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AN APPLICATION OF THE THEORY OF EXTREME VALUES FOR ESTIMATING THE DELAY IN THE DISCOVERY OF A FIRE

by

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SUMMARY

The economic benefits from fire detection measures depend upon the reduction they can give in the delays in discovering fires. For evaluating these benefits data on the times of origin of fires are required. These data are not available from the official reports of the fire brigades.

To supplement the information given in fire reports a team from the Station has visited a series of fires in Hertfordshire and Buckinghamshire. These were fires that required the use by the fire brigades of one or more jets. The team has been able to make estimates of maximum possible delays in 14 recent fires in industrial buildings. The buildings involved in these fires were not provided with detectors. From these data it appears that, in the absence of detectors, the average delay in the discovery of all fires in industrial buildings would be of the order of 110 minutes. For the small sample of 14 fires visited the expected delays ranged from 6.1 minutes to 96.7 minutes. These results are based on an application of the statistical theory of extreme values. It has been assumed that the (parent) probability distribution of delays in discovery is of a simple exponential form.

The object of this work is to demonstrate the use of extreme value theory in analysing data, such as those collected by the Station's Fire Visiting Team, for providing results useful for practical purposes and not at this stage to present specific conclusions. There may be other areas in fire research for which extreme value techniques would be useful.

KEY WORDS: Fires, industrial, discovery, delays.

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Introduction

The statistical theory of extreme values has found practical applications in many fields. The oldest problem concerned with this theory arise from the construction of dams for controlling floods in rivers. Extreme pressures, temperatures, rainfalls, etc are examples in the meteorological area. Fracturing of metals, textiles and other materials (ie breaking strength under applied force) are problems in materials testing and quality control requiring the application of extreme value theory. There are many more applications including actuarial problems in life insurance. In general, the theory is likely to be useful in cases where only extreme (ie maximum, large, minimum or small) values of variables under study are available or could be estimated.

Only recently have attempts been made to apply extreme value theory to actuarial problems in fire and other non-life fields. The possibility of applying this theory to the analysis of large losses due to fire is also being studied[1,2]. There may be other problems in fire research that could be classified as extreme value phenomena Fire resistance of building structures falls into this category and could be the subject of later study. In this paper an attempt is made to estimate the expected delay in the discovery of a fire using extreme value theory and certain realistic assumptions. Data are available only for maximum possible delays for a small batch of fires. Expected delay is likely to be important in the economics of automatic detectors.

The parent distribution

In any application of extreme value theory the first step is to identify the nature of the parent probability distribution of the variable under consideration. This is the distribution of the probabilities with which various values of the variable are attained. The known values are extremes from this distribution.

Consider the variable 't' which is the duration between the time of ignition of a fire and the time of its discovery. The fire is on "trial" during every second of its existence. At the end of the trial it is either discovered or not discovered. Let $P_0(t)$ be the probability that the fire is not discovered during a small time interval $(t, t+h)$ and $P_1(t) \left(= 1 - P_0(t)\right)$ the probability that it is discovered. It may be assumed that

$$\frac{1 - P_0(t)}{t} \longrightarrow \lambda \qquad \text{........ (1)}$$

where λ is a positive constant, and represents the probability of discovery per unit time. In essence, the process of being discovered follows a "Poisson" law. As for such processes it may be postulated that the probability of discovery during $(t, t+h)$, where h is small is independent of t (ie the time that has elapsed since ignition). This appears to be a reasonable assumption.

Feller[3] and others have studied the "Poisson" process in detail. They have formulated a system of differential equations for $P_n(t)$ ($n = 0$ or 1 in our case). In particular for $n = 0$,

$$P_0(t+h) = P_0(t)(1 - \lambda h) \qquad \text{........ (2)}$$

approximately which leads to

$$\frac{d}{dt}\left(P_0(t)\right) = -\lambda P_0(t) \qquad \text{........ (3)}$$

From (3), integrating

$$P_0(t) = e^{-\lambda t} \qquad \text{........ (4)}$$

$$P_1(t) = 1 - e^{-\lambda t} \qquad \text{........ (5)}$$

Expression (4) denotes the probability of the fire remaining undiscovered after a lapse of time 't' from ignition, while (5) gives the probability of its being discovered before 't'. The probability of its being discovered increases with time, which is true. The value of λ may vary from one set of conditions to another.

Hence, as a first approximation, the probability distribution of the delay in discovering a fire may be assumed to be of a simple exponential form as in (4) and (5). This distribution is one of the distributions belonging to the exponential family which also contains the well known distributions logistic, gamma, chisquare, normal and log normal.

Extreme value distribution

Consider, now, a particular fire. The variable t, ie the delay in discovering the fire, could possibly assume a number of values ξ_n. The sequence ξ_n consists of independent random variables having the same probability distribution function given by (5) ie

$$F(t) = 1 - e^{-\lambda t} \qquad \text{........ (6)}$$

Consider, in particular, the maximum term of the sequence (ξ_n) given by

$$x = max(\xi_1, \xi_2, \cdots \xi_n) \qquad \text{........ (7)}$$

If we take a number of samples (fires) each with a sequence of values (ξ_n) of t, the variable x pertaining to these samples has a probability distribution of its own, depending on its parent F(t) and sample size n. Conceptually n is large, ie x is the maximum possible delay in discovery of a fire among a large number of possible delays. Since F(t) has been assumed to be of the exponential type the cumulative distribution function of x is

$$\Phi_y = e^{-e^{-y}} \qquad \text{........ (8)}$$

with its derivative

$$\chi_y = e^{-y-e^{-y}} dy \qquad \text{........ (9)}$$

as the density function[4,5]. In (8) and (9) the reduced variate y is given by

$$y = a(x - b) \qquad \text{........ (10)}$$

where b is known as the characteristic largest value and a the value of the "intensity function" of the parent distribution F(t) at the point b. Again, since F(t) has been assumed to be a simple exponential distribution as in (6), the value of the parameter a is equal to the constant value λ for all t[5].

Application to data

The actual time of ignition of a fire is rarely known. However, it is possible to estimate the maximum delay in discovering a fire. For example, if it is known that the workers of a factory left the factory premises at 6 p.m. and that the fire was discovered by the night watchman at 10 p.m. the likely maximum delay in discovering this fire would be 4 hours. Using such techniques in conjunction with a scientific appreciation of the fire incident the Fire Visiting Team have made estimates of the maximum delays in 14 recent fires in industrial buildings in Hertfordshire and Buckinghamshire. These were fires that required the use by the fire brigades of one or more jets and hence large fires in that sense. The buildings involved in these fires did not have detectors. The data furnished by the team are given with the estimates arranged in ascending order, in Table 1.

Table 1

Maximum delays and reduced variates

Rank or Order (i)	Maximum delays (mts) (x)	Cumulative relative frequency ($i/N+1$)	Reduced variate (y)
1	13	0.0667	- 0.9944
2	14	0.1333	- 0.7018
3	15	0.2000	- 0.4759
4	30	0.2666	- 0.2780
5	59	0.3333	- 0.0950
6	60	0.4000	0.0874
7	61	0.4666	0.2726
8	62	0.5333	0.4633
9	120	0.6000	0.6717
10	170	0.6666	0.9040
11	180	0.7333	1.1692
12	240	0.8000	1.4999
13	360	0.8666	1.9469
14	361	0.9333	2.6686

Consider, now, the estimate of maximum delay with the i^{th} order or rank. If it is denoted by $x_{(i)}$ the empirical value of the cumulative relative frequency of $x_{(i)}$, according to Gumbel[5], is

$$\Phi(x_{(i)}) = i/N+1 \qquad \text{........ (11)}$$

where N = 14 ie the number of maximum values available. These cumulative probabilities are shown in Col. 3 of the table. Since percentiles are preserved under linear transformations.

$$\Phi y_{(i)} = i/N+1 \qquad \text{........ (12)}$$

where $y_{(i)}$ is the reduced variate given by (10).
From (8) and (12)

$$y_{(i)} = -\log_e(-\log_e i/N+1) \qquad \text{........ (13)}$$

These reduced values, corresponding to $x_{(i)}$, are shown in Col. (4). The values of y could be calculated from (13) but they were extracted from Table 2 of "Probability Tables for the analysis of extreme value data" published by the National Bureau of Standards[6].

Using the method of least squares the straight line

$$x = b + \frac{y}{a} \qquad \text{........ (14)}$$

was fitted to the pairs of values $x_{(i)}$, $y_{(i)}$.
Expression (14) is merely another form of (10).
The following results were obtained

$$b = 67.88 \qquad \text{........ (15)}$$

$$\frac{1}{a} = 111.31 \qquad \text{........ (16)}$$

The most probable (modal) maximum delay for the sample of fires considered was 67.88 minutes (b). As discussed earlier $\lambda = a$ in (6).
The linear relationship between the observed values $x_{(i)}$ and the theoretical values $y_{(i)}$ is also apparent from Fig 1.

Average delay

Using the foregoing results we could rewrite (6) as

$$F(t) = 1 - e^{-0.0089t} \qquad \text{........ (17)}$$

approximately, where t is measured in minutes. The density function of t is

$$f(t) = \frac{d}{dt} F(t) = \lambda e^{-\lambda t} \qquad \text{........ (18)}$$

Hence the average value of t is

$$\bar{t} = \int_0^\infty \lambda e^{-\lambda t} t \, dt$$

$$= \frac{1}{\lambda} = \frac{1}{a}$$

$$= 111.31 \qquad \text{........ (19)}$$

Hence the average delay in discovering all fires is about 110 minutes. The median (50 per cent point) is 77 minutes ($\log_e 2/a$). Therefore in 50 per cent of the fires the delay is more than 77 minutes. These estimates relate to fires in industrial buildings, without automatic detectors, requiring the use of one or more jets.

Discussion

From theoretical considerations the probability distribution of the delays in discovering fires appears to be of a simple exponential distribution, as a first approximation. Hence, according to extreme value theory, estimates of maximum possible delays would have a probability distribution of the form shown in (8) and (9). This form is known as the "First asymptotic distribution" true of the largest values from exponential type parents.

The straight line relationship between observed maximum values and theoretical maximum values was good as revealed by the correlation coefficient with a value of 0.959. Hence the assumption that the parent distribution is of exponential type is legitimate. But, within this family, it could also be a normal, log normal, gamma, chisquare or a logistic distribution. As an approximation the simple exponential form is easy to handle. The precise nature of the probability distribution of delays could be established if data were available for actual delays instead of maximum delays.

Using the data on maximum delays and the tools furnished by extreme value theory it has been possible to make an estimate of the average delay in discovering all industrial fires. This average value is based on the assumption that the delays in fires occurring in industrial buildings could vary from zero (no delay) to infinity (very large).

The parameter λ has another practical use besides the estimation of average delay in all fires. Suppose there is reason to believe that the delay (t) in discovering a fire in a particular industrial building could have a maximum value of x minutes. For this fire, the density function of t is

$$K \lambda e^{-\lambda t} dt \qquad \text{.........} \quad (20)$$

The value of the integral of (20) over (o, x) should be unity. Hence it is easily seen that

$$K = \frac{1}{1 - e^{-\lambda x}} \qquad \text{.........} \quad (21)$$

The expected value (average) of t over the range (o, x) is given by

$$\begin{aligned} E(t, x) &= \bar{t}(x) \\ &= K \int_0^x \lambda t e^{-\lambda t} dt \\ &= \frac{1}{\lambda} - \frac{x e^{-\lambda x}}{1 - e^{-\lambda x}} \qquad \text{.........} \quad (22) \end{aligned}$$

Using the above result, for fires with maximum delays as in Table 1, the estimates of expected delays are as in Table 2 and Fig 2. The graph could be used for estimating the expected delay in a fire with known maximum delay in the absence of detectors. If it were possible to determine experimentally or otherwise the time taken by a detector to operate, the expected reduction in the delays in the discovery of fires could be estimated. Economic benefits due to detectors depend upon this expected reduction in detection time.

Table 2

Expected delays

Maximum delays (mts)	Expected delays (mts)
13	6.1
14	6.7
15	7.3
30	14.7
59	26.8
60	27.4
61	27.8
62	28.1
120	49.5
170	64.2
180	66.6
240	79.8
360	96.7
361	96.7

Incidentally for $x \rightarrow \infty$, expression (22) tends to the value $\frac{1}{\lambda}$ as obtained before (expression (19).

The estimates obtained in this paper could be improved if data on maximum delays were collected for more fires. The methods of estimation would also require refinement in the light of further research concerning the parent probability distribution of delays.

Another area of research in this field could be to determine the influence of factors causing the delays ie absence of detectors, security patrols etc. But such studies would require a large sample of fires.

Conclusion

Estimates of maximum possible delays in discovering fires in industrial buildings ranged from 13 minutes to 361 minutes in a sample of 14 fires. These buildings were not provided with detectors and required the use by the fire brigades of one or more jets. For such fires the overall average delay appears to be of the order of 110 minutes. In 50 per cent of these fires the delay could be more than 77 minutes. The estimated values of expected delays in the 14 fires considered ranges from 6.1 minutes to 96.7 minutes. The above mentioned results are based on an application of the statistical theory of extreme values and certain realistic assumptions.

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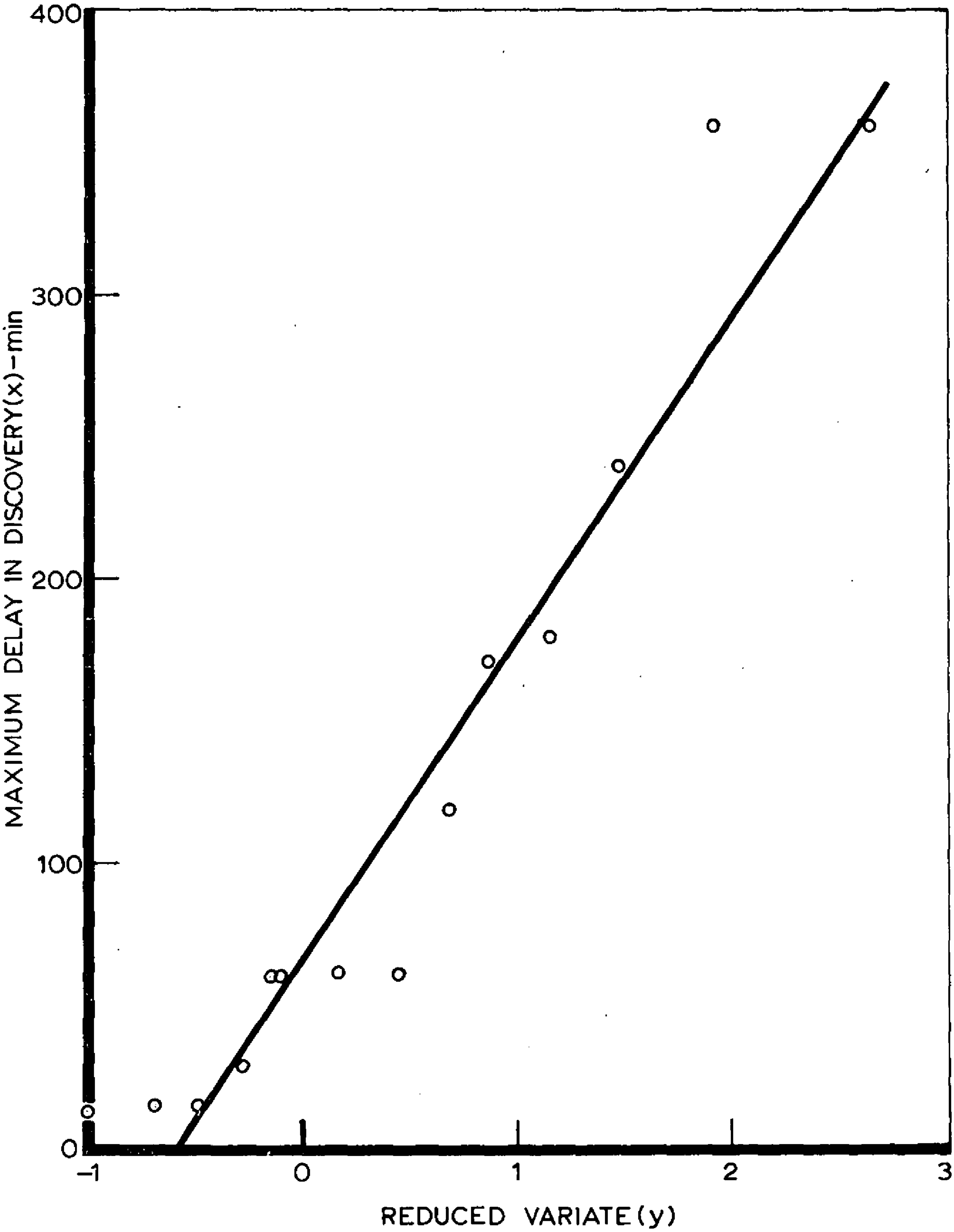

FIG.1 MAXIMUM DELAY AND REDUCED VARIATE

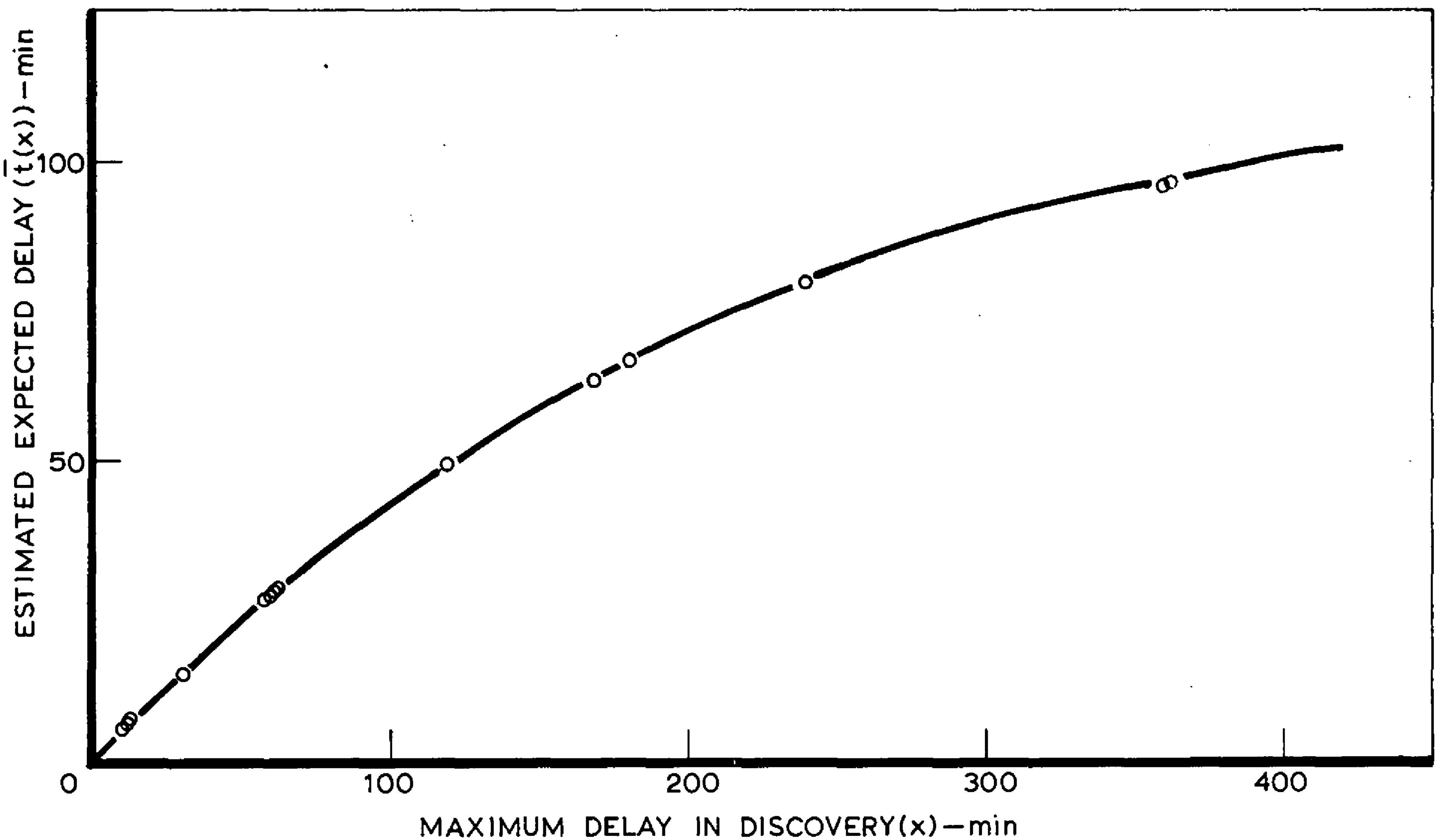

FIG. 2 MAXIMUM DELAY AND EXPECTED DELAY