# Principles for Calculation of Load-Bearing and Deformation Behaviour of Composite Structural Elements under Fire Action

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## ABSTRACT

The fire behaviour of composite steel-concrete members was especially investigated by testing as well as by using calculation models for the temperature rise and the load-bearing capacity including deformations and restraints. This combined procedure resulted in well designed members within a comparatively short period. The thermal and structural responses of composite structures are studied theoretically-numerically. Analytical models for determining thermal material properties for concrete and steel are described. Temperature-dependent stress-strain relationships for concrete and structural steel are presented. Finally the application of the analytical models in a computer program is shown.

## KEYWORDS

Composite structures, computer design, computer programs, concrete, fire behaviour, fire protection, heat transfer, material behaviour, reinforcing steel, structural analysis, structural steel, thermal analysis.

## INTRODUCTION

Composite steel-concrete systems are structural elements, which are made of concrete, structural steel and reinforcing steel. In recent years extensive experimental and theoretical-numerical research has been carried out on the load-bearing and deformation behaviour of composite uniaxial structures under fire action by the "Institut für Baustoffe, Massivbau und Brandschutz der Technischen Universität Braunschweig". Fig. 1 shows some typical cross-sections of investigated structures.

The computer Program "STABA-F" was developed to support the investigations theoretically and numerically. Both material and geometric nonlinearities are considered. In order to describe the behaviour of composite structures in fire it is necessary to analyse the thermal and structural response under fire action.

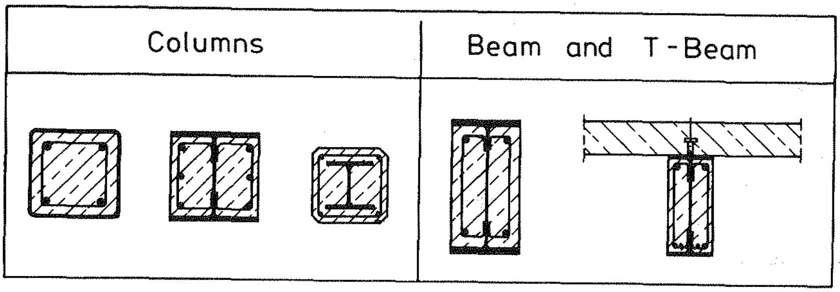


Fig. 1. Typical cross-sections of composite structures

THERMAL ANALYSIS

The heat transfer from the fire to the structural element depends on the type of material and the surface of the member, the colour of the flames, the geometry and the material properties of the furnace walls and the ventilation conditions in the furnace.

Extensive investigations in the heating of structures in the furnaces for columns and beams of the "Institut für Baustoffe, Massivbau und Brandschutz der Technischen Universität Braunschweig" showed that there is sufficient correspondence between measured and calculated temperature distribution in a section, assuming for the convection coefficient of heat transfer $\alpha = 25$ W/m$^2$K and for the resultant emissivity $\varepsilon = 0.3 - 0.7$ for concrete and $\varepsilon = 0.5 - 0.9$ for steel. Heat conduction is described by the well-known equation from Fourier, valid for homogeneous and isotropic materials. Applied to composite structures some simplifactions are necessary:

- water vaporizes as soon as reaching the boiling-point,
- movement of the steam is put together with other effects,
- consumption of energy for vaporizing the water and other peculiarities are taken into account in a simplified way by suitable design values for the specific heat capacity of concrete up to 200 °C,
- concrete is taken into account in a simplified way as a homogeneous material, the heterogeneous structure, as well as capillary pores and internal cracks, are lumped together.

A finite element method in connection with a time-step integration is used to calculate the temperature distribution in the cross-section /1/. The time steps have to be chosen quite small ($\Delta t = 2.5 - 5$ min), because the characteristic values of the thermal conductivity $\lambda$, specific density $\rho$ and specific heat capacity $c_p$ are very much dependent on temperature. Figs. 2 and 3 show the temperature-dependence of the thermal material properties for concrete with predominantly siliceous aggregates and for steel. To determine the temperature distribution a rectangular network is preferred with a maximum width of less than 20 mm. In the region of the structural steel it is advantageous to reduce the width of the network to the thickness of the structural steel profile. The elements of the cross-sectional discretization have corresponding thermal material properties of steel or concrete.

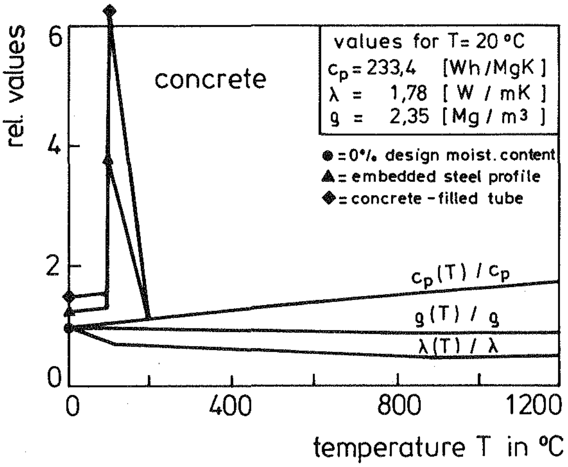


Fig. 2. Design values for thermal material properties for concrete with siliceous aggregates

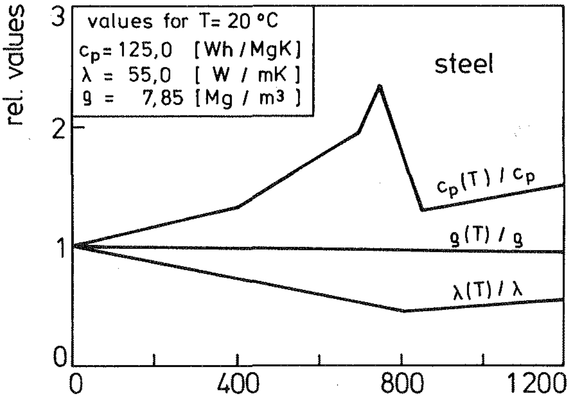


Fig. 3. Design values for thermal material properties for steel

Due to the element temperatures, the thermal strains for the cross-section elements are derived by using the temperature dependent thermal strain for concrete and steel as shown in Fig. 4.

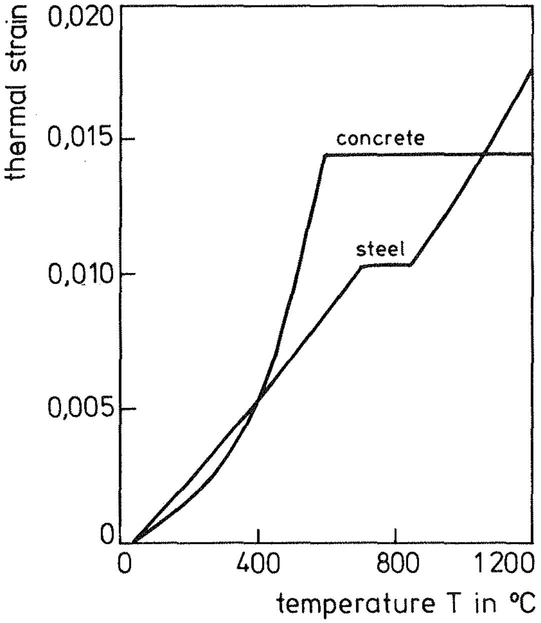


Fig. 4. Design values for thermal strains of concrete with siliceous aggregates and steel

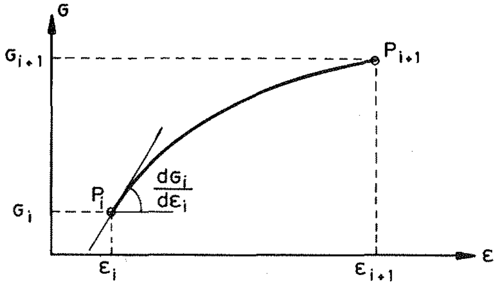
STRESS-STRAIN RELATIONSHIPS AT ELEVATED TEMPERATURES

At a fire situation the material is normally subjected to a transient process with varying temperatures and stresses. To get material data of direct relevance for fire, transient creep tests are carried out. During the test the specimen is subjected to a certain constant load and a constant heating rate. From these tests uniaxial stress-strain characteristics are obtained, which include the temperature-dependent elastic strains and the comparatively large transient creep strains. Fig. 5 shows the analytical model for determining the temperature-dependent stress-strain relationships for concrete and structural steel. The model is also valid for other materials such as reinforcing steel and prestressing steel. A complete stress-strain curve consists of different segments described by equation (1) in Fig. 5. Material constants ($\varepsilon_o$, $\beta_o$, $\overline{E}_o$) are used as starting-points for the parameters $\varepsilon_i$, $\sigma_i$ and $d\sigma_i/d\varepsilon_i$. The temperature dependence is introduced by varying these parameters with respect to temperature. The coefficients $a_k$, $b_k$ and $c_k$ for calculating the material temperature dependence (see Fig. 5) are available at the "Institut für Baustoffe, Massivbau und Brandschutz der Technischen Universität Braunschweig".

Theoretical stress-strain relationships for concrete and structural steel are shown in Figs. 6 - 7. The input parameters for the calculation are listed in Table 1.

| Material constants | Concrete | Structural steel |
|---|---|---|
| $\varepsilon_o$ [-] | $-10^{-3}$ | $10^{-3}$ |
| $\beta_o$ [N/mm²] | $f'_c$ | 240 |
| $\bar{E}_o$ [N/mm²] | $\beta_o \cdot 10^3$ | $2.1 \cdot 10^5$ |
| $f'_c$: specified compressive strength at T = 20 °C | | |

Table 1. Material constants applied to calculate the stress-strain relationships at elevated temperatures for concrete and structural steel (see Figs. 6 - 7)



For $\varepsilon_i \leq \varepsilon < \varepsilon_{i+1}$ is

$$\boxed{\sigma(\varepsilon) = \left(1 - \left(\frac{\varepsilon_{i+1}-\varepsilon}{\varepsilon_{i+1}-\varepsilon_i}\right)^n\right) \cdot \left(\sigma_{i+1} - \sigma_i - \frac{d\sigma_{i+1}}{d\varepsilon_{i+1}} \left(\varepsilon_{i+1}-\varepsilon\right)\right) + \sigma_i} \qquad (1)$$

$$\text{where } n = \frac{\frac{d\sigma_i}{d\varepsilon_i} \cdot (\varepsilon_{i+1}-\varepsilon_i)}{\sigma_{i+1} - \sigma_i - \frac{d\sigma_{i+1}}{d\varepsilon_{i+1}} \cdot (\varepsilon_{i+1}-\varepsilon_i)}$$

$$\boxed{\begin{aligned} \varepsilon_i(T) &= \varepsilon_o \cdot \sum_{k=o}^{3} a_k \cdot T^k \\ \sigma_i(T) &= \beta_o \cdot \sum_{k=o}^{3} b_k \cdot T^k \\ \frac{d\sigma_i(T)}{d\varepsilon_i(T)} &= \bar{E}_o \cdot \sum_{k=o}^{3} c_k \cdot T^k \end{aligned}} \left.\vphantom{\begin{aligned} a \\ b \\ c \\ d \end{aligned}}\right\}$$

Material constants
$\varepsilon_o$, $\beta_o$, $\bar{E}_o$ (see Table 1)
Coefficients for calculating material temperature-dependence
$a_k$, $b_k$, $c_k$
Temperature T [°C]

Fig. 5. Principles for temperature dependent stress-strain relationships

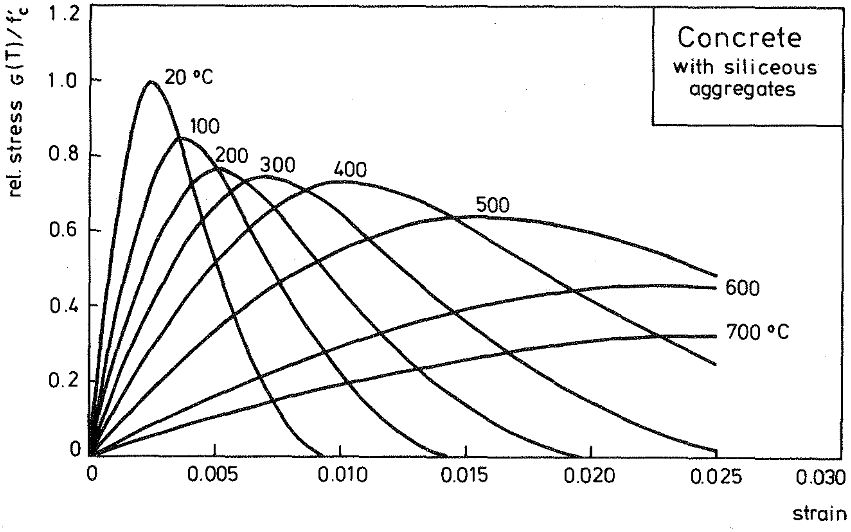


Fig. 6. Stress-strain relationships at elevated temperatures for concrete with siliceous aggregates

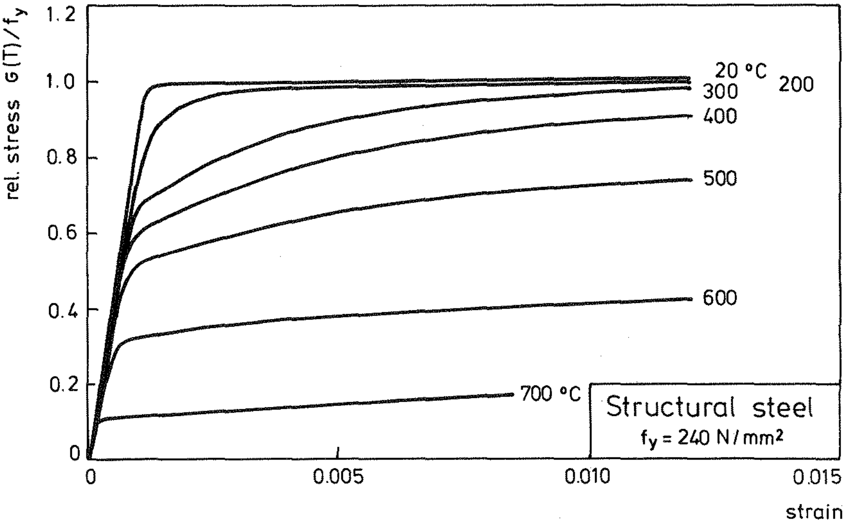


Fig. 7. Stress-strain relationships at elevated temperatures for structural steel

## STRUCTURAL ANALYSIS

The determination of load-bearing and deformation behaviour of composite structural elements under fire action can be described in four parts independent of each other:

- geometric description of the composite structural cross-section,
- determination of the temperature distribution under fire action,
- determination of the nonlinear interaction between bending moment M and curvature 1/r dependent on normal force N and temperature distribution,
- determination of all forces and deformations in accordance with 2nd order theory and any boundary conditions and no matter how loaded.

The first step to determine a composite structural element should be the discretization of the cross-section. In order to ensure a detailed specification of different parts of the cross-section (structural steel, concrete and reinforcement) rectangular and triangular elements are used /1/. The calculated temperature distribution of a composite cross-section, composed of a rolled shape HE 240 B embedded in concrete and by 0.5 % reinforcement, after 30 minutes fire exposure according to ISO 834 is shown in Fig. 8.

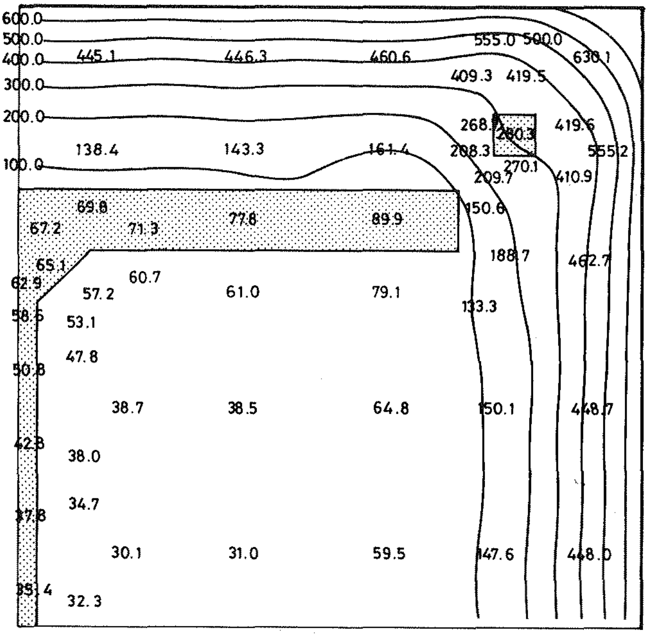


Fig. 8. Calculated temperature distribution of a quarter of a cross-section, consisting of a rolled shape HE 240 B embedded in concrete and 0.5 % reinforcement, after 30 minutes fire exposure according to ISO 834

Knowing the temperature distribution and with assumption of the following simplifications

- the Bernoulli-Navier hypothesis,
- only uniaxial stresses are taken into account, shear stresses are neglected,
- there is no slip between concrete and steel,
- the stress-strain relationships are nonlinear elastic,

it is possible to determine the relations between loads and deflections of a bar. The same network as in the calculation of temperature is used. The stress causing strain $\varepsilon_i^\sigma$ at the location i follows from

$$\varepsilon_i^\sigma = \varepsilon^o + 1/r_y \cdot z_i + 1/r_z \cdot y_i - \varepsilon_i^{th} \tag{2}$$

With the actual temperature $T_i$ the stress $\sigma_i$ can be linked by the temperature dependent stress relationships (see Figs. 6 - 7). Fig. 9 shows the calculated temperature, the strain and the stress distribution in section A-A for a composite cross-section consisting of a concrete filled hollow-section with 3 % reinforcement after 30 minutes fire (ISO 834) exposure.

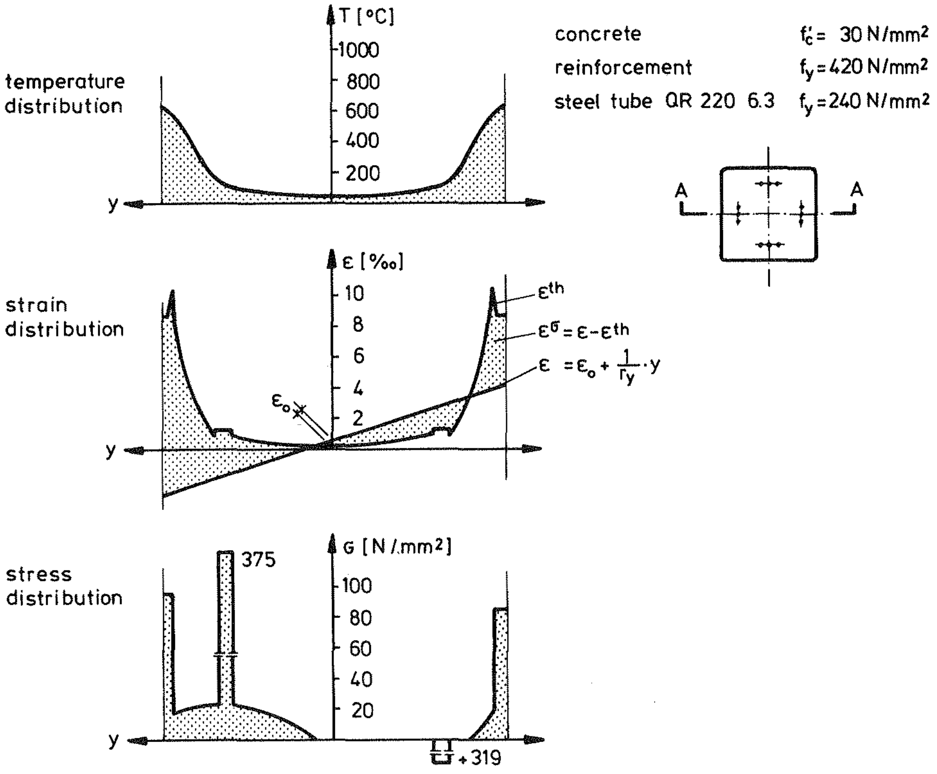


Fig. 9. Temperature, strain and stress distribution in section A-A of a composite cross-section composed of a concrete filled hollow-section after 30 minutes fire exposure (N = 100 kN, $M_y$ = 25 kNm)

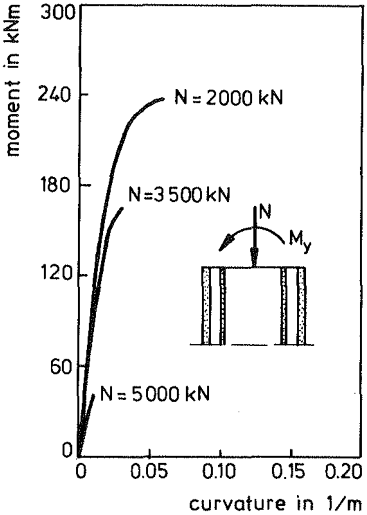
The integration of the stress distribution yields the following forces according to the temperature

$$N = \int_A \sigma \cdot dA \cong \sum_{i=1}^{n} \sigma_i \cdot \Delta A_i \qquad (3)$$

$$M_y = \int_A \sigma \cdot z \cdot dA \cong \sum_{i=1}^{n} \sigma_i \cdot z_i \cdot \Delta A_i \qquad (4)$$

$$M_z = \int_A \sigma \cdot y \cdot dA \cong \sum_{i=1}^{n} \sigma_i \cdot y_i \cdot \Delta A_i \qquad (5)$$

The Relations between the bending moment $M_y$ and the curvature $1/r_z$ of a composite cross-section is shown in Fig. 10. Every curve is dependent on the actual applied normal force N.



concrete $f'_c$ = 40 N/mm²
reinforcement 0.5 % $f_y$ = 420 N/mm²
rolled shape HE 240 B $f_y$ = 240 N/mm²

Fig. 10. Relationship between bending moment $M_y$ and curvature $1/r_z$ dependent on the normal force N.
Cross-section and temperature distribution shown in Fig. 8

An accurate evaluation of the load-bearing behaviour takes into account the influence of mechanical (nonlinear moment/curvature relationship) and geometrical nonlinear interaction (2nd order theory) between load and deformation. To determine bending Moment M, shear force Q, slope of the bar and deflection w the method of transferrring these values from one division to the next is used. The initially unknown forces or deformations at the beginning of the structural element have to be determined by integration in that way that the compatibility condition at the end of the structural element is fulfilled. The definition of the stiffness as the gradient of the partially linear moment/curvature relationship results into a quick converging calculation algorithm. It is possible to determine the load-bearing and deformation behaviour with different moment/curvature relationships along the axis of the bar. Fig. 11 shows evaluated moments and deformations of a composite column under ultimate load after 30 minutes fire exposure. Recalculation of tests showed a good agreement of measurements and determined values /3/. In the research report /4/ the calculated ultimate loads are summed up in charts and diagramms.

concrete $f_c' = 40\ \mathrm{N/mm^2}$
reinforcement 0.5 % $f_y = 420\ \mathrm{N/mm^2}$
rolled shape HE 240 B $f_y = 240\ \mathrm{N/mm^2}$

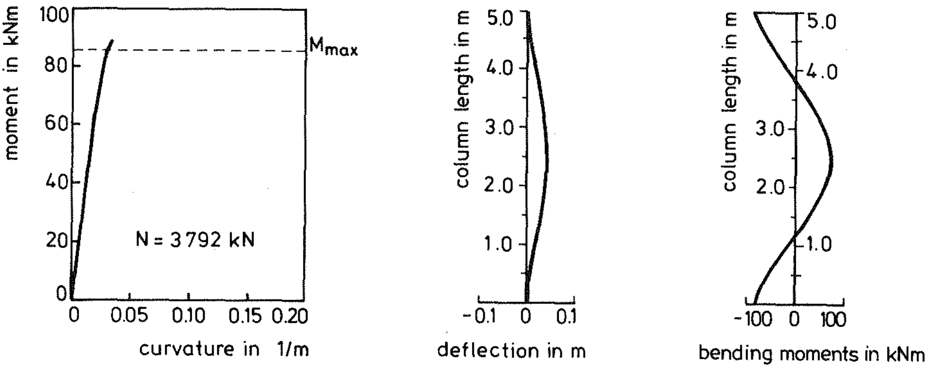


Fig. 11. Ultimate limit state of a composite-column consisting of a rolled shape HE 240 B embedded in concrete and 0.5 % reinforcement after 30 minutes fire exposure

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