

# Initial Reactions to a Fire from a Simple Robotic Device

J. J. BREAUX

Advisor in Computation and Planning  
Asturiana de Zinc, Spain

## 1. INTRODUCTION

It can be argued that an understanding of human response when confronted with catastrophic events should provide more than post hoc rationales and explanations of people's actions. Although an important facet in the interpretation of a given event, emphasis on clarifying the past does not in itself result in methods aimed at, or capable of, resolving potential behaviour in future or hypothetical events of a similar nature. Such future-oriented systems should encompass actual human behaviour to a large degree and as such should not be confused with global simulations in which people are cast as uniform objects in the evaluation of person-hazard outcome (e.g., Stahl, 1975; Melinek & Booth, 1975; Berlin, 1981). Clearly, simulations of this type seek to answer specific questions as, for example, whether a given structure can cope with a given flow of people. Accordingly, the assumption that the target population can be regarded as homogeneous performs the service of allowing one's equations to fully assess worst-case events in which the structure's integrity is being "pushed to the limit" under generalised gasflow hypotheses. However, should one wish to project or understand the behaviour of an individual or isolated group under these models, forthcoming extrapolations or accounts are bound to the collective flow and movement premises inherent in the method. Although these systems might mimic global patterns of action when the target population is sufficiently large and constrained, as in traffic problems (Baerwald, 1965) or large-scale disasters (Fritz & Mathewson, 1957), they can tell the investigator little or nothing at all about motives and actual behaviour. That this latter information is critical to an understanding of how events transpire and, further, how individuals in fact influence the development of the hazard has been noted elsewhere for the case of fires (Breux et al., 1977; Canter, 1980; Keating & Loftus, 1984). In part, the purpose of the present contribution to the symposium is to emphasise the importance, utility and extended scope of predictive models designed to mimic or otherwise emulate the inferred reasoning and ensuing behaviour of people subjected to stressful events.

The present paper is the result of a project to develop a fully automated system capable of generating both "intelligent" analysis of an hazard and coherent activity in response to it. Of further importance at the design stage was the ability to tune or configure the model to reflect the circumstances and behavioural tendencies of hypothetical individuals with the aim of assessing the impact of such factors on the reasoning and activity generated. A specification of the entire model and associated algorithms is due for publication in late 1985 (Breux, in press).

In what follows a specific component of the system is discussed. This unit, referred to as the "Priority controller", has been selected for two reasons. First, it illustrates the design feature of adjusting the logic to simulate a given type of person or persons. Second, the implications for behaviour embodied in this component relate well to the observation that in fires a person's reactions and ultimate fate are often a function of more than just the hazard and the individual's desire to evade it. Often, other objects, animate or otherwise, can be shown to have "driven" the behaviour under observation.

The style of exposition to be followed is intentionally "wordy" with the relevant mathematics inserted where required by the narrative. This follows from the fact that what often appears "obvious" is, upon closer scrutiny, highly complex in both derivation and execution. The reader will also note a tendency to antropomorphise and explain "from the ground up". The author regrets these conventions but considers their use beneficial in conveying a "feeling" for the subject.

## 2. PRELIMINARY CONSIDERATIONS

The global model initially assumes that an individual's response to a known fire (or similar hazard) is a function of one or several objects perceived as threatened. These objects can vary in number and include the self, other people, pets and material possessions. It is assumed that for any given context such an implicit list of objects exists and is brought into awareness by the appreciation of a generalised threat. The relative importance of each object, the degree to which it is perceived as threatened and the effort associated with the reduction of that threat contribute to a decision in which one of the objects (or a number in succession) become the locus of primary concern in terms of which activity is subsequently planned. It is obvious that such objects can be defined as important in different ways across a variety of situations. The model requires access to a ranked list. This list is passed to the decision making Priority controller from a subcomponent called the Major priorities unit.

That people can rank a set of objects in this manner seems reasonable although one might question whether an implicit list has any basis in reality. Many young fathers, for example, indicate little difficulty in ranking objects of this nature when the criterion is "general importance assuming a risk to all objects". Typically, children come first followed by or tied with wife. Lower in the hierarchy one finds car, possessions and house, among others. The model assumes the existence of such an ordered list prior to the fire which is "activated" once a threat is perceived. Its importance is sufficient to merit detailed consideration.

There are numerous ways of deriving and representing ranked priority lists ranging from unweighted rank orders (Diamond, 1959), to adjusted ratio scales (Phillips, 1977) and magnitude models (Curry, 1977). For reasons to become apparent below the modelling process is best served by ratio scale vectors yielding a fair representation of reality given minor inconsistencies in derivation. With few exceptions the first stage in establishing a ratio scale for a set of  $n$  objects or elements consists of obtaining a fundamental number of pairwise comparisons ( $l$ ) which in turn are used to construct a two-dimensional matrix. The cells of this matrix will have values which reflect the relationship between any two objects with respect to the scale or dimension used to compare them. Certain ground rules are imposed on this matrix. Thus, the comparison of an object with itself should constitute an identity relation (not always the case with correlation matrices). Further, comparisons should be consistent. That is, if an object  $\alpha$  is regarded as more important than an object  $\beta$  then  $\beta$ , on its own, must be regarded as less important than  $\alpha$ . This also implies that for more than two objects the matrix should exhibit transitivity. Accordingly, if  $A$  is regarded as more important than  $B$  and, further,  $B$  is regarded as more important than  $C$ , then the matrix must indicate that the cell relating  $A$  and  $C$  shows the former as being more important. This

is the condition of qualitative transitivity. One can also posit quantitative transitivity. If strictly adhered to and if A were regarded as twice as important as B then, logically, B should be regarded as half as important as A. This condition can and should be relaxed insofar as one cannot expect respondents to be absolutely consistent in the quantitative sense. Fortunately, it has been demonstrated that the eigenvector associated with a comparison matrix of moderate size is relatively insensitive to minor consistency violations (Wilkinson, 1965). This has been put to advantage in a subsequent demonstration indicating that the eigenvector corresponding to the maximum eigenvalue of such a matrix yields a cardinal ratio scale for the elements compared (Saaty, 1976; see also Keyfitz, 1968). This type of scale has several advantages not the least of which is that it can be readily concatenated with other indices thereby allowing for the adjustment of scales and set functions, a capability required by the model. Indeed, it is this type of vector which characterises output from the Major priorities unit for eventual use by the Priority controller.

A specification of such a vector can be illustrated using data obtained from a 48 year old widowed woman with a pet dog and a number of valued possessions. Following a short discussion it was found that three objects were a cause of primary concern to her; the dog, a number of documents including photos (kept in a box in the kitchen) and several pieces of jewelry given to her by her parents and late husband (kept in the main bedroom). Employing a ten-point scale for gauging relative importance (Saaty & Khouja, 1976) she was asked to make a series of comparisons involving the three objects assuming an equivalent threat to each. This produced the following:

	Dog	Documents	Jewelry
Dog	1	4	6
Documents		1	3
Jewelry			1

The entries reflect the fact that the dog is regarded as more important to her than documents and demonstrably more important than jewelry. Similarly, the documents are judged as weakly more important than jewelry. The "1's" in the main diagonal indicate identity relations, that is, the assumption that an object when compared with itself, given the same criterion, is neither more or less than itself. Clearly, such a matrix is not symmetrical and the missing values, intentionally excluded from the comparison task, are assigned the corresponding reciprocals of the principal values. This gives:

	Dog	Documents	Jewelry
Dog	1	4	6
Documents	1/4	1	3
Jewelry	1/6	1/3	1

One is now in a position to solve the eigenvector problem which requires a solution to the following equation:

$$Ax = \lambda_{\max} x ,$$

where A is the square matrix, x a column vector and  $\lambda_{\max}$  the maximum eigenvalue. In fact one is primarily interested in x. Some readers will be more familiar with the form,  $(A - \lambda_{\max} I)x = 0 ,$

where I is the identity matrix, that is,  $AA^{-1} = I.$  For the above,

$$x = \begin{pmatrix} .690 \\ .218 \\ .092 \end{pmatrix} .$$

This provides a robust, one-dimensional indication of relative importance. It is this relative assessment of objects which shall be assumed to reside in a potential victim's head, being cued upon the perception of threat. Of course, such a precise specification will not characterise a real person but should nonetheless adequately reflect subjective relative importance. In any event, the vector per se is intended for the modelling process.

### 3. PRIORITY CONTROLLER - SPECIFICATION

The main task of the controller is to provide a revised ranking of relevant objects given new information which is a function of the hazard. That object with the highest adjusted rank will subsequently become the focus of short or long-term planning. In order to effect this adjustment of the vector derived in the preceding section, the controller processes data through a two-phased procedure. First an integration of hazard-related information is accomplished by evaluating the objects on a number of criteria specific to the situation. The second phase consists of postulating a decision function which places the criteria in perspective and merges these to obtain a final revised ranking. Crucial to this undertaking is the representation of objects in a given criterion. In the model this involves the use of inclusion functions.

An inclusion function allows one to specify, for a given object, the degree to which it "participates" or shares in a criterion. The notion of degree is important in this context and requires a specialised approach.

Traditionally, a criterion such as threat might have been applied to an object according to classical set theory. This would result in discrete appraisals, for example, "the object is threatened" or "the object is not threatened". The all-or-none status of these statements often poorly describes how people view a goal or object. Items may be perceived as only partly represented in a set (the set of threatened objects, say). This is often apparent in the study of people's behaviour in fires. They frequently indicate differential concern or worry for a variety of objects important to them and, further, perceive the hazard as posing a differential threat to these objects. It can be argued that only in special cases does one encounter totally dichotomised threat perception, that is, a mental allocation of objects to either the threatened or non-threatened category. The importance of obtaining an adequate and reasonable representation of this process for the model has resulted in the inclusion of techniques normally associated with soft or "fuzzy" set theory (Zadeh, 1971). This allows for a graded specification of how much an object belongs to a set which itself may be vague. By stressing the degree of inclusion one is more likely to avoid over-aggregated models and solutions based on functions with coarse discrimination.

To make the use of these functions somewhat more apparent, the above mentioned woman can be placed in an hypothetical event. Assumed is a fire in the main bedroom which is next door to her living room (sitting room). Her dog is in the guest bedroom. The kitchen is at some remove from this latter location. (This example, including plans of the apartment, is discussed at length elsewhere, Breaux, in press).

The model assumes that once a threat is perceived to exist two functions are subjectively evaluated. The first of these assesses the degree to which each of the objects in the Major priorities vector is threatened. This vector may include the person making the assessment as well, but for simplicity this will be treated somewhat differently in the present paper. Accordingly, an inclusion function for "threat to object" can be advanced. Continuing with the above example one might have:

$$F_{\text{threat to object}} = \left\{ \frac{.6}{\text{Dog}}, \frac{.2}{\text{Documents}}, \frac{.9}{\text{Jewelry}} \right\} .$$

The metric for evaluation is a scale ranging from 0 (no threat) to 1 (absolute threat). Given this event and its corresponding threat function it can be seen that since the fire is in the main bedroom, jewelry is most threatened. The close physical proximity of the guest bedroom to the hazard source places the dog next, with documents (in the kitchen) least threatened. The ability to postulate this function presupposes outside information which in the model is provided by the External features component. For the moment this information is assumed as given. Further, when automated, the system assigns threat values which are inversely proportional to the object's distance from the source of the hazard.

The second function to be advanced assumes that the individual assesses the difficulty of reducing threat to each of the objects. In order to render this function compatible with the decision process to be discussed below, it is stated in terms of the 'ease' with which threat reduction can be effected. Given the context and perceived degree of threat, our respondent's "ease of threat reduction" function might be:

$$F_{\text{ease threat reduction}} = \left\{ \frac{.5}{\text{Dog}}, \frac{.9}{\text{Documents}}, \frac{.1}{\text{Jewelry}} \right\} .$$

The metric ranges from 0 (least ease, that is, greatest difficulty) to 1 (least possible difficulty). Thus, the threat associated with documents is seen as the easiest to relieve (they are far from the fire, the kitchen near an exit), followed by dog (nearer the fire). The threatened status of jewelry is most difficult to alter since this object is in the same room as the fire and would incur considerable risk. In the model this function is not simply correlated with distance from the fire. A Resource component attenuates the function given such factors as availability of fire-fighting materials as well as proximity to the hazard.

At this point it should be stated that there are a number of ways whereby one could attempt to alleviate threat to objects whether these include the self or not. This is the subject of the Plan generator. Presently the primary concern is with the assumption that if an individual values a number of objects and these are perceived as threatened then an improvement in their status is contemplated.

Both functions must now be adjusted for object importance. That is, degree of inclusion in the threat and ease functions are weighted for relative importance insofar as the objects are of unequal salience to the victim. How this might occur in reality is questionable. For the purpose of the model a multiplicative process is assumed and, further, is considered a reasonable approximation. This is accomplished by post-multiplying the inclusion functions by the Major priorities vector thereby adjusting the value of the former, that is,

$F_x = F' .$   
Continuing with the above example:

$$F_{\text{threat}} = \left\{ \frac{.6}{\text{Dog}}, \frac{.2}{\text{Documents}}, \frac{.9}{\text{Jewelry}} \right\} \cdot \begin{pmatrix} .690 \\ .218 \\ .092 \end{pmatrix} =$$

$$F'_{\text{threat}} = \left\{ \frac{.414}{\text{Dog}}, \frac{.044}{\text{Documents}}, \frac{.083}{\text{Jewelry}} \right\} .$$

And,

$$F_{\text{ease}} = \left\{ \begin{array}{ccc} .5 & .9 & .1 \\ \text{Dog} & \text{Documents} & \text{Jewelry} \end{array} \right\} \cdot \begin{pmatrix} .690 \\ .218 \\ .092 \end{pmatrix} =$$

$$F'_{\text{ease}} = \left\{ \begin{array}{ccc} .345 & .196 & .009 \\ \text{Dog} & \text{Documents} & \text{Jewelry} \end{array} \right\} .$$

The new functions (F') can be regarded as placing in perspective the concern associated with objects given perceived hazard impact. This method of adjusting the "raw" inclusion functions realises or mimics two phenomena often detected in the accounts of those who have been through such an ordeal. First, the degree to which a valued object is regarded as threatened cannot truly be understood as entailing an objective process. The role or value of that object in one's life will colour or distort its status in more circumscribed events. This can be expected to be especially so relative to children and other loved ones. In the model the relative importance of objects, in general, is given by x. The impact of the event is given by the inclusion functions which are "personalised" by x. In deriving the F'<sup>threat</sup> function there is the implicit assumption that perceived threat depends on sensory data<sup>threat</sup> (including extrapolations) interacting with an object's meaning to the perceiver. In the example, this has resulted in dog being relatively most threatened.

The second phenomenon brought out by the above concerns the "ease of threat reduction" function. In the example the unadjusted function for ease of threat reduction places documents in the best position. Were one's actions relative to an object based solely on this dimension it might be expected that documents would be an initial target. By adjusting this function for object importance (x) a distortion is postulated which improves the position of dog (F'<sup>ease</sup>). In this case the highly disproportionate importance of an object has attenuated the level of difficulty associated with improving its threatened status.

In real life there is usually a rapid appreciation of the true difficulty in reducing threat to an object. This occurs once action is taken relative to that object. Such statements as, "I didn't expect it to be so hot" or "the smoke didn't seem so bad until you got into it" reflect the subsequent reappraisal. In the model the ease function is highly sensitive to elapsed time which in turn is correlated with the fire growth functions employed to drive the event.

#### 4. PRIORITY CONTROLLER - DECISION STAGE

As noted previously, the role of the adjusted F functions (the criteria) is to provide a basis for a subsequent decision. This will result in one of the objects being nominated as an immediate focus of primary concern. Other objects may follow or take its place but at a given instant the recognises but one. (The Plan generator can specify a temporal series of such objects.) Because the model employs multiple criteria, their relative importance prior to the decision must be clarified. This can be hypothesised or based on interviews. For example, the respondent asserted that she would take "great risks" to save the dog if it were threatened. This stated predisposition could be taken to imply that threat was considerably more important than ease as a predecision variable. This might be represented as a matrix in which criteria are scaled for relative influence, for example:

	Threat	Ease
Threat	1	7
Ease	.143	1

for which the primary eigenvector is:

$$\begin{pmatrix} .88 \\ .12 \end{pmatrix} .$$

This vector becomes the model's representation of relative importance. It now remains to adjust the  $F^i$  functions for differential influence. By effecting this adjustment now the difficulty in projecting the subsequent decision is reduced. This is because irrespective of the decision function used to combine the criteria, these (the criteria) will already have been equated or made comparable.

One method of placing the criteria in perspective derives from the technique of exponential quantification in fuzzy set theory (cf. Zadeh, 1976). Accordingly, concepts are regarded as definable in terms of exponential relations. Applying an extension of this approach (cf. Yager, 1977) the preceding vector  $\begin{pmatrix} .88 \\ .12 \end{pmatrix}$  can be used to define a scalar,  $\alpha \geq 0$ , such that,

$$F^i \alpha = \left\{ \frac{x \alpha}{\text{object a}} , \frac{y \alpha}{\text{object b}} , \frac{z \alpha}{\text{object c}} \right\} .$$

Where  $x$ ,  $y$ , and  $z$  are the values for objects  $a$ ,  $b$ , and  $c$  in criterion  $F^i$ . The values of  $\alpha$  are given by:

$$nx = \begin{pmatrix} nx_1 \\ nx_2 \\ \vdots \\ nx_n \end{pmatrix} ,$$

where  $n$  is the number of criteria (in this case two, threat and ease) and  $x_i$  the vector value corresponding to the  $i^{\text{th}}$  criterion. For the present example this yields:

$$\begin{pmatrix} 2 \cdot .88 \\ 2 \cdot .12 \end{pmatrix} = \begin{pmatrix} 1.76 \\ .24 \end{pmatrix} .$$

The resulting  $\alpha$ 's, one corresponding to  $F^{\text{threat}}$  (1.76) and the other to  $F^{\text{ease}}$  (.24), are the operative exponents used to adjust the criteria for decision influence. The role of this manipulation with respect to exponential quantification is best understood by regarding the criterion importance vector  $\begin{pmatrix} .88 \\ .12 \end{pmatrix}$  as reflecting a superordinate criterion (impact on decision) in terms of which the  $F^i$ 's are expressed. The effect of  $\alpha > 1$  is to further accentuate an important object as one might expect if the criterion is relatively influential in reaching a decision. Conversely, for  $\alpha < 1$  the effect is to level or deaccentuate object values for that criterion. For  $\alpha = 1$ , the inclusion function is unaltered.

Continuing, for  $F^{\text{threat}}$  and  $F^{\text{ease}}$  one has:

$$F^{\text{threat}} (1.76) = \left\{ \frac{.218}{\text{Dog}} , \frac{.004}{\text{Documents}} , \frac{.013}{\text{Jewelry}} \right\} .$$

$$F^{\text{ease}} (.24) = \left\{ \frac{.775}{\text{Dog}} , \frac{.676}{\text{Documents}} , \frac{.323}{\text{Jewelry}} \right\} .$$

Since the criteria are now fully adjusted their contribution to the decision can be based on a simple conjunctive algorithm. The final selection of an object is based on a decision function, D, in which the highest ranked element is chosen. The algorithm most suitable to produce D given the criteria is a matter of debate. Initially appealing would be D based on additive worth. This has been avoided and instead a maximin strategy selected. For the above this implies:

$$D = \text{Min}(F'_{\text{threat}} \cap F'_{\text{ease}}) \text{ for all objects, viz.}$$

Threat	(.218 , .004 , .013)
Ease	(.775 , .676 , .323)
D =	(.218 , .004 , .013) .

It is apparent that the dog receives the highest rank. In fact, both the additive worth and maximin approaches select the dog given these inclusion functions. However, under more complex conditions the maximin strategy can be used to introduce an element of "hedging" when acting on minimal information, as when one is a victim in a strange environment. Given this approach one is essentially basing a decision on a set (D) of least attractive values. In the present example this has resulted in the highest ranking for the dog and, given this, the model would now start to generate activity in the service of that object.

Over the course of the event we can expect D to change given a variety of circumstances. The present exercise constitutes a single pass through the logic. For a given fire numerous cycles can be expected and an example indicating this is presented elsewhere (Breux, in press).

## 5. CLOSING COMMENTS

The design of robotic devices capable of exhibiting coherent behavioural structure, if only at the level of action strings on a computer printout, requires a departure from classical statephase transition schemes based on inherent Markovian principles. Even where semi-Markov assumptions are employed, thereby incorporating time as a factor, it can be argued that such methods are better suited to the retrospective analysis of an event as opposed to its projection in future time.

It is likely that the manner in which human beings make decisions, as well as the implications of these for related activity, bear little resemblance to systems based on a finite (even if long) history. It is here where the present paper has attempted to indicate, in highly simplified terms, a way out. This need not imply that the use of transition spaces be disregarded. However, at the very least, it indicates that the successive selection of such spaces is arrived at in a manner more consistent with what is known about human potential. Although not treated herein, the present model does make use of low-level state transition matrices where applicable (associated with a Plan generator and Plan executor). These inclusions can be regarded as locally valid manifolds whose necessity is irreducible given spatial and 'logical' constraints on the individual(s) modeled. That is, in certain cases they can be regarded as primitive sets whose coherence does not require derivation. (For example, to get from one point to another might require a series of directed steps of an invariant nature.)

In terms of computation the model discriminates two processes which can be run in parallel. The target's logic and behaviour, and the nature of the hazard. For fires the latter is given by inputted parameters of a fire growth function whose derivative is used to progressively invalidate use of the building or area. Because the target and hazard processes are reciprocally contrient, people (that is, hypothetical entities) can influence the progression of the hazard, for better or for worse. Similarly, depending on circumstances, the hazard parameters will account for target behaviour.



Parts of this system have been executed on an Hewlett-Packard 3040 and an IBM-XT. A real-time robotic device would require somewhat greater sophistication. However, this would not necessarily imply a LISP or PROLOG machine insofar as the relevant algorithms indicate structural integrity. This is considered to be an important point. Even as concerns string processing it can be argued that matrix localisation of string components is subsidiary to the manner in which these are pointed to (in computer memory). Contrary to popular belief, "If-Then" rule systems might not require list or logical processing systems for their realisation. The "bottom line" is who or what can do it in time.

Footnote

(1) Experience with the present and alternative comparison techniques has indicated to the author that people find it easier (and are more consistent) when making "greater than" comparisons than "less than" judgements where these additionally require some numerical or verbal quantification. For this reason comparisons characterised by the former distinction are taken as the base data with converse instances being assigned the reciprocal of the corresponding value. This has the further advantage of limiting the number of comparisons to be made which in certain circumstances can be more than the respondent is willing to tolerate. In the example this number, C, is limited to:

$$C = \binom{n}{2},$$

where n is the number of objects. This is equivalent to saying that the number of essential comparisons is given by:

$$\frac{n!}{[(2!)][(n-2)!]} = \frac{6}{2} = 3$$

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