BECOMING LARGE AND THE CHANCE OF FIRE SPREADING BEYOND THE ROOM OF ORIGIN

by

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SUMMARY

Data are presented which show that the chance p_L of a fire becoming large (e.g. fire loss greater than £10 000) is correlated with the chance p_s of fires spreading beyond the room of origin, so that approximately $p_L = p_s^3$. A simple stochastic model of fire spread supports this correlation, and indicates that the observed large fires correspond on average to fires involving four or more rooms (a result which is supported by other observed data.) This correlation shows that the probability of spread beyond the room of origin is a useful measure of the chance of a fire becoming large and suggests that large fires as a group may be studied by research on those factors which influence whether fires spread beyond the room of origin. This is of considerable practical importance because there are many more data on smaller fires.

KEY WORDS: Fire spread, loss, fire statistics, probability, model.

MINISTRY OF TECHNOLOGY AND FIRE OFFICES' COMMITTEE
JOINT FIRE RESEARCH ORGANIZATION

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INTRODUCTION

The annual fire loss is currently of the order of £120 M[1], of which over half is attributable to a comparatively small number of large fires[9]. A reduction in the number of large fires could lead to a substantial reduction in the annual fire loss, and it is therefore of importance to study those factors which influence whether or not fires become large. However, because the numbers of such fires are small, (and because the possible combinations of events are large,) a statistical study is difficult, and one should attempt to find some other means of studying these fires.

One approach is to study the factors influencing fire growth at some earlier stage, where the fire is still relatively small and data are consequently more numerous. However, such an approach is valid only if there exists some link between the earlier events and the incidence of large fires. It is this latter problem which is the subject of this paper.

Large fires are usually defined in terms of the loss, or sometimes in terms of the fire fighting force necessary to bring them under control. Thus large fires are usually taken to be those with losses in excess of £10 000, or those in which 5 or more jets were used. In fire brigade reports, collated and processed by the Fire Research Station, the only measure of size for smaller fires in multi-compartment buildings, other than those fires confined to the item first ignited, is whether the fire spreads beyond the room of origin. These fires constitute about 15 per cent of all fires, whereas only about 1 per cent become large. Fire reports may be used to estimate a fundamental probability associated with buildings sub-divided into rooms or compartments, namely p_s, the chance of a fire spreading beyond the room of origin. This statistic has already been studied for various occupancies and for different circumstances surrounding the fire[2-6]. In the present paper the relationship between the chance of a fire becoming large and the chance of fire spreading beyond the room of origin is examined both statistically and by means of a simple model describing the spread of fire by statistical laws.

EMPIRICAL CORRELATION

Published data resulting from an analysis of brigade fire reports are presented in Table 1, giving p_L and p_s in relation to occupancy for fires in multi-compartment buildings in 1963 which spread beyond the item first ignited. For these data

p_s = proportion of fires which spread beyond the room of origin

p_L = proportion of fires fought with five or more jets.

The data for the number of large fires are from Ref.7 and those for the total number of fires and the number spreading are from Ref.8. Fires confined to common service spaces or exterior components were excluded. Single compartment buildings were excluded from p_s because the concept of spread beyond the room of origin has no meaning for these buildings. The number of large fires occurring in single compartment buildings was not available but is thought to be relatively small. p_L was therefore calculated on the assumption that most large fires occur in multi-compartment buildings.

In order to obtain more homogeneous data, values of p_L and p_s were obtained for a single occupancy (distributive trades) for the years 1963-1968. These values are shown in Table 2, with the number of large fires[9] taken to be those costing over £10 000.

A regression analysis of the data for the different occupancies (excluding residential) yields the regression equation

$$n_L = 1.84\, n_s^{2.75}\, N^{-1.88}$$

where n_L, n_s are the numbers of large fires and fires spreading beyond the room of origin respectively, and N is the total number of fires, for each occupancy. This may be rearranged to give

$$p_L = \frac{n_L}{N} = 1.84 \left(\frac{n_s}{N}\right)^{2.75} N^{-0.13}$$

$$= 1.84\, p_s^{2.75}\, N^{-0.13}$$

The index of N, -0.13, is not significant, so that p_L is related only to the value of p_s, and the value of N has no meaning by itself, i.e. there is a relationship between probabilities, rather than between absolute numbers.

When N is excluded and the data for each occupancy is weighted by the number of fires in that occupancy, the regression equation becomes

$$p_L = Ap_s^{\alpha}$$

where $A = 1.23 \pm 0.08$, $\alpha = 3.19 \pm 0.32$

The index of p_s is very nearly a cube, and it will be shown below that this has some theoretical significance.

The data for residential occupancies were not included because most of these buildings are relatively small and consequently a large fire, within the definition in this paper, is not possible. This is reflected in the small value of p_L which lies well below the regression line fitted to the remaining data.

The regression line and all the data are plotted in Fig.1. As can be seen, the data for distributive trades for different years, although covering a smaller range of the variables, closely follows the regression line. Since these are for more homogeneous data, this gives additional credibility to the correlation.

The remainder of the paper will discuss the implications of this correlation. First, however, we introduce a theoretical model of fire spread in buildings in order to examine possible mechanisms leading to an equation of this kind.

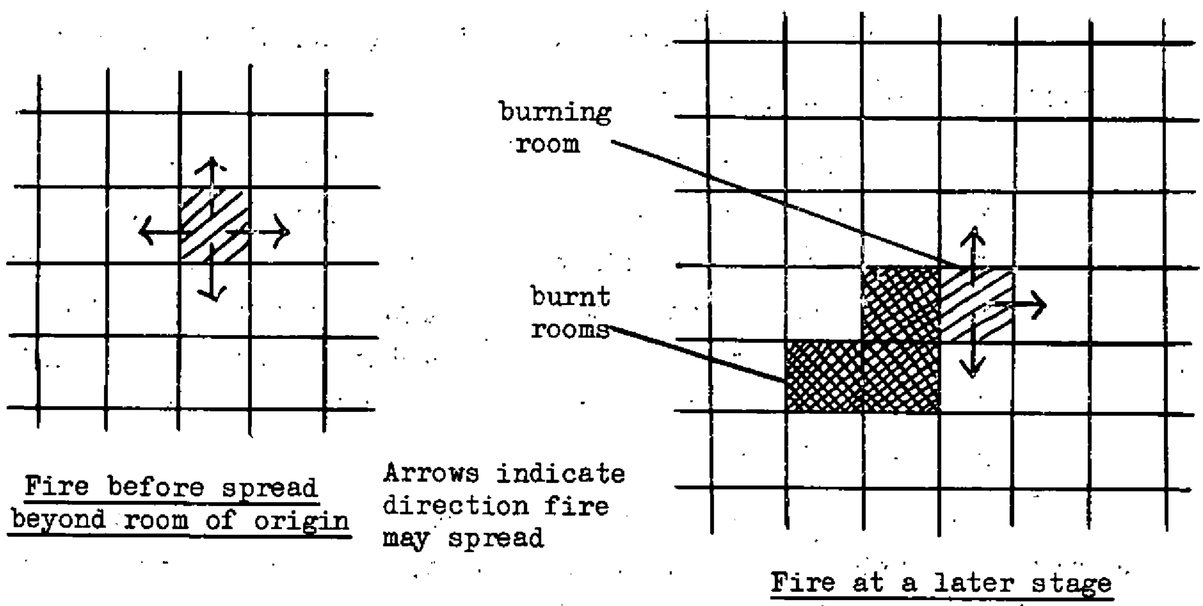


FIG.2.

THE MODEL

We suppose the building to be represented by an infinite array of cells, each representing a room or compartment, so that single-storey buildings are represented by a two-dimensional array, and multi-storey buildings by a three-dimensional array. Fire is started in one of the cells and then, at random, spreads to any number of its immediate neighbours (ignoring diagonal spread), or it may be extinguished before spreading, and similarly at any later stage of development. A diagram of this model is shown in Fig.2. The fire is considered as a random walk on the cells of the array, and because, we assume here, rooms once extinguished cannot be re-ignited, the walk must be self-avoiding. Furthermore, at each stage, fire may spread into more than one neighbour, so that after each step the random walk can produce a new generation of random walks.

We must now define the probabilities of extinction, and spread to one or more neighbours. Few data are available and the exact form of these probability laws is not known. We shall therefore examine two plausible hypotheses or models.

i) Discrete-time model

The life-times of the fire in each compartment are assumed to be equal. To each boundary a probability s is attached, representing the chance of crossing the boundary in this life time. In this simplified version of the model we suppose this probability p_s of spreading beyond the room of origin is $1 - (1 - s)^N$, which for small s can be expanded to give approximately $p_s = Ns$.

ii) Continuous-time model

In this model it is not assumed that rooms burn for a fixed time. The probability of the fire spreading across any boundary is then an increasing function of the duration of the fire in that room. There is some evidence[2] that to a first approximation the chance of a fire spreading across a given boundary in a small time interval dt is λ dt, and the chance of extinction is μ dt where λ and μ are constants.

The probability of a fire in a room being uncontrolled after time t decreases approximately exponentially with time, as shown by the data in Fig.3, although the rate of decay appears to decrease at long times, possibly due to the heterogeneity of the data. Therefore the number of rooms being put out at time t is approximately proportional to the number alight at that time. Thus the probability of a burning room being put out in any interval, dt, is a constant, $\mu\ dt$. It was assumed that the probability of the fire spreading to any given neighbour is also a constant, dt. It can be seen that the probability s, of the fire spreading to any given neighbour before going out is then given by

$$s = \frac{\lambda}{\mu + \lambda} \qquad \ldots\ldots(1)$$

$$p_s = \frac{N\lambda}{\mu + N\lambda}$$

$$= \frac{Ns}{1 + (N-1)s} \qquad \ldots\ldots(2)$$

For small values of s, when $(N-1)s \ll 1$,

$$p_s \simeq Ns$$

The discrete-time model is a first approximation which does not take account of the increasing chance of spread for fires of longer duration. The continuous-time model takes into account the effect of duration, although the way in which spread depends on time is an approximation. It differs from the discrete-time model in that a fire which crosses a given boundary has more chance of crossing other boundaries. Thus, in the continous-time model, fires which spread are more likely to cross several boundaries, although for small values of s, $p_s \simeq Ns$, the same as for the discrete-time model.

The model discussed in this paper is a simplification of what may happen in reality. Firstly, a building consists generally of rooms of different shapes and sizes and with different fire resistance ratings, and correspondingly the chance of spread will be different for different rooms, so that p_s itself has some form of statistical distribution. Secondly, the model ignores the effect of corridors, service ducts etc. which will enable the fire to spread to rooms other than immediate neighbours. Thirdly, the probability laws are purely hypothetical at this stage, although they are plausible, and based on some statistical evidence.

Generally, more data and measurements are required to build a more precise model. The model embodies the simplest hypotheses which describe the main features of fire spread in buildings and are consistent with data available, and there is little justification for a more detailed treatment at this stage.

ANALYTIC SOLUTION

In simple cases the probability of a given extent of spread can be obtained analytically. Formulae are given in Table 3 for the case when the chance of spread is small. Only in the case of one-dimensional arrays (rows) can analytic solutions be obtained for the probability of any extent of spread. Results for rows are therefore briefly given in the Appendix.

SIMULATED SOLUTION

It is difficult except in simple cases to find analytically the probabilities of different extents of spread. A computer model was therefore used in which the spread from room to room was simulated by use of random numbers. A 25 x 25 and a 21 x 21 x 21 array of rooms were taken and the frequency of different extents of spread and the average number of rooms burnt $\bar{n}$ found for different values of s. The centre room was taken to be alight initially.

Whether spread occurred was determined by a series of pseudo-random numbers X_i where $0 \leqslant X \leqslant 1$. For the discrete time model spread was taken to occur if $X < s$.

In the continuous time model, following the earlier discussion, the probability, p_1, of a fire in a room being alight at time t after ignition is given by

$$dp_1 = -\mu\, p_1 dt$$

Therefore, integrating

$$p_1 = \exp(-\mu t) \qquad \ldots\ldots(3)$$

The probability, p_2, of the fire, if still alight, having spread to a given initially unignited neighbour is given by

$$dp_2 = \lambda(1-p_2)\, dt$$

whence

$$p_2 = 1-\exp(-\lambda t) \qquad \ldots\ldots(4)$$

For the continuous time model, therefore, the life time t, of the fire in a room was taken to be $-\ln(X_1)/\mu$, which has the probability distribution represented by equation (3). Spread was then taken to occur if $X_2 > \exp(-\lambda t) = X_1^{\lambda/\mu}$, which satisfies equation (4).

At high values of s, $\bar{n}$ would be limited by the size of the arrays. However, this limitation was not important for the values taken.

AVERAGE EXTENT OF SPREAD

The first room ignited can spread to N other rooms. Subsequent rooms ignited can spread to up to $N-1$ other rooms. Thus the average extent of spread is given approximately by

$$\bar{n} = 1 + Ns\,(1 + (N-1)s + ((N-1)s)^2 + \ldots)$$

$$= (1 + s)/(1 + s - Ns) \qquad \ldots\ldots(5)$$

Values of $\bar{n}$ obtained by simulation are shown in Fig.4 together with the curves given by equation (5). It can be seen that for Ns up to 0.7, equation (5) is in good agreement with the simulated solution.

$\bar{n}$ increases steadily up to $Ns = 0.7$, which corresponds approximately to $p_s = 0.4$. At higher values of s, $\bar{n}$ increases much more rapidly, indicating the steeply rising chance of a large fire for increases in s (or p_s). For most practical cases, p_s is less than 0.4 (see Tables 1 and 2). The variation of $\bar{n}$ at high values of p_s of interest in connection with conflagrations.

PROBABILITY OF FIRES BECOMING LARGE

In physical terms large fires can be represented on average by a large area, or in terms of the present model by a large extent of spread. For present purposes we therefore define a large fire as one in which the number of rooms n is not less than some fixed number n_o, so that $n \geqslant n_o$. We now calculate the probability of a large fire in terms of the chance of spread beyond the room of origin.

APPROXIMATE FORMULAE

A fire spreads across $n-1$ boundaries, where n is the number of rooms burnt. Thus, approximately, when p_s is sufficiently small,

$$p(n \geqslant n_o) = p_s^{\,n_o - 1} \qquad \ldots\ldots(6)$$

Approximate analytic formulae for the probabilities of the number of rooms being burnt being greater than or equal to two, three or four are given in Table 3. These formulae are for infinite two and three dimensional arrays ignited at the centre or surface and using the discrete-time model. The average values at low p_s are

$$p\ (n \geq 3) = 1.22\ p_s^2 \qquad \text{......(7)}$$

$$p\ (n \geq 4) = 1.68\ p_s^3 \qquad \text{......(8)}$$

It was found that the precise analytic equations agree with equation (7) to within 18 per cent for all values of p_s and with equation (8) to within about 20 per cent, when $p_s < 0.5$.

In Fig.5 the probabilities, p_L of fires for which $n \geq 3$, 4 and 5 are plotted against the probability, p_s, of spread beyond the room of origin. The values of p_L taken are those given by equation (6). Observed data for the probabilities of fires in various occupancies becoming large are replotted for comparison.

COMPARISON OF MODELS USED

Values of $p\ (n \geq 4)$ obtained by the different models are shown in Fig.6. The values shown are for:

i) approximate solution, equation (6)

ii) average analytic solution (equation (8)) for infinite two and three-dimensional arrays using the discrete-time model

iii) analytic solution for 3 x 3 array ignited at the centre using the discrete-time model

iv) simulated solution for ignition at the centre of an effectively infinite two-dimensional array using the continuous-time model. The values of p_s plotted are calculated values.

The results show that the variation of p_L with p_s is similar whichever model is taken. In particular, for the discrete-time model, infinite two-and three-dimensional arrays ignited at the centre, edges or corners give very similar results.

DISCUSSION

Since the regression line derived in the introduction is very nearly a cube, comparison with the theory suggests that the large fires correspond to $n \geq 4$. In Fig.5 the data on large fires are superimposed on the lines calculated from the model, and it can be seen that the data and the calculated lines follow the same trend. The data correspond closely to the calculated line for four or more rooms and thus, on the basis of the model, large fires correspond to fires involving four or more rooms.

In 1963 there were 464 fires in buildings fought with five or more jets and 597 costing more than £10 000, of which about 360 cost more than £20 000. Thus fires fought with five or more jets appear to correspond to those costing more than about £15 000 in 1963 or about £20 000 at present. The average financial loss in large fires is about of the order of £15 per square foot[11]. Thus a large fire costing £20 000 must involve about 1300 ft^2.

This compares with data given by Thomas[12] which indicate that the average size of five jet fires is about 2000 ft^2, but this estimate is probably too high since the sample was biased in favour of large fires. If, we take a large fire as having an area of the order of 1300 ft^2 (120 m^2) or more and large fires as corresponding to four rooms or more, we have 30 m^2 as an average 'room size', which in absence of actual data is not an unreasonable value. The model can be used to provide information about the way in which the duration of a fire varies with its size. Because there is no time scale involved in the method of simulation in the present paper, the duration can be expressed only in terms of the number of generations required to produce a fire involving a particular number of rooms. It was found that within the range of interest, with $0.05 < s < 0.3$, the average duration $\bar{t}$ varied little with s, and the variation of $\bar{t}$ with the number of rooms n could be represented approximately by an equation

$$\bar{t} = 1.23\, n^{0.71} \qquad \text{......(9)}$$

This equation can be transformed into a relationship between time and area by assuming that the average area of a room is 30 m^2, as derived above, and that the duration of a fire generation is 9 min, as indicated by the data in Fig.3 for fires in non-residential buildings. The resulting data and regression time are plotted in Fig.7 and compared with the regression.

$$T = 1.66\, A^{0.56}$$

obtained by Baldwin[13], following Thomas[12], for published data[14]. In this equation T is the time taken to control a fire in min and A is the area of the fire in m^2. These simulated data compare reasonably well with observed data, and lie within the scatter of the observed data. The duration of the simulated fires is longer for a fire of given area, but this is to be expected since the values of T for observed data are for the period after the arrival of the brigade. Thus the simple model discussed in this paper leads to quite reasonable estimates of observed quantities.

CONCLUSIONS

The model supports the existence of a correlation between the observed occurrence of large fires and the chance of spread beyond the room or origin, and it seems likely therefore that this correlation is real. We may therefore draw the following important conclusions:

i) The probability of spread beyond the room of origin, which is a property of a class of fires, has been found to be a useful measure of the chance of fires becoming large.

ii) Large fires as a group may be studied by research on those factors which influence whether fires spread beyond the room of origin. This is of considerable practical importance because there are many more data on smaller fires.

iii) The probability of a fire becoming large increases rapidly with the probability, p_s, of spread beyond the room of origin. Quite small reductions in the chance of a fire spreading beyond the room of origin could therefore result in relatively large reductions in the chance of a large fire. The chance of fire spread can be reduced in several ways, for example, by providing more adequate walls and floors separating compartments, by protecting the compartments with sprinklers or other devices which will ensure early detection or control. This paper provides a tool for studying such questions, which will be the subject of a further report.

iv) We may identify the events most likely to lead to large fires. For example, the chance of spread beyond the room of origin for fires started by malicious ignition or by rubbish burning is about 0.4, and thus fires with these causes have a very high chance of becoming large.

v) p_L varies in a similar manner with p_s for the observed data and for the models taken. At low p_s, the precise model used is not important. The average extent of spread, obtained by simulation, is found to increase steadily with p_s. The increase is more rapid when $p_s > 0.4$.

REFERENCES

1. Fire Protection Association Journal, 86 (1970) 59.

2. THOMAS, P. H. The spread of fire in buildings : a statistical approach. Ministry of Technology and Fire Offices' Committee Joint Fire Research Organization F.R. Note No.694, 1967.

3. THOMAS, P. H. Fires in old and new non-residential buildings. Ministry of Technology and Fire Offices' Committee Joint Fire Research Organization F.R. Note No.727, 1968.

4. BALDWIN, R. and THOMAS, P. H. Spread of fire in buildings - the effect of the source of ignition. Ministry of Technology and Fire Offices' Committee Joint Fire Research Organization F.R. Note No.729, 1968.

5. BALDWIN, R. and THOMAS, P. H. The spread of fire in buildings - the effect of the type of construction. Ministry of Technology and Fire Offices' Committee Joint Fire Research Organization F.R. Note No.735, 1968.

6. BALDWIN, R. and THOMAS, P. H. The spread of fire in buildings - the effect of varying standards of fire cover. Ministry of Technology and Fire Offices' Committee Joint Fire Research Organization F.R. Note No.789, 1969.

7. DUNN, JENNIFER E. and FRY, J. F. Fires fought with five or more jets. Fire Research Technical Paper No.16, London 1966, Her Majesty's Stationery Office.

8. U.K. Fire Statistics, 1963, Her Majesty's Stationery Office.

9. RAMACHANDRAN, G. and KIRSOP, P. A brief analysis of large fires during 1965-68. Ministry of Technology and Fire Offfices' Committee Joint Fire Research Organization F.R. Note No.792, 1969.

10. A study of large fires in 1962 and 1963. Fire Protection Association Journal, 63, 306-13, 1964.

11. RAMACHANDRAN, G. Private communication.

12. THOMAS, P. H. Use of water in the extinction of large fires. Institution of Fire Engineers Quarterly, 19 , 130-2, 1959.

13. BALDWIN, R. Use of water in the extinction of fires by brigades. Ministry of Technology and Fire Offices' Committee Joint Fire Research Organization F.R. Note No.803/1970.

14. LABES, W. G. Fire department operational analysis, final report. Illinois Institute of Technology Research Institute. Contract No. N0022867 C0701, OCD Work Unit 2522F, 1968.

APPENDIX

ANALYTIC SOLUTIONS FOR INFINITE ONE-DIMENSIONAL ARRAY

i) Ignition at one end:

$$p_s = s$$

$$p(n) = s^{n-1}\,(1-s)$$

$$p(n \geq n_o) = s^{n_o-1}$$

$$\bar{n} = (1-s)^{-1} \qquad \text{(a)}$$

ii) Ignition at centre:

from equation (a)

$$\bar{n} = 1 + 2s(1-s)^{-1} = (1+s)(1-s)^{-1}$$

For the discrete-time model

$$p_s = 2s - s^2$$

$$p(n) = n\,s^{n-1}(1-s)^2$$

For the continuous-time model

from equation (2)

$$p_s = 2s\,(1+s)^{-1}$$

Table 1

Probabilities of fires becoming large and of spread beyond the room of origin in different occupancies in 1963

Occupancy	Number of fires	Probability of fire becoming large* p_L	Probability of spread p_s
Manufacturing industries excluding agriculture, forestry, fishing	6948	0.0330	0.319
Distributive trades - retail	2072	0.0121	0.246
Distributive trades - other	808	0.0569	0.413
Financial, professional and miscellaneous services	3088	0.0071	0.210
Catering, hotels, etc.	2318	0.0108	0.215
Transport and communications	464	0.0453	0.315
Places of public entertainment	478	0.0356	0.335
Public administration and defence	632	0.0174	0.231
Residential	37118	0.00092	0.143

*i.e. 5 or more jets

Table 2

Probability of fires becoming large and of spread beyond the room of origin for the distributive trades (1963-1968)

Year	Number of fires	Probability of fire becoming large* p_L	Probability of spread p_s
1963	2880	0.0274	0.293
1964	2918	0.0469	0.334
1965	2758	0.0482	0.331
1966	2808	0.0605	0.350
1967	2844	0.0573	0.344
1968	3128	0.0553	0.345

*i.e. over £10 000

Table 3

Probabilities of n or more rooms being ignited in infinite array when $s << 1$, discrete-time model

Configuration	Position of ignition	n		
		2	3	4
Two-dimension	corner	2s	$5s^2 = 1.25\ p_s^2$	$16s^3 = 2.00\ p_s^3$
	side	3s	$10s^2 = 1.11\ p_s^2$	$39s^3 = 1.44\ p_s^3$
	centre	4s	$18s^2 = 1.13\ p_s^2$	$88s^3 = 1.38\ p_s^3$
Three-dimension	corner	3s	$12s^2 = 1.33\ p_s^2$	$61s^3 = 2.26\ p_s^3$
	edge	4s	$20s^2 = 1.25\ p_s^2$	$86s^3 = 1.34\ p_s^3$
	side	5s	$31s^2 = 1.24\ p_s^2$	$197s^3 = 1.58\ p_s^3$
	centre	6s	$45s^2 = 1.25\ p_s^2$	$380s^3 = 1.76\ p_s^3$

p_s = probability of spread beyond room of origin

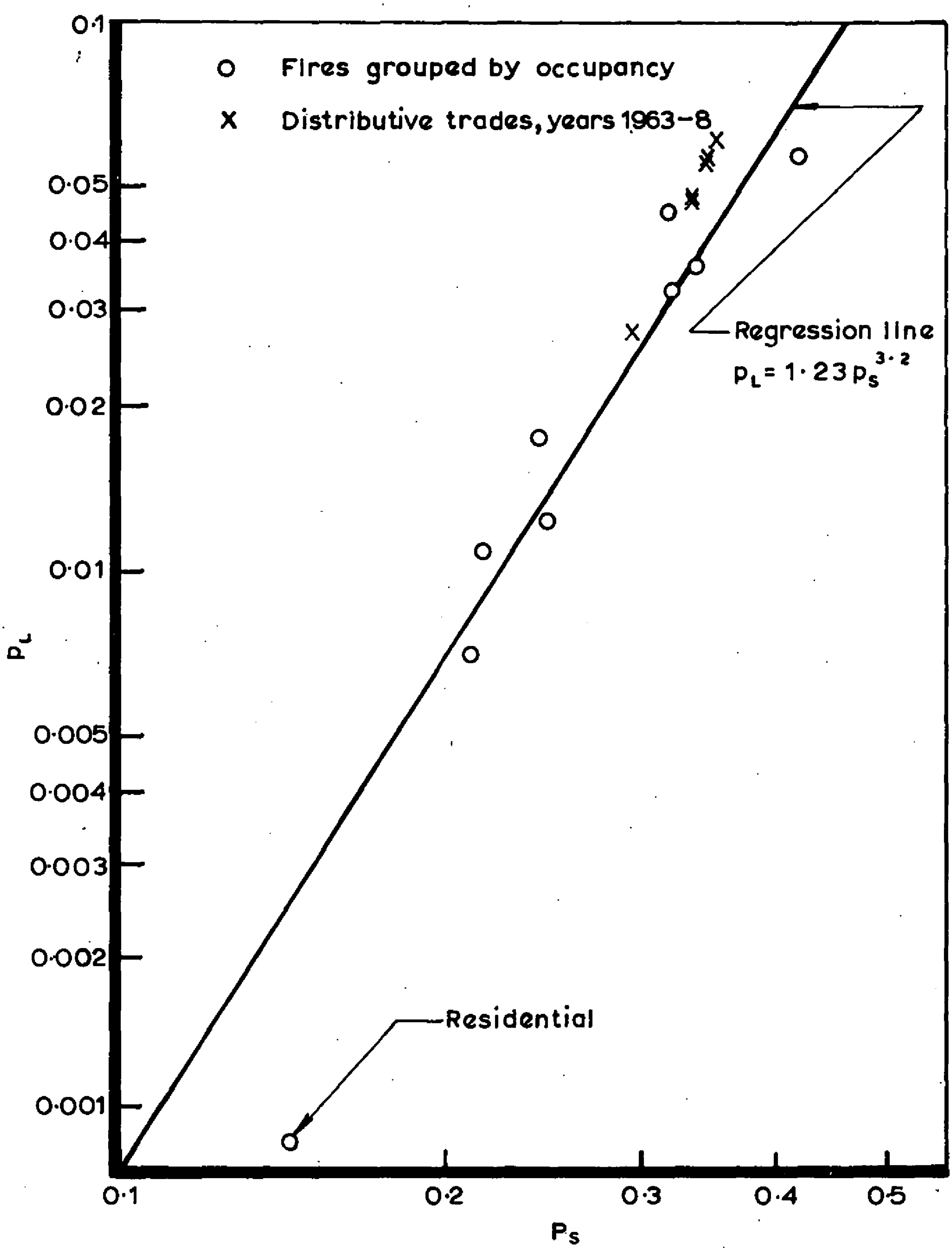


FIG.1 OBSERVED PROBABILITIES OF FIRES BECOMING LARGE

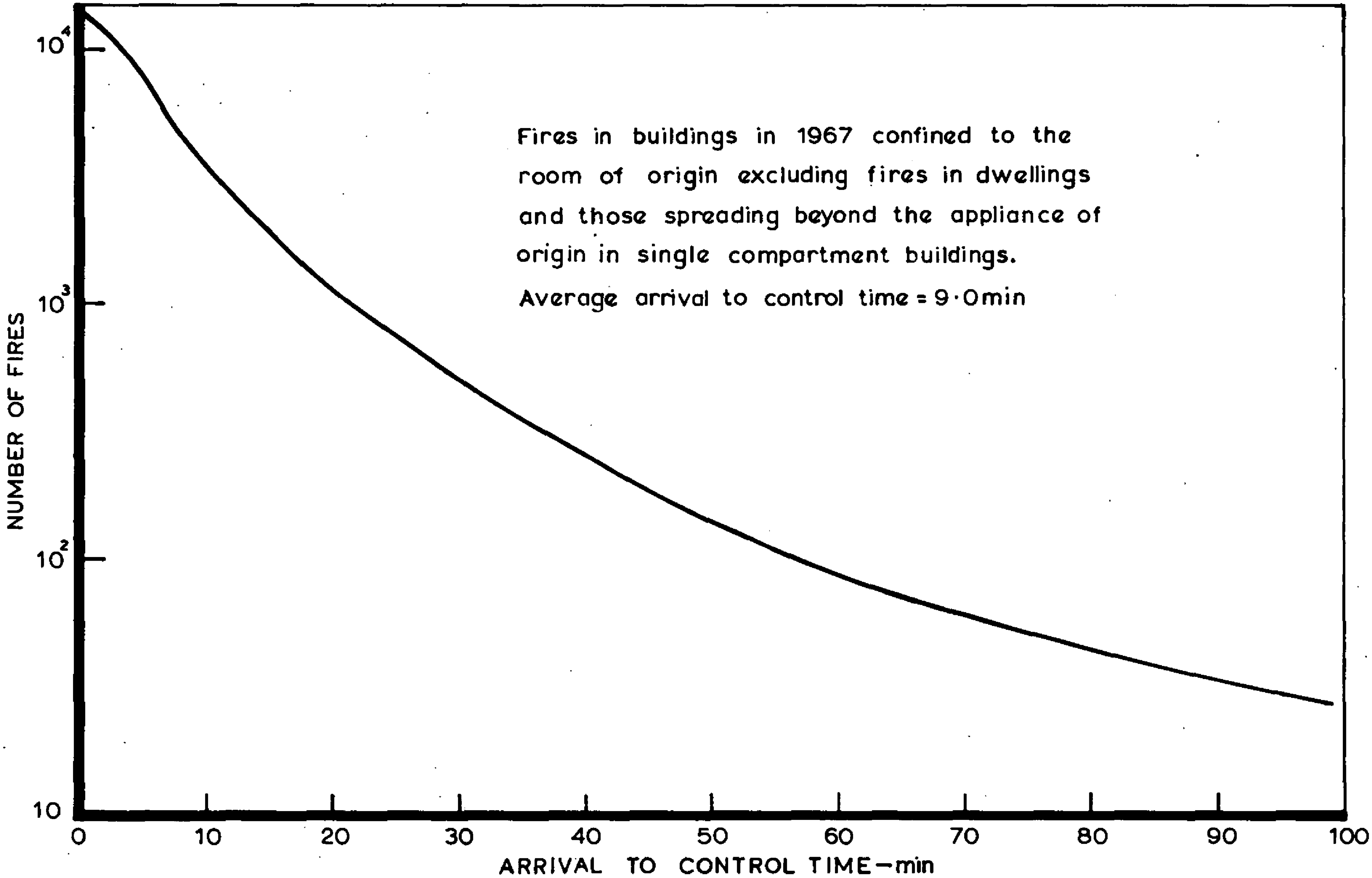


FIG.3 NUMBER OF FIRES REMAINING UNCONTROLLED

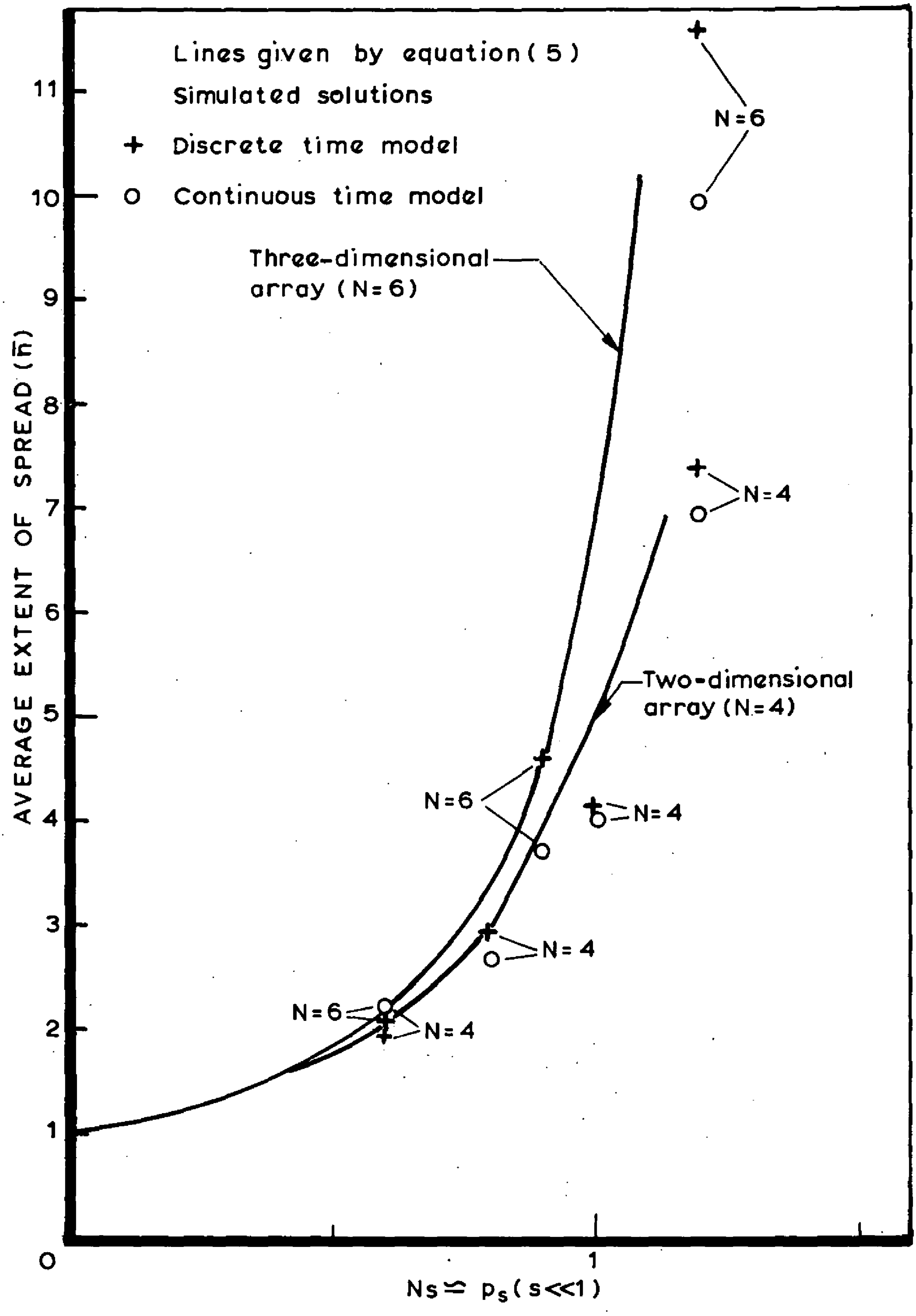
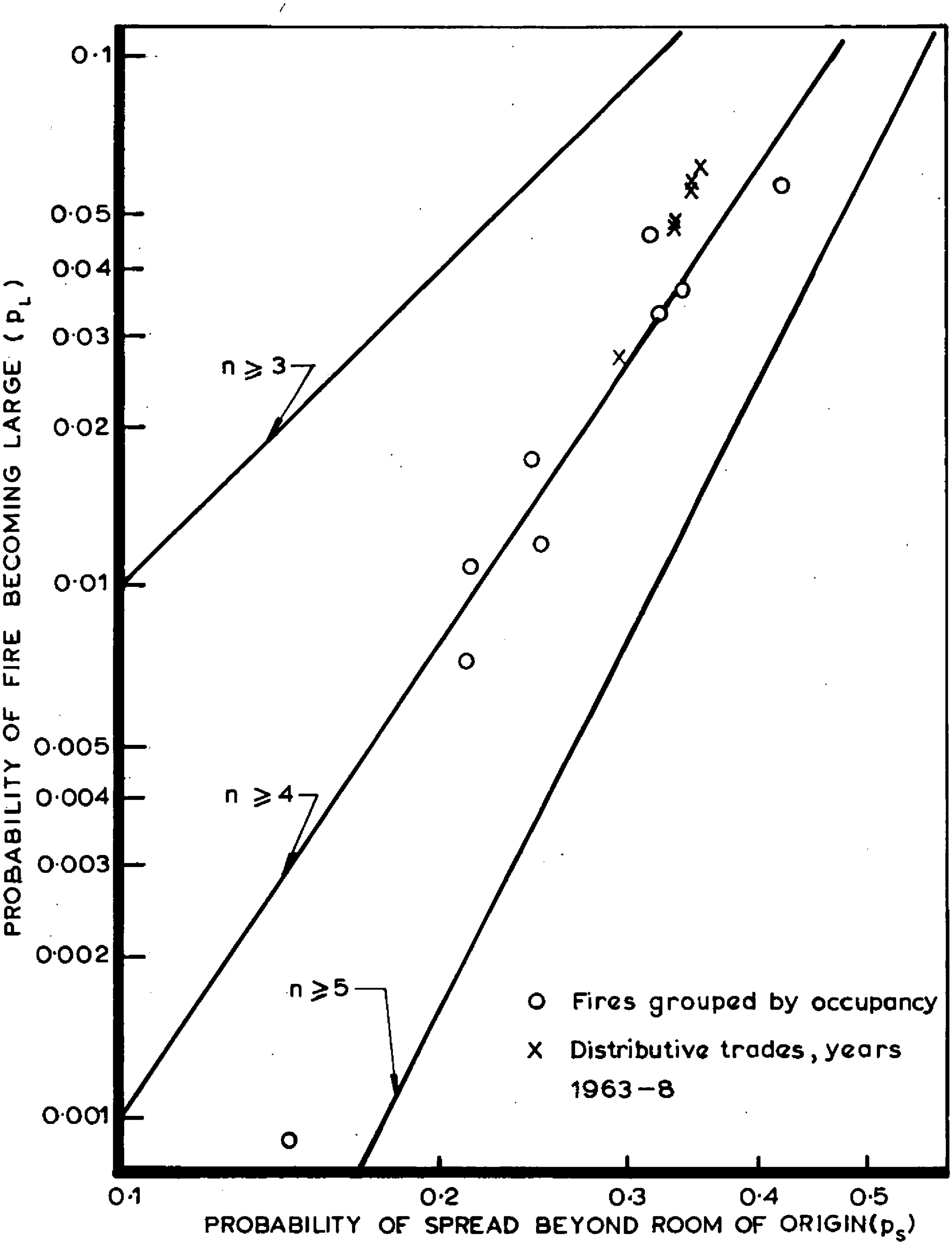


FIG.4 AVERAGE EXTENT OF SPREAD, INFINITE ARRAY IGNITED AT CENTRE



The lines are approximate analytic solutions given by equation (6)

FIG. 5 PROBABILITY OF FIRES BECOMING LARGE

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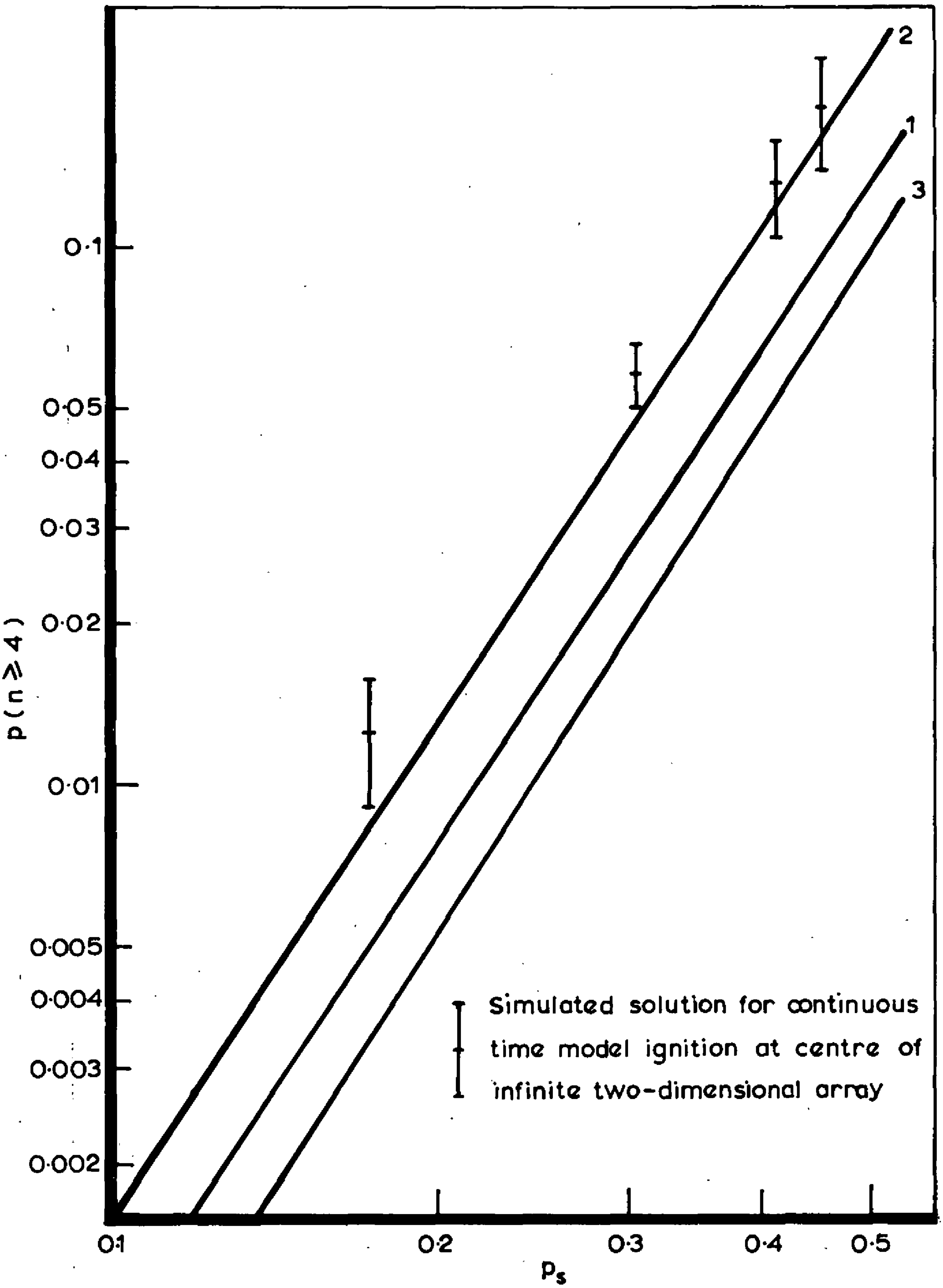
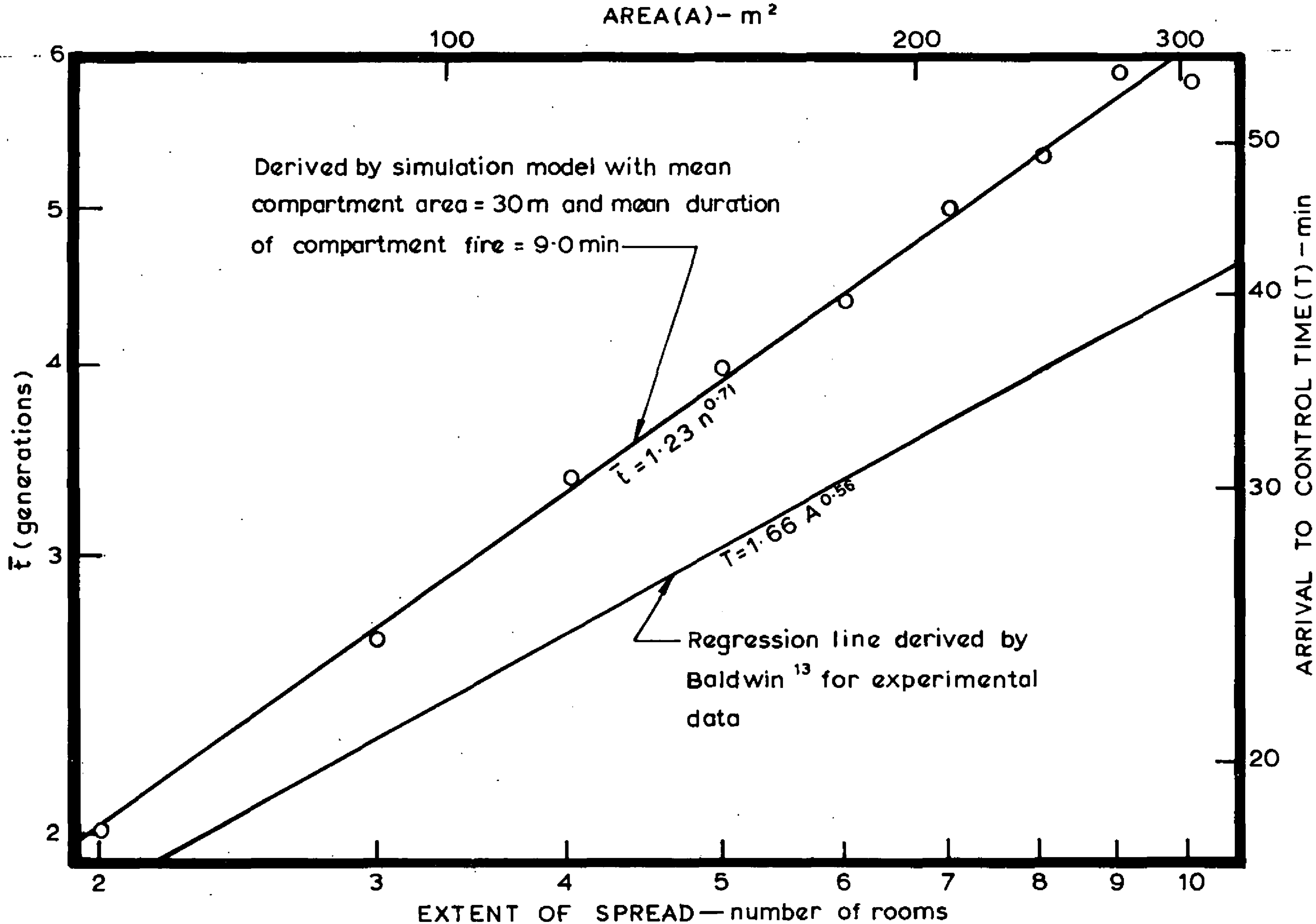


1) Approximate analytic solution, equation (6)

2) Average analytic solution for infinite two and three-dimensional arrays, discrete time model, equation (8)

3) Analytic solution for 3×3 array ignited at centre, discrete time model

FIG. 6 COMPARISON OF DIFFERENT MODELS



Infinite two-dimensional array ignited at centre, discrete time model

FIG.7 VARIATION OF AVERAGE DURATION OF FIRE WITH EXTENT OF SPREAD

FIG.7 VARIATION OF AVERAGE DURATION OF FIRE WITH EXTENT OF SPREAD

Infinite two-dimensional array ignited at centre, discrete time model