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SOME NOTES ON THE MATHEMATICAL ANALYSIS OF SAFETY

by

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SUMMARY

This note summarises some of the more important methods that have been proposed in the literature for assessing the safety of a structure on a quantitative basis and for determining what constitutes an acceptable risk. These methods have hitherto been applied in branches of engineering such as design of aircraft and dams and design of buildings against wind loads, live loads, earthquakes etc: the object of the note is to discuss the relevance of these methods in the design of fire protection.

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INTRODUCTION

Research on the physics of fires has now reached the stage where the fire resistance necessary to protect a structure against fire may be calculated from a knowledge of the configuration of a compartment, its contents and the ventilation conditions[1,2]. However, legislation for the control of building design specifies requirements for a wide range of buildings, and in these circumstances, it is necessary to conduct a survey of the contents and configuration of buildings in order to estimate realistic fire resistance requirements. Such a survey is being undertaken by several different countries, and the Building Research Station has now completed a survey of floor loads in modern office buildings[3]. Joint Fire Research Organization has made a preliminary analysis of a small sample of these data[4], which combined with experimental results leads to a frequency distribution for the fire resistance necessary to protect structural steel (shown in Fig.1)[5]. This distribution is very skew and covers a wide range of fire resistance, and it should be noted that similar distributions would result from surveys even of individual buildings. Clearly, the fire resistance necessary to protect a given column has no single value, but must be regarded as a variable subject to statistical variation over a wide range, i.e. a random or stochastic variable with the distribution of Fig.1.

The application of this result in the choice of a single value of fire resistance for this occupancy for the purpose of building regulations presents some difficulties. According to the conventional philosophy of fire grading the fire resistance should be sufficient to withstand a 'burn-out', but even within this narrow range of building occupancy, the maximum (75 minutes) will grossly overprotect most of the buildings of this type (whose predicted mean fire severity is 25 minutes) and any lesser value will underprotect a few. Indeed there is no guarantee that the maximum of this sample is in fact the maximum of the population, so that the maximum of the sample does not guarantee complete protection. Clearly whatever value of fire resistance is specified, within a practical range, some degree of risk is implicit.

This discussion raises an important question, namely, what constitutes an acceptable risk, that is, an acceptable degree of safety. Essentially, this is a non-engineering question, and indeed the fire engineer is not equipped to decide what risk to life is acceptable - this is a matter for the community at large. The function of the fire engineer is to advise on cost and social implications associated with any specified degree of safety, and to ensure that any given expenditure is used to the greatest advantage.

The problem of finding an acceptable degree of safety, is not unique to fire problems; it has long been recognised, in the design of aircraft[6], for example, that strength and loading are random variables, and that design under these circumstances involves an acceptance of a risk that in a very small number of instances the loading will exceed the strength. This philosophy has now found its way into the field of structural engineering, with the realisation that the traditional 'factor of safety' itself embodies implicitly acceptance of a degree of risk.

In this paper we review some of the main developments in these other fields and comment on their suitability in the context of fire safety. The problems of fire are rather different to those of other engineering fields, because not only are loads and strengths subject to variability but also ventilation, room shapes and sizes, open doors etc. More important, the duration of the fire is curtailed by chance discovery, and by the action of brigades, sprinklers etc, and these act to reduce the risk inherent in variability of loads and strengths. The various theories described will be reviewed with these additional factors in mind, and where possible adapted to include them.

ANALYSIS OF STRUCTURAL SAFETY

a. Reliability analysis

We review first some methods of calculating the risk implicit in the choice of a particular value for fire resistance, using the techniques of reliability analysis, following Freudenthal et al[7]. The risk is measured by the probability of failure of a building element designed to have fire resistance R. Because of the variability of materials, uncertainty of testing etc, R is a random variable, as is the fire severity S, because of variability of fire loads, ventilation, discovery and arrival of brigades, etc. Failure occurs if $R < S$, and the probability of failure, p_f is then given by

$$p_f = P\{R < S\}$$

Define the statistical distributions of R and S as follows:

$$F_R(x) = P\{R < x\} = \int_0^x f_R(z)\,dz$$

$$F_S(x) = P\{S < x\} = \int_0^x f_S(z)\,dz$$

Then

$$p_f = \int_0^\infty F_R(x)\, f_S(x)\, dx$$

Now to a first approximation fires occur at random and independently. Furthermore, only a small fraction grow to a sufficient size to cause damage to the structural elements of the buildings. Let p be the annual probability of occurrence of such a fire, so that we are implicitly making a generous allowance for the beneficial effects of early discovery and brigade action. Then if p is small, the probability of failure p_f during the design lifetime of the building L is given approximately by

$$p_F = L p p_f$$

Table 1

The annual chance of a fire outbreak for various occupancies

Hazard	Number of buildings*	Number of fires annually+	p_a
Industry	183,377	8,075	4.4×10^{-2}
Houses	14,202,359	38,142	2.7×10^{-3}
Commercial - shops	664,817	5,574	8.4×10^{-3}
Commercial - office	152,430	866	5.7×10^{-3}
Assembly - entertainment	12,540	1,446	1.2×10^{-1}
Assembly - non-residential	143,019	2,810	2.0×10^{-2}
Residential - clubs, hotels etc	36,609	1,352	3.7×10^{-2}
Residential - institutions	-	803	-
Storage	199,612	2,420	1.2×10^{-2}

*Source: 108th Report of the Commissioners of H.M. Inland Revenue

+Fires during 1967

b. Application to fire resistance calculations

In order to apply these results to fire resistance calculations, it is necessary first to have data for estimating the quantities p , F_R (x), f_s (x). The quantity p can be estimated from the frequency of fires and the number of buildings at risk, leading to an estimate of p_a the chance of a fire occurring annually. Baldwin and Allen[8] have estimated that only 10 per cent of fires involve damage to the building fabrics, so that

$$p = \frac{1}{10} p_a$$

Some estimates of p_a for various types of building are given in Table 1.

At present, there are no data that will lead to an estimate of F_R (x), although on intuitive grounds one would expect it to be a skew distribution of the log-normal type. At this stage it will be more profitable to regard R as a constant, say $R = R_o$ and then $p_f = 1 - F_S(R_o)$.

The distribution of fire severity, S , can be estimated from surveys of contents of buildings, together with experimental results. One example has been given in Fig.1 for modern office buildings; at the moment these are the only available data.

The probability of failure p_F is calculated in Fig.2 for modern office buildings on the basis of these data, assuming a design lifetime of 50 years. For R = 60 min, the highest fire resistance specified by current regulations for offices, p_F = 0.002.

c. Extended reliability concept

The calculations of the preceding paragraphs have been on the basis of reliability theory, and several authors have pointed out the disadvantages of this method[9,10,11,12,13].

i. Reliability analysis is usually applied in a manufacturing environment, where a product is manufactured under controlled conditions, and where reliability data are readily available. To apply the theory a complete statistical analysis of loads, resistances, damage etc must be available, and even then the tail of the distribution is of greatest interest (c.f. application to offices where p_F = 0.002) where fewest data are available and where uncertainty is greatest.

ii. There are some non-statistical variables which do not fall within the reliability framework. Principal amongst these is the uncertainty of the mathematical model used for analysis, and the errors introduced thereby. In the context of fire there are many examples of this, for example the role and definition of window openings and ventilation, comparison of experimental fires with furnace tests etc.

iii. Difficulties arise when subjective elements enter the problem, making it difficult to draw on existing expertise and experience gained in the application of existing regulations and codes of practice.

Various schemes have been proposed to overcome these disadvantages, some of which will be discussed later in the paper. A modified form of reliability analysis has been proposed by Ang and Amin[12], who point out that the probability of failure based on reliability analysis is a basically sound measure of structural safety, whose basic format should be preserved as far as possible. To combat the disadvantages listed above they propose on "extended reliability concept", as follows.

Reliability failure postulates that failure occurs when $R < S$, so that $p_f = P(R < S)$. In the extended reliability concept a 'factor of uncertainty', V, is introduced, with $V > 1$, and we say that the event $R/S < V$ constitutes a state of danger or failure, so that $p_f = P(R/S < V)$ where p_f is now interpreted as the probability of danger.

The choice of V requires judgement on the part of the engineer, and depends on the reliability of the underlying mathematical model of the system, and the quality of the available information.

e.g. if R is correct to 10 per cent, the reliable resistance should be taken as $0.90\ R$, and similarly if S is known to 15 per cent, then the design S should be $\frac{S}{0.85}$

Then $V = \frac{1}{0.9 \times 0.85} = 1.30$

Note that the values of R and S have been corrected in the direction of greater safety.

In calculating factors of safety in traditional methods, it is usual to consider the maximum possible severity (S_q, say) and the minimum resistance (R_p, say).

Let
$$p = P(R < R_p)$$
$$q = P(S > S_q)$$

The requirement for structural safety is then

$$\frac{R_p}{S_q} \geqslant V$$

$$p_f = P\left(\frac{R}{S} < V\right) \leqslant \alpha$$

where α is the allowable or acceptable value of the probability of failure. The uncertainty factor V is required to account for the indeterminable errors and inadequacies in the calculations, whereas the acceptable risk, α, is required to ensure that the chance of any unsafe state of a structure, arising from statistical variabilities, is sufficiently small.

Ang and Amin show that if $\bar{V} = \frac{R_p}{S_q}$, then

$$pq < P\left(\frac{R}{S} < \bar{V}\right) < p + q - pq$$

Hence, if $\alpha = pq$, then $\bar{V} > V$

i.e. $\frac{R_p}{S_q} \geqslant V$ is automatically satisfied.

A sufficient basis for design is therefore

$$P\left(\frac{R}{S} < V\right) = pq$$

They assert that $\alpha = pq$ is the most suitable value of the risk, because it is a somewhat conservative value for a required value of V, and in any case the choice of R_p and S_q (usually on practical grounds) already implies a risk of approximately pq.

The extended reliability concept is more flexible and retains the basic characteristics and results from standard reliability analysis. It has the advantage that it separates purely statistical variation from errors and uncertainty in the basic mathematical model, so that specific allowance can be made for engineering experience and data limitations. In addition the authors demonstrate that the analysis is less sensitive to the type of distribution used for R and S (important in an environment of incomplete data), and that it can be fairly easily calibrated to existing practice.

ii. The possibility exists of the loss of hundreds of lives in one fire in a building. This is, at least at first sight, different from the loss of the same number of lives in individual incidents, such as in road accidents. Many building regulations can be traced back to disasters, i.e. a reflection of the <u>extreme</u> tail of a distribution function.

iii. Death by fire is universally regarded as one of the worst kinds of death, implying some form of personal weighting factor to be applied to costs of life in fire accidents.

In view of these problems, and others associated with escape and the effects of failure of fire resistance on the population of a building, it is not possible to pursue the economic arguments involving public safety at the present time. However, if loss of property is the only consideration, then economic arguments are obviously appropriate and directly applicable. Following standard decision theory, we attempt to minimise the sum of costs and expected loss, T, given by

$$T = I + D p_F$$

where I is the initial costs

D is the damage in the event of failure, discounted to present day values

p_F is the annual probability of failure.

Maskell and Baldwin[16] have shown that if R_o is the fire resistance then I is a linear function of R_o (for steel columns this applies only for $R_o \geqslant 30$ min). For office buildings, we may use the earlier calculations for p_F, so that

$$T = A + BR_o + Dp(1 - F_s(R_o))$$

where $\frac{B}{A} = \frac{1}{20}$ approximately for protected steel columns.

This function is plotted in Fig.3 as a function of R_o, for various levels of D. Evidently T has a minimum within the practical range of R_o, but the difference in costs between $R_o = 30$ min and $R_o = 60$ min is not very great, partly because costs, I, increase very slowly with R_o. However, Fig.2 shows that 30 min fire resistance corresponds approximately to an order of magnitude in the value of p_F. Hence on the basis of the present analysis, the costs are relatively insensitive to safety levels; at very little increased cost the probability of failure can be reduced by an order of magnitude.

CHOICE OF FAILURE PROBABILITY

a. Cost - benefit analysis

The theories discussed above derive what are effectively design rules, so that the statistical variation and uncertainty in resistance and severity may be taken into account, in order to achieve a given degree of safety, as measured by the probability of failure. There remains the difficult choice of an acceptable degree of safety. In the extended reliability theory this choice is made effectively by transferring the decision to a choice of the maximum severity and the minimum expected resistance, which may be an easier choice on practical grounds, but the decision embodies implicitly a choice of failure probability.

Building controls exist largely to ensure the safety of life, but in some situations the decision to install a given degree of fire protection rests entirely on economic grounds based on cost and the expected saving of property loss. However, even in the consideration of public safety, economics cannot be completely ignored, if only because there is a practical limit to the funds that can be set aside for the provision of safety. The optimum level of fire protection on economic grounds is that which minimises the sum of costs and expected losses, a relatively simple calculation, but one which demands that a price must be placed on the value of human life. This approach, whilst repugnant to many people, is now fairly widely practiced, particularly in the field of road accidents[14]. The technique is largely based on an estimate of the loss of production to the national economy, ignoring almost entirely the social aspects, grief and pain. Unfortunately, little work has been done in the context of fire accidents, but certain problems arise from applying the road accident approach.

i. People dying in fires are usually very young or old and infirm[15]. The loss of production (or in the case of the young, money invested in education etc) is thus very small. The problem is not so much one of evaluating economic loss to the community, but a social one, in evaluating an acceptable sum to be spent on protecting the very young, old and infirm from death by fire. A considerable sum is spent by the community in providing child welfare, hospitals, pensions etc.

A similar approach has been suggested for structural design by Johnson[17], Turkstra[18], Karman[19] and others. Karman states the following conclusions:

i. A common loadbearing structure can be regarded as being optimally designed when the costs incurred by any breakdown of the structure during service time amounts to 2-4 per cent of the original costs of the structure.

ii. The average value of total loss and damage due to structural collapse amounts to about 100-150 times the original building costs. When failure or damage does not result in catastrophic collapse but calls for a comprehensive reinforcing of the structure, the total amount can be approximately 2-4 times the initial costs. These conditions are equivalent to ultimate and non-ultimate limit states.

Hence $150\ p_{\text{ult}} + 4\ p_{\text{non-ult}} = 0.02$

where p_{ult} and $p_{\text{non-ult}}$ mean the probabilities of ultimate and non-ultimate limit states. The ratio $p_{\text{ult}}/p_{\text{non-ult}}$ depends on the type of structure; for brittle structures it is near to one, whilst for plastic structures it might be 1/500. Karman suggests for an average structure

$$\frac{p_{\text{ult}}}{p_{\text{non-ult}}} = \frac{1}{100}$$

so that

$$p_{\text{ult}} = 3.6 \times 10^{-5}$$

$$p_{\text{non-ult}} = 3.6 \times 10^{-3}$$

b. Return periods

In design against earthquake, wind, flood etc it is common practice to refer to an acceptable or design <u>return period</u> instead of a probability of failure. The return period of a phenomenon is the average period elapsing between occurrences of the phenomenon. If T is the return period in years and p the probability of occurrence per year, then

$$Tp = 1$$

Ligtenberg[20] refers to a 10,000 year return period for the design of dams in Holland, i.e. the design level of flood is that which exceeded on average only once in 10,000 years. Ferry Borges[21] refers to a 1,000 year return period for earthquake and wind, although the British Standard Code of Practice for wind loads uses a 50 year return period[22], because of lack of suitable data; structural safety is assured by an additional ad hoc safety factor. In Fig.4 the return period for fire severity is plotted against fire resistance based on the calculations of the preceding paragraphs. It can be seen that 30 mins fire resistance corresponds to a return period of ≈5,000 years.

Note that 'return period' in this context has no real meaning, but is merely a useful way of putting failure probabilities in perspective. Clearly an extrapolation of existing records, covering a period of no more than 20 years, to a period of ≈5,000 years, is not realistic unless one makes some sweeping assumptions about the future and fire hazard being manmade might be regarded as inherently different from natural phenomena such as wind and flood.

So far as fire is concerned, the object is to take a design fire severity corresponding to a return period of T years, T reflecting the seriousness of the consequences of failure. Where the consequences are less serious (in serviceability, for example) T could be considerably smaller, particularly where failure is a result of two or more uncorrelated events. From a national point of view, it may be advisable to take into account the number of a given type of building at risk. If there are N buildings at risk and the return period is T, then the expected annual number of failures is $\frac{N}{T}$. If N is large compared with T this could lead to serious professional and political consequences, and one would conclude that it is desirable to make T of the same order as N. However, as can be seen from Table 1 the number of buildings in a group is of the order 10^5 in many cases (10^6 in the case of houses), thus demanding an extraordinarily high reliability from fire protection, and leading to uneconomic buildings. Hence, at a lower reliability, because of the large number of buildings at risk, we must accept a number of failures of fire protection annually; this is a consequence of economics.

Comparing fire with other hazards, it seems likely that fire is a more serious hazard than wind or earthquake - there are only a few instances of severe wind each year, but 80,000 fires in buildings[23], of which 8,000 approximately are sufficiently serious to cause damage to the building fabric. Furthermore, except in exceptional circumstances wind does not lead to loss of life. It seems reasonable to take the return period appropriate to flood design, i.e. 10,000 years. This leads to quite reasonable demands on the reliability of fire protection, as shown in Table 2.

is the calculus of extreme values[24]. Johnson[17], for example, considers the variable $\alpha = \frac{R-S}{\sigma}$, where σ is the standard deviation of the variable $X = R - S$, and supposes that it has the distribution of extreme values $\phi(-\alpha)$, given by

$$\phi(-\alpha) = \frac{\pi}{\sqrt{6}} \quad \exp\left(-\frac{\pi}{\sqrt{6}}\,\alpha - c - \exp\left(-\frac{\pi}{\sqrt{6}} - c\right)\right).$$

However, to make any progress with this approach the way in which σ varies with R must be known. This approach warrants further study.

APPROXIMATE METHODS OF ASSESSING SAFETY

Some of the difficulties of an exact analysis of safety through the medium of reliability analysis have already been mentioned. In addition there are the formidable difficulties of specification of an acceptable risk, with its attendant legal problems, and the technical difficulties of exploring the tail of a distribution through surveys, extreme value theory, etc. These problems are common to all branches of structural engineering, where exact mathematical models are not always available, there is incomplete data on strengths and loads and where a fairly simple code of practice is the desired end product. Several approximate methods have been proposed which avoid some or all of the difficulties, by removing the necessity for exact specification of failure probabilities, distributions etc. One such method has already been described above (the extended reliability concept), in which a 'factor of uncertainty' was included to allow for the quality of data and the underlying mathematical model, and the specification of a failure probability was avoided by transferring the decision to the stage where characteristic values of R and S were chosen.

Cornell[13] suggests that uniformity of risk is more important, and that existing engineering experience should be relied upon to set the standard. The method is based on the observation that in reliability analysis safety is governed largely by the number of standard deviations of the mean value $R - S$ from zero, denoted by β so that

$$\beta = \frac{\overline{R-S}}{\text{st dev }(R-S)}$$

β is defined as the 'safety index', and it is asserted that uniformity of risk follows if β is held constant. It is convenient to express results in terms of a safety factor θ, defined by

$$\theta = \frac{\overline{R}}{\overline{S}} \qquad \text{then} \qquad \beta = \frac{\theta - 1}{(\theta^2 V_R^2 + V_S^2)^{\frac{1}{2}}}$$

Table 2

Design reliability of fire protection base on a return period of 10^4 years

		Design probability of failure in a given fire
Industry		.023
Houses		.37
Commercial	- shops	.12
Commercial	- offices	.18
Assembly	- entertainment	.008
	- non-residential	.05
Residential	- clubs, hotels etc	.027
Residential	- institutions	-
Storage		.083

CHOICE OF PROBABILITY DISTRIBUTIONS

A further point of difficulty that arises in the estimation of failure probabilities, is the estimation of the distribution functions of fire severity and fire resistance. There are no data at present to estimate the distribution of fire resistance, but as demonstrated above, one can estimate the distribution of fire severity from data on fire loadings and experimental results. However, a statistical difficulty arises because failure is a very rare event, so that the chief interest is in the tail of the distribution, where the data are few and uncertainty is greatest. One possible technique is to fit a known distribution to the data, but the result of this is usually indecisive, giving two or three distributions with equally good fit, but differing sometimes by one or two orders of magnitude in the tail[9].

A further philosophical difficulty arises because of the interpretation of the function of building controls. Consider a population of buildings of a given type, and suppose we agree on a probability of failure, ϕ_F , as defined earlier. As the population size increases the expected number of failures will also increase, so that if the function of building controls is to protect a population of buildings rather than individual buildings, then there should be a higher standard of safety in the larger populations. Clearly one is interested in the way in which the safety margin $R - S$ varies with population size, and it is evident that as the number of buildings at risk increases so the smallest value of $R - S$ will decrease. The technique for this problem

In both of these theories, the parameters β , and the partial safety factors may be derived by calibration with existing codes of practice.

From a practical point of view these methods are attractive, relying only on the first and second moments of the statistical distribution, or on the likely error in calculation. Furthermore, the need to specify a probability of failure is eliminated, and design is through the more familiar factors of safety. The theory requires some adaptation, however, for application to fire problems, which lead to some slight complications. It is proved above that

$$p_F = L_p \, p_f$$

Now, if the risk due to fire is to be the same as the risk due to other structural design features, p_F must be the same, and may be derived from the value of β once a distribution of $R - S$ is given. However, the safety index, β , refers only to the probability of failure <u>given</u> that a fire has occurred so that β takes no account of fire frequency.

But $p_f = \dfrac{p_F}{L_p}$

and since $L_p < 1$, $p_f > p_F$ and therefore the acceptable value of β for fire is less than for structural design for everyday loads.

For example, Cornell gives $\beta = 4$ for structural design, so that if $R - S$ is normally distributed,

$$p_F = 3 \times 10^{-5}$$

Now for offices, $L_p = 0.03$

$$p_f = \frac{p_F}{L_p} = 10^{-3}$$

Hence, assuming once again a normal distribution for $R - S$, for fires in offices,

$$\beta = 3.1$$

Incidentally, to achieve this degree of safety,

$$V_R < \frac{1}{\beta} = \frac{1}{3}$$

so that fire resistance ratings of materials including allowance for variability of materials, testing, workmanship etc must be known to an accuracy of better than 33 per cent.

where V_R , V_S are the coefficients of variation of R and S .

One interesting result that emerges from this treatment follows if we invert the equation to make θ the subject of the equation.

Then $$\theta = \frac{1 + \beta(V_R^2 + V_S^2 - \beta^2 V_R V_S^2)^{\frac{1}{2}}}{1 - \beta^2 V_R^2}$$

This equation has a solution if and only if

$$V_R \leqslant 1/\beta$$

implying that some levels of safety cannot be achieved if the variability of the fire resisting protection is too great. This result is obvious on intuitive grounds, but, it is useful to have an analytic proof.

Cornell shows that V_R and V_S may be calculated on the basis of the variability of different elements of the problem.

If $R = c\,MFP$, then $V_R = (V_M^2 + V_F^2 + V_P^2)^{\frac{1}{2}}$

and $S = k\,TE$, then $V_S = (V_T^2 + V_E^2)^{\frac{1}{2}}$

where M represents material strength (e.g. variability of materials)

F represents fabrication (e.g. standards of workmanship)

P the influence of professional assumptions (e.g. variability of fire tests)

T represents the variability of loads

E is an error term presumably associated with errors of mathematical model.

Ravindra, Heaney and Lind[25] show that the specification of the safety index β and the calculation of the coefficient of variation on the basis of variability of materials, workmanship, loads etc is approximately equivalent to defining a safety factor θ , which is derived as the product of a number of partial safety factors, each associated with one of the variables M , F , P , T and E defined above, so that

$$\theta = \theta_R\,\theta_S$$
$$= \theta_M\,\theta_F\,\theta_P\,\theta_T\,\theta_E$$

Characteristic loads and strengths are calculated on the basis

Characteristic load = mean load + K_1 x standard deviation.

Characteristic strength = mean strength $- K_2$ x standard deviation, where K_1 , K_2 are chosen so that the chance of exceeding the characteristic load or of a strength less than the characteristic strength is small.

The characteristic values take into account the expected variations, but do not allow for loads significantly different from those assumed in design, lack of precision, in design calculations, inadequacy in the method of analysis used, dimensional errors in construction which alter loads or effects of loads or variations in strength due to deteriorations with age; variations of quality etc. Partial safety factors are, therefore, introduced for each limit state, so that the design load is the characteristic load multiplied by a partial safety factor γ_ℓ and the design strength is the characteristic strength divided by a partial safety factor for the material (γ_m) appropriate to that limit state. In principle, the overall or global load factor is obtained as the product of the two partial safety factors, so that global load factor $= \gamma_\ell \ \gamma_m$. The partial safety factors take on different values for each limit state and for different materials etc, and are adjusted to take some account of the seriousness of a particular limit state being reached.

SAFETY OF LIFE

Building controls are based almost entirely on laws to protect life safety, and whilst the engineer has a clear responsibility in the field of economy and serviceability, the main preoccupation of any theory of structural safety to be used in legislation must be the analysis of life safety. Unfortunately, this analysis is not easy to perform for a given degree of structural safety (i.e. a given p_F), more so, in fire, where death can result from a variety of causes such as fire spread, asphyxiation, building collapse, and whereas collapse through excess loads is likely to be sudden and catastrophic, fires last often some hours and means of escape are provided, apart from the rescue operations of brigades. At this stage of the research it is not very fruitful to attempt to analyse the hazard to life due to varying standards of fire resistance, although such an analysis may be possible in tall buildings where evacuation is impracticable.

In the meantime it may be useful to measure the risk of death or injury due to fire against the many other risks that human beings run. This approach has long been recognised in the aircraft industry in connection with structural safety, and in hazard analysis in the chemical industry, where effort is concentrated on eliminating those hazards with the highest accident rate, An alternative approach is to re-calibrate β against existing regulation on fire, thus sacrificing uniformity with other risks in building design. For the distribution F_S (R_O) of Fig.1 for office buildings, the mean severity $\bar{S}$ = 25 min, and the fire resistance specified by regulations is R_O = 60 min. Hence, the factor of safety

$$\theta = \frac{\bar{R}}{\bar{S}}$$

$$= 2 \text{ approximately}$$

$$\therefore \beta = \frac{\theta - 1}{\left(V_R^2\theta^2 + V_S^2\right)^{\frac{1}{2}}}$$

$$= \frac{1}{\left(4V_R^2 + V_S^2\right)^{\frac{1}{2}}}$$

From Fig.1, $V_S = \frac{2}{3}$, and if we assume $V_R = \frac{1}{10}$

$$\beta = 1.5$$

a value much lower than that derived earlier. Even with this approach there must be a rational procedure for modifying β to allow for varying frequencies of fire in different occupancies, and this requires some assumptions about the statistical distributions of R and S.

LIMIT STATE DESIGN

Limit state design is widely employed in European countries[26,27,28,29] as a basis for codes of practice for reinforced and prestressed concrete. An excellent review of this approach has been given by Rowe[10]. In this field, the engineer requires to provide safe and serviceable structures at an economic price, that is with due regard for economy, there must be a reasonable expectation that the structure will not become unfit for its intended purpose during its life. Unfitness for use will occur when a part of the whole of the structure fails, suffers excessive deflection or sustains excessive local damage such as cracking; the stage at which this occurs is described as the limit state. Limit states may also be reached as a result of excessive vibration, fire, deterioration or fatigue, but it should be noted that so far there appear to have been no attempts to define the limit states for fire. The object of design in this approach is to avoid any limit state being attained.

or those which exceed some critical level. It is found in military aircraft[6], for example, that an acceptable structural accident rate is in the region of 1 per 10^7 flying hours; under war conditions this rose somewhat, but pilots and crew regarded types structurally dangerous when the accident rate was greater than about 5 in 10^7.

Kletz[30], following Sowby[31], gives the following fatal accident rate per 10^8 exposed hours.

Staying at home	3
Travelling by car	57
Motor cycling	660

The risk to an able-bodied man at home is much lower - probably about 1 in 10^8.

Fry has presented fire data, from which the number of deaths from fire per 10^8 exposed hours may be calculated:

Hotels	1
Dwellings	0.1

It is clear that the risk of death due to fire is much less than the many other risks that the public take in their everyday lives and occupations. However, one may question whether it is valid to compare accident rates in different pursuits and occupations. One travels by car because of the benefits conferred, such as convenience, saving of time, economy etc, and implicitly accepts the higher risk of death to enjoy these benefits. Hence the acceptable risk is influenced by the benefits - effectively an implicit cost-benefit approach. The same argument might apply to risk of death from different causes within the home, although the cost-benefit aspect is not so well defined. However, recent public disquiet about the incidence of deaths from fire in hotels indicates that the risk of 1 per 10^8 exposed hours (an order of magnitude higher than in dwellings) is not acceptable, and this could be taken as a measure in other occupancies.

DISCUSSION AND CONCLUSIONS

This paper has been devoted to a summary of some of the more important methods that have been proposed in the literature for assessing the safety of a structure and for determining what constitutes an acceptable risk. In fire there are many basic phenomena which must be considered in design:

i. Fire loads, room shapes, size etc are all variable.

ii. The severity of a fire is curtailed by the brigade, but the time of discovery of the fire, and hence the time at which the brigade arrive, is a random variable.

iii. The expected frequency of fires varies over a considerable range for different types of hazard.

iv. Fire protection devices such as sprinklers or detectors may be present and act to reduce fire severity.

v. The fire performance is assessed in a standard furnace test, and this assessment is itself variable, reflected in the variability of the furnace test.

vi. The properties of the materials used in practice are variable, and workmanship is not always subject to strict control.

vii. The assessment of fire severity depends on experiments and calculations which may not only be inaccurate, but also represent reality with differing degrees of accuracy.

viii. Knowledge of fire behaviour depends on materials used, design of buildings etc, and these are changing rapidly. The lifetime of a building is probably about 50 years, so changes in material and design have to be predicted over a considerable period.

ix. For use by designers, and in codes of practice, it is essential that methods of assessing risk should be fairly simple and easy to use.

It is clear that there are two kinds of variability to be considered - statistical and non-statistical (but probabilistic). Statistical variation can be measured by collecting data, and includes fire loads, fire frequency, effects of brigade action etc. Non-statistical variation includes the various uncertainties of design, such as relevance of the calculations, future trends and to some extent the relevance of fire testing. Reliability analysis clearly only copes with variation which can be measured statistically, and it is not easy to incorporate in codes of practice. However, the extended reliability approach of Ang and Amin offers a tool for coping with the remaining uncertainty, with viable alternatives in the first order approach of Cornell, Ravindra et al. It seems clear from modern practice and recommendations by authoritative bodies that the most acceptable approach to engineers lies through the definition of characteristic values of severity and resistance on a statistical basis, in conjunction with a factor of safety expressing the degree of uncertainty in the design. The factor of safety may be calculated as a product of a number of partial safety factors each assessing a different source of uncertainty. This is the basic approach of limit state theory, now widely accepted and to be incorporated in standard codes of practice. The approximate methods offer a way of calculating these safety factors, and the work of Ravindra et al justifies the use of independent partial safety factors to allow for basically inter-related sources of uncertainty.

The object of engineers is to design and manufacture structures which are both safe and economic, and this philosophy should also be appropriate in fire engineering. However, the definition of safety and the specification of acceptable risks presents some problems which are discussed briefly in the paper. The economics are also somewhat uncertain, particularly where the safety of life is concerned; little is known about the interaction of life safety and structural safety or the effectiveness of existing life safety and structural safety or the effectiveness of existing life safety measures. One approach is to accept existing practice as representing the best practical solution, particularly where traditional buildings are concerned, and to calibrate safety factors, probabilities of failure, i.e. risk, against existing designs. This approach has been suggested by other authors and has the virtue of ensuring uniformity of risk, so that future designs may be no more or less safe than traditional designs.

These problems have still to be solved in fire engineering, and many avenues of approach yet remain to be explored. The object of this note is to introduce the subject and to examine some of the methods available for the study of safety on an analytical basis.

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FIG.1 FREQUENCY DIAGRAM FOR PREDICTED FIRE RESISTANCE REQUIREMENTS

MEMO 50 + FR009

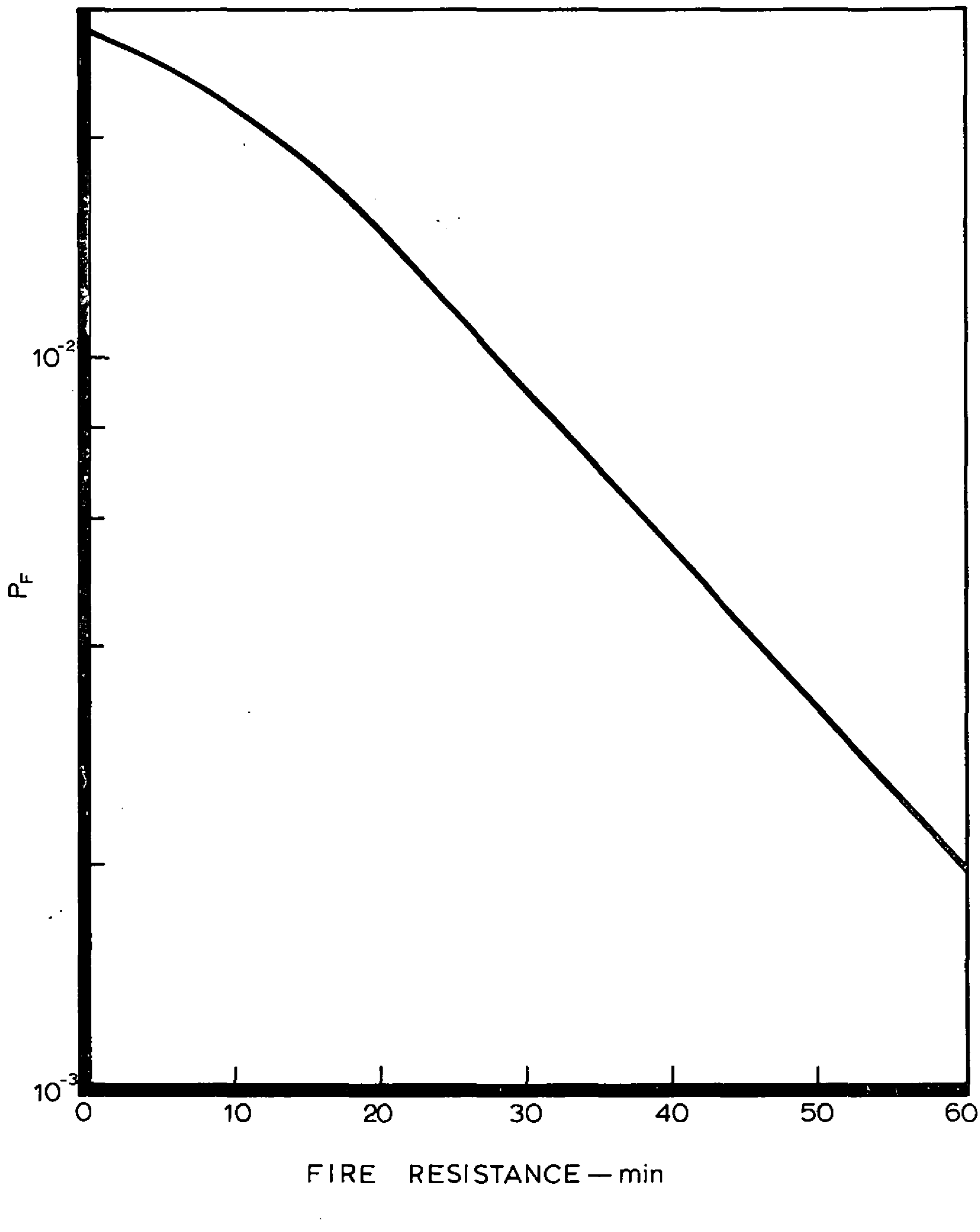

FIG. 2. PROBABILITY OF FAILURE DURING LIFETIME (50 YEARS) OF BUILDING, P_F

1116 FR 909

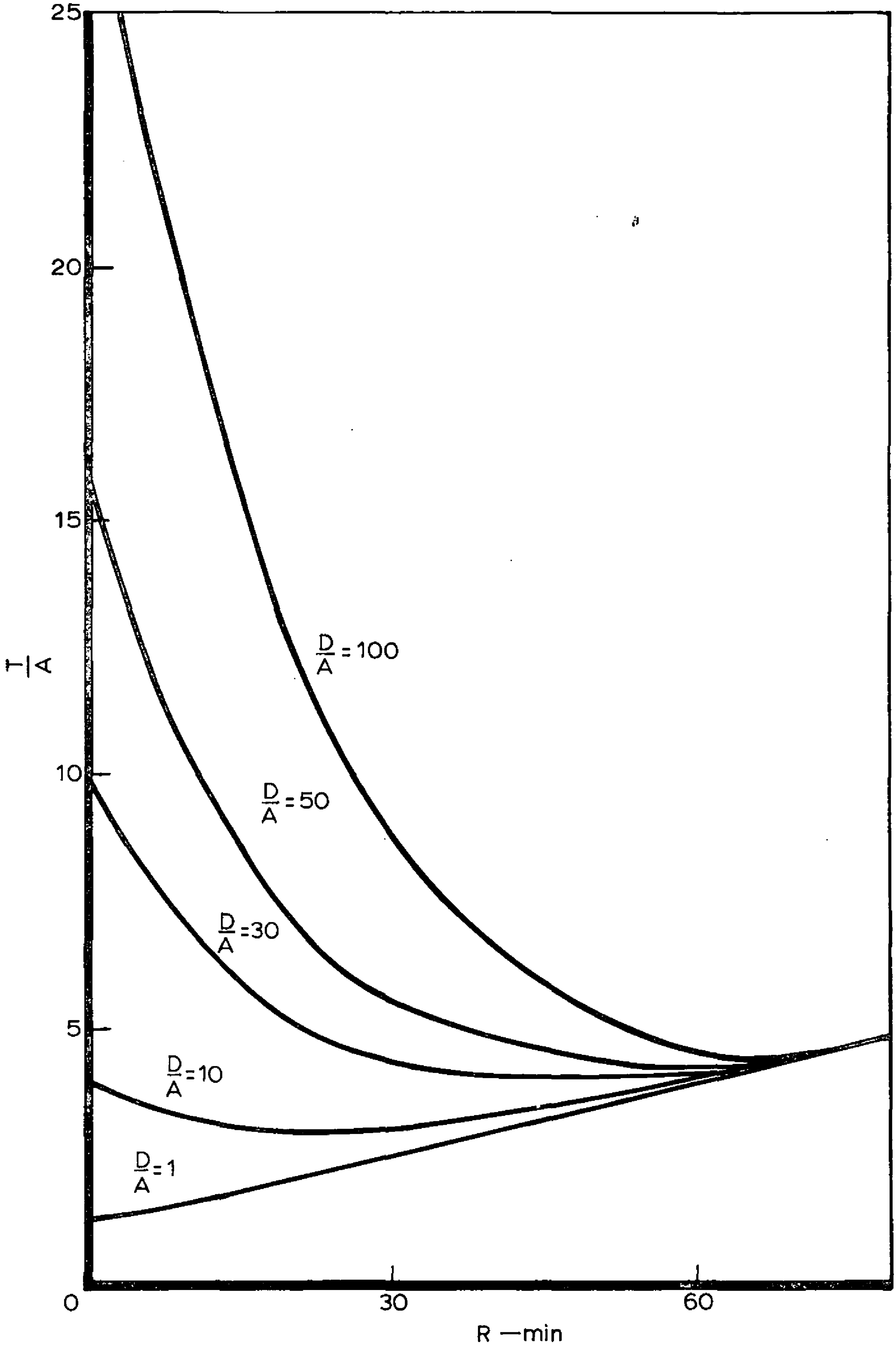

FIG. 3. TOTAL COST AS A FUNCTION OF FIRE RESISTANCE

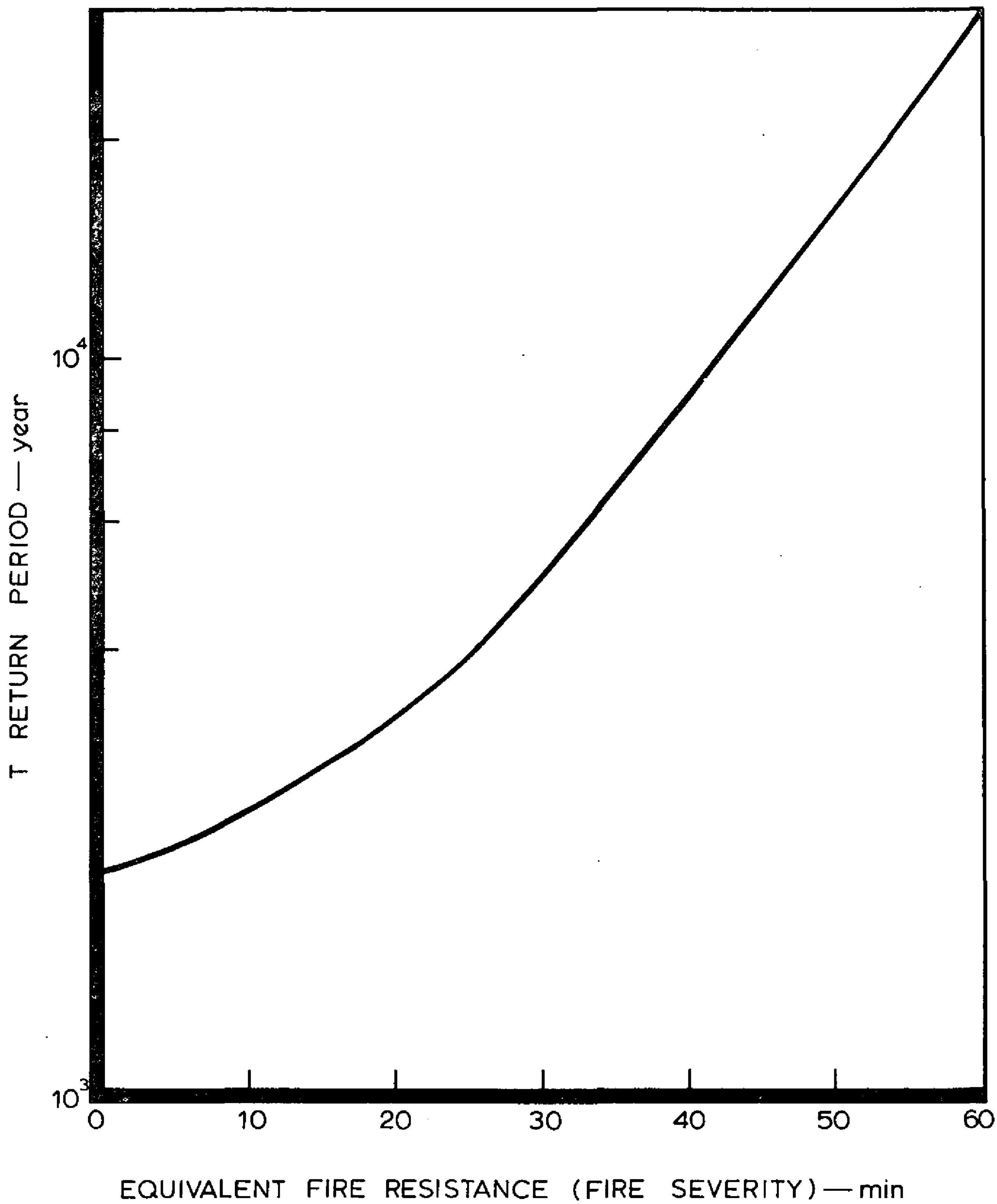

FIG. 4. RETURN PERIOD OF FIRE SEVERITY

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