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PASSIVE AND ACTIVE FIRE PROTECTION - THE OPTIMUM COMBINATION

by

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SUMMARY

It is argued that sprinklers and fire resistance can be used in combination but as they serve different functions they are not freely interchangeable. This note presents a model which shows how to trade off one against the other. The economic optimum combination is that which minimises the sum of costs and expected loss. A major cost parameter for fire protection systems is identified for which few data are available at present. A diagram is presented which shows the principle under which one system or the other is preferred in isolation, where a combination of systems is preferred, or where no protection is justified.

KEY WORDS: Fire resistance, fire protection, sprinkler, economics, cost benefit.

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INTRODUCTION

Building controls normally rely on passive fire protection, and make little or no allowance for the beneficial action of sprinklers or other active measures. However, sprinklers reduce fire severity and hence the risk of large fire losses, and it could be argued that passive protection could be relaxed in favour of active protection in many cases. The means of trading off one against the other on a rational basis is the subject of this paper.

The optimum level of fire protection is that which minimises the sum of costs and expected fire losses. If life loss is to be included in the same sum a value must be placed on saving life. The losses arise from failure of the protection system, either through inadequate design and installation or through component failure. Sprinklers are known to be prone to failure, partly because they require maintenance and are then liable to be turned off, and statistics are available for estimating the risk[1]; passive fire protection is also fallible, and there is a finite risk of it proving inadequate in the event of a fire[2]. The risk of failure can be reduced by more exacting specifications or maintenance requirements, but this increases the cost of the installation. A balance has to be struck between the risk of failure, the ensuing damage, and the cost of reducing this risk or damage: the optimum minimises the total expected loss.

THEORY

1. Role of active and passive fire protection

Passive protection usually consists of structural measures to protect the frame of the building or to prevent spread and is usually measured in terms of its fire resistance. This protection will only be of value in situations where the fire has grown large enough to otherwise damage the building fabrics[2]. The effect of introducing increased fire resistance is to raise the threshold of fire severity at which structural damage or significant loss occurs. The primary effect of sprinklers is to reduce the frequency of fires above a more or less fixed threshold, which is usually smaller than the severity necessary to overcome fire resistance. This is illustrated in Fig 1. The threshold for sprinklers is, of course, altered by changing their sensitivity and spacing.

We shall assume in the following model that if the sprinklers or detectors fail or are inadequate the resulting situation is little different from what would have happened in their absence.

2. Probability

Suppose that fires occur in a building with annual probability p, and that a proportion p_o grow sufficiently large to cause damage to the structure; it is these fires that will benefit from passive protection.

If active protection is installed we assume that there is a probability p_2 that it will fail, and then the fire proceeds as if none were installed so that a proportion p_o are big enough to damage the structure. If the active protection does not fail then let a proportion p_1 damage the structure. P_1 is included for generality but will later be taken as zero for convenience.

Suppose now that passive fire protection is also installed and that if a fire is sufficiently large to cause damage to the structure, then the probability of failure of the passive protection is p_3.

The active protection may fail or operate successfully, and the passive protection may also operate or fail and we assume these events are independant, and that passive protection only influences those fires sufficiently large to damage the structure. The four possible events arising from failure or operation of protection are shown in Table 1, together with the probability of the event. The definitions of the probabilities are summarised in Table 2.

Table 1. Probabilities and damage for combined action of active or passive fire protection

Active	Passive	Probabilities	Expected damage (Utilities)
Operate	Operate	$p(1-p_2)\ (1 - p_1\ p_3)$	D_1
	Fail	$p(1-p_2)\ \ p_1\ p_3$	D_2
Fail	Operate	$pp_2\ (1 - p_op_3)$	D_3
	Fail	$pp_2\ p_op_3$	D_4

Table 2. Definitions of the probabilities

Probability	Definition
p	Probability of a fire occurring per building per year
p_o	Probability of a fire becoming sufficiently large to damage the structure when active protection is either not installed or fails
p_1	Probability of a fire becoming sufficiently large to damage the structure when active protection is installed and operates
p_2	Probability of failure of active protection given that a fire has occurred
p_3	Probability of failure of passive fire protection given a fire that is sufficiently large to damage the structure

3. Damage and expected losses

In the situations defined by Table 1 let D_1, D_2, D_3, D_4, be the expected loss of utility usually but not necessarily expressed in direct money terms. These are the expected losses given that a fire occurs and given that the protection operates or fails as defined. The expected losses include direct and consequential losses and loss of life may be included[3] or considered separately. If life loss is not included; D_1, D_2, D_3, and D_4 then represent damage to building and contents.

Combining the probabilities with expected damage we have an expression for the expected losses (or more strictly of utility)

$$\begin{aligned} E = \; & p(1 - p_2)(1 - p_1p_3)\, D_1 \\ & + p(1 - p_2)\; p_1p_3D_2 \\ & + p\; p_2(1 - p_op_3)D_3 \\ & + pp_op_2p_3D_4 \end{aligned}$$

4. Costs

The expected losses, E, can be reduced if the probabilities of failure p_2 and p_3 are reduced, by, for example, increasing the fire resistance or duplication of active systems. It is possible also that the proportion p_1, damaging the structure when the active system does not fail, could be reduced by a more effective system, but this possibility will be ignored for the time being.

However, p_2 and p_3 can be reduced only at a cost, and unfortunately, little information is available on the way in which the cost varies with specific reductions in failure probability. However, consideration of the effect of duplication of active systems would suggest a cost relationship of the form

$$C_A = -C_2 \log p_2$$

where C_2 is a constant

The results of Kawagoe and Saito[4], Baldwin[2], Maskell and Baldwin[5], suggest that such a relationship holds for passive protection, so that

$$C_p = -C_3 \log p_3$$

where C_3 is a constant.

It is apparent that costs of systems should be assessed in relation to the change in frequency of failures, and the absence of such information makes any detailed analysis of combination of systems unrealistic. The relationships described above will be used for the purposes of illustration and to outline the arguments in general terms.

5. Optimisation

The total expected losses include the sum of costs and expected losses, and then

$$\begin{aligned} E = & - C_2 \log p_2 - C_3 \log p_3 \\ & + p(1 - p_2)(1 - p_1 p_3)\ D_1 + p(1 - p_2) p_1 p_3\ D_2 \\ & + p p_2 (1 - p_o p_3)\ D_3 + p p_o p_2 p_3\ D_4 \end{aligned} \qquad (1)$$

The optimum values of p_2 and p_3 must be such that $E\ (p_1, p_3)$ is a minimum, given by the solution of the simultaneous equations

$$\frac{\partial E}{\partial p_2} = \frac{\partial E}{\partial p_3} = 0$$

Since E is infinite when p_2 or p_3 is zero the solution must represent a minimum.

It should be noted that the third, fourth and fifth terms of equation (1) are asymmetric with respect to p_2 and p_3. This lack of symmetry reflects their different roles in fire protection and ensures restriction on complete interchangeability. The solution of these equations is straightforward but leads to complicated algebraic expressions. We can make considerable simplifications if we assume p_1 = , that is if sprinklers operate successfully, then no damage occurs to the structure. This is probably not too far removed from the truth.

The optimum combination then occurs when $\frac{\partial E}{\partial p_2} = \frac{\partial E}{\partial p_3} = 0$

giving

$$p_2 = \frac{C_2 - C_3}{p(D_3 - D_1)} \tag{2}$$

$$p_3 = \frac{C_3(D_3 - D_1)}{p_o(C_2 - C_3)(D_4 - D_3)} \tag{3}$$

Note that this solution is valid only if

$$0 \leqslant p_2, p_3 \leqslant 1$$

<u>Optimum solution when only one system is installed</u>

(i) Active system

If active protection only is installed, the total expected loss is

$$E_1 = -C_2 \log p_2 + p(1-p_2) D_1 + p\, p_2(1-p_o) D_3 + p\, p_o\, p_2\, D_4 \tag{4}$$

The optimum occurs when $\frac{\partial E_1}{\partial p_2} = 0$

and then

$$p_2 = \frac{C_2}{p(D_3 - D_1) + pp_o(D_4 - D_3)} \tag{5}$$

(ii) Passive system

If passive protection only is installed, the total expected loss is

$$E_2 = -C_3 \log p_3 + p(1 - p_o p_3) D_3 + pp_o p_3 D_4 \tag{6}$$

and the optimum occurs when

$$p_3 = \frac{C_3}{pp_o(D_4 - D_3)} \tag{7}$$

Note that active protection is justified only if $o < p_2 < 1$
and passive protection is justified only if $o < p_3 < 1$.

DISCUSSION AND CONCLUSIONS

The solution given in equations 2, 3, 5 and 7 provide the optimum values of p_2 and p_3 when active and passive protection systems are installed in isolation or in combination. They provide quantative decision criteria for the best system or combination of systems, taking into account risk, cost, damage and losses, by minimising the total expected loss.

The solutions are valid only for $o \leqslant p_2, p_3 \leqslant 1$, because p_2, p_3 are probabilities. A solution outside these bounds, in particular, $p_2, p_3 > 1$ indicates that the particular system (or combination of systems) is not justified and should not be installed. We may use this fact to examine the situations under which active systems in isolation are preferred, passive systems only, a combination of these two, and where neither active nor passive systems are justified.

The conditions derived from equations 5 and 7 for active or passive systems in isolation involve either C_2 or C_3 balanced against expected damage, but the conditions for a combination of systems derived from equations 2 and 3 involve both C_2 and C_3. We therefore examine the relationship between the decision criteria and C_2 and C_3 in a graph (Fig 2): we shall subdivide the area into regions in which one system or the other, a combination or none is justified.

Active systems in isolation are justified or not justified according as

$$C_2 \lesseqgtr p(D_3 - D_1) + pp_o (D_4 - D_3) \qquad (8)$$

This condition is derived from equation 5 by putting $p_2 \lesseqgtr 1$.
Similarly, passive systems are justified or not justified according as

$$C_3 \lesseqgtr pp_o (D_4 - D_3) \qquad (9)$$

In the diagram (Fig 2), the boundary for active systems is denoted by the line AEG, and the boundary for passive systems by the line CEF. Active systems are justified only in the region DOAG, passive systems in the region BOCF. Note that $C_2, C_3 > o$ and we assume $D_3 > D_1$, $D_4 > D_3$ as would usually be the case. Note also that in the region FEG neither system is justified.

Now from equations 2 and 3, a combination of systems is only justified if

$$C_2 - C_3 < p\,(D_3 - D_1) \tag{10}$$

and

$$\frac{C_3}{pp_o\,(D_4 - D_3)} < \frac{C_2 - C_3}{p\,(D_3 - D_1)} \tag{11}$$

These boundaries are marked by the lines HE and OE respectively in the diagram. The region below HE (and HE continued) represents conditions given by inequality 10, and the region above OE (and OE continued) represents inequality (11). Hence a combination of active and passive protection is justified in the region OHE, in fact in this region a combination is preferred to either system in isolation.

In the remaining regions, it is easy to show by examining the losses, that in region HAE active protection is preferred to either passive protection or a combination of systems, and in region OEC passive protection is preferred.

It is worth noting that if a particular system cannot be justified in isolation then it cannot be used in combination with the other, and if neither system can be justified in isolation then neither can any combination. In the region where both systems can be justified, it does not follow automatically that a combination is best, but provided $D_3 > D_1$ a region always exists where a combination is preferred to either system in isolation.

The size of the region OHE, where a combination is preferred, cannot be determined without data, but in the notation of the diagram,

$$\frac{\text{area OHE}}{\text{area OHEC}} = \frac{a}{a + b}$$

If b is small compared with a, then the combination will be preferred for most of the region where both forms of protection are justified, but if a is small compared with b, then one system or the other in isolation will be preferred in most cases.

Before these criteria can be put to practical use in assessing the trade-off of fire resistance against sprinklers or detectors it will be necessary to undertake an analysis of the costs of sprinklers and detectors and fire resistance. The analysis in this note shows that the important factor which has to be considered is the cost of reducing the probability of failure, that is the cost of increasing the effectiveness of the system, $\frac{\partial c}{\partial p}$. This task has already received some attention so far as structural protection is concerned, but little has been done in defining improvements to active protection and the costs.

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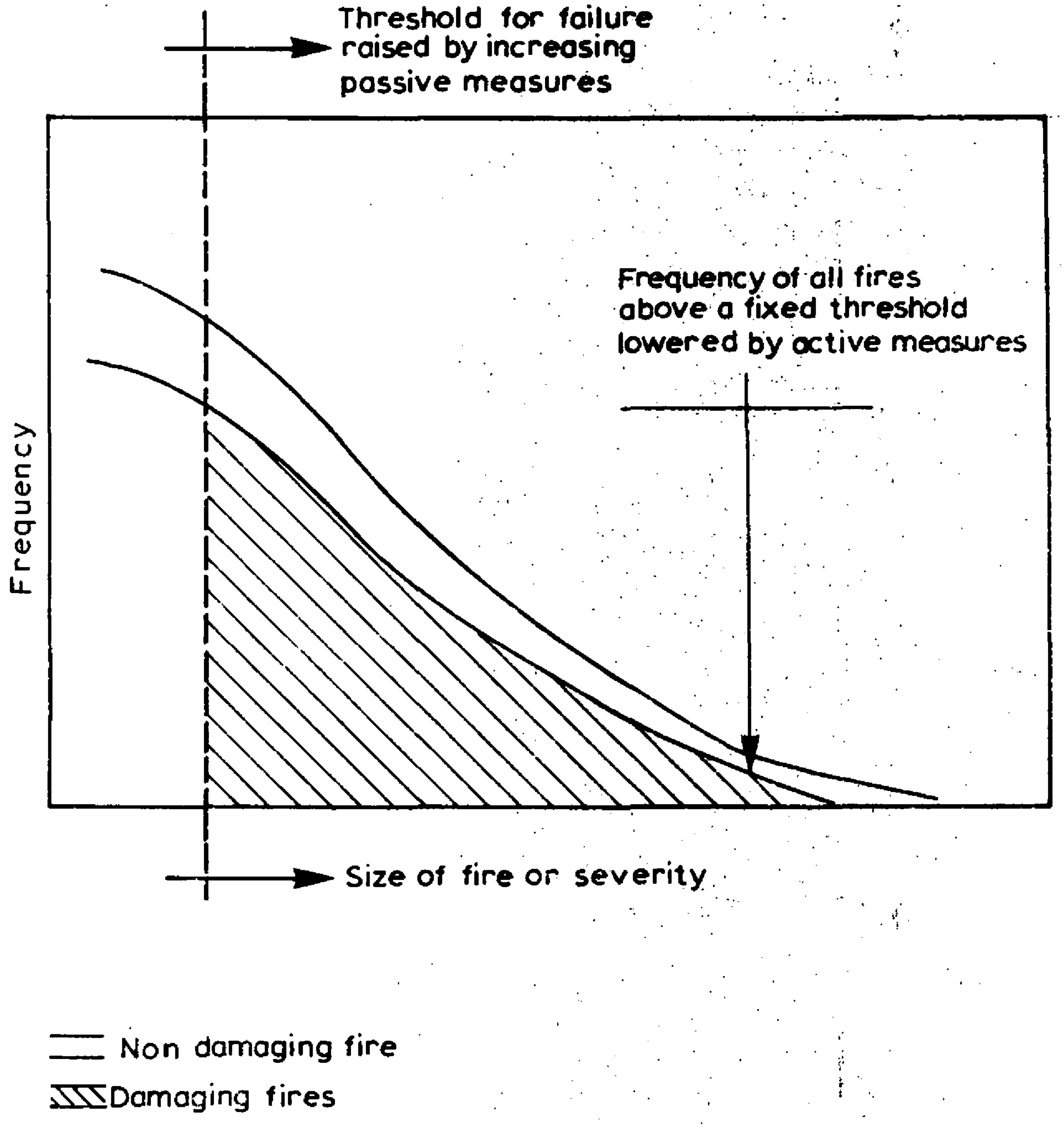
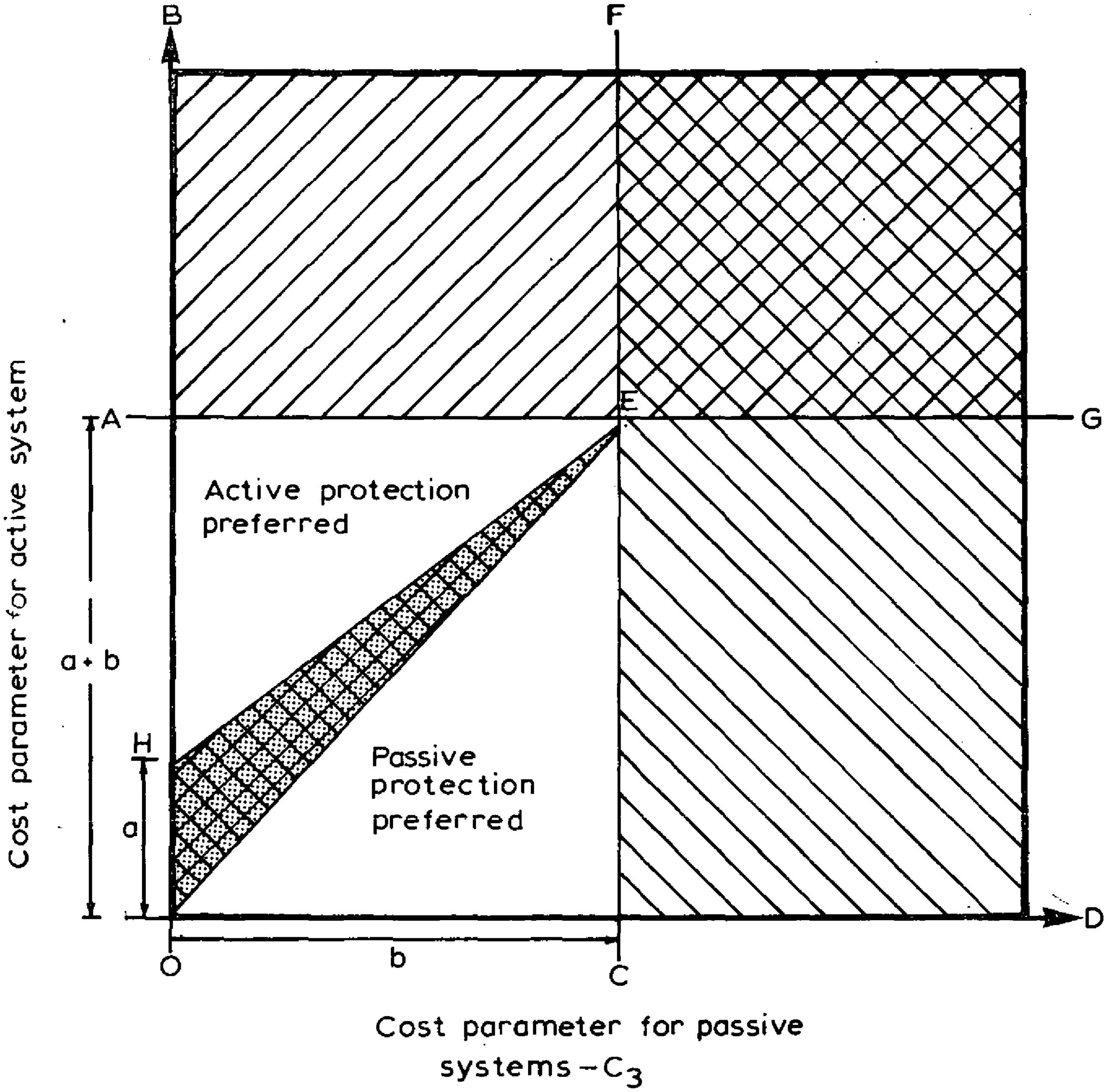


FIG 1 ROLE OF ACTIVE AND PASSIVE MEASURES



$a = p\,(D_3 - D_1)$

$b = pp_o\,(D_4 - D_3)$

Combination of active and passive protection preferred

Active protection not justified

Neither system justified

Passive protection not justified

FIG 2 THE OPTIMUM SYSTEM AS FUNCTION OF PARAMETERS OF EXPECTED DAMAGE & COST