Exponential Model of Fire Growth

G. RAMACHANDRAN
Fire Research Station
Borehamwood, Herts, WD6 2BL, United Kingdom

ABSTRACT

This paper discusses a statistical model for estimating the rate of fire growth and doubling time from data on actual fires attended by fire brigades. The model is based on the assumption that the area damaged in a fire has an exponential relationship with duration of burning. Results of application are presented for a few groups of industrial buildings, three major areas of fire origin and two materials ignited first.

Keywords: Exponential Model, Fire, Growth, Actual Fires, Statistical Analysis.

INTRODUCTION

Rate of fire growth plays a central role in planning the evacuation of a building in the event of a fire and determining effective methods of detection and suppression. According to scientific theories supported by experimental evidence, the heat output of a fire increases as an exponential function of time. This implies that the area damaged by direct burning has an exponential relationship with duration of burning. Assuming this relationship, it is shown in this paper how the rate of fire growth can be estimated by a logarithmic regression analysis of data on actual fires attended by fire brigades. Applying this statistical method fire growth rates and doubling times in terms of area destroyed were estimated for eight groups of industrial buildings, three areas of fire origin and two (early and later) periods of fire development.

For four groups of industrial buildings, growth rates for the early period were also estimated for two materials ignited first. Using the doubling time in terms of area destroyed, doubling times in terms of volume destroyed were calculated under three assumptions regarding the relationship between horizontal and vertical rates of fire spread. The expected value and confidence limits of these parameters would provide a framework for comparing growth rates based on actual fires with rates determined from experimental fires.

THE MODEL

According to a simple deterministic model[1]

$$q = q_o \exp (at) \tag{1}$$

where

q = heat output of fire at time t
q_o = heat output of fire at time zero
a = growth factor

This model has been postulated for fire growth in a building with radiation as the dominant mode of heat transfer. Inside an enclosure there will be re-radiation from building walls, smoke as well as from flame under the ceiling. The exponential model in equation (1) is supported by experiments on spread of fire in a building Labes[2], for example, has discussed this model for home fires in terms of volume involved in fire.

It is, therefore, reasonable to assume that

$$A(T) = A(o) \exp(\beta T) \tag{2}$$

where

$A(T)$ = floor area damaged in T minutes since ignition
$A(o)$ = floor area initially ignited
β = fire growth parameter

Equation (2) is for flame damaged area only and does not include smoke and water damage.

The duration of burning T, can be divided into the following five periods:

T_1 - ignition to detection or discovery of fire
T_2 - detection to calling fire brigade
T_3 - call to arrival of brigade at the scene of fire (attendance time)
T_4 - arrival to the time when fire is brought under control by brigade (control time)
T_5 - control to extinction

T_4 denotes the duration up to the time when the fire has been effectively controlled or surrounded by the brigade and a message is sent to the fire station to stop the despatch of further reinforcements. The growth of fire is practically negligible during T_5 and hence only the periods T_1 to T_4 need to be considered in a statistical investigation. It must be noted that $A(T)$ is not the area damaged or burning during the T^{th} minute; it is the area damaged in T minutes.

A fire detected soon after ignition will be in its early stage of growth when fire fighting by brigade commences and hence can be controlled quickly. A reduced detection time(T_1) would therefore shorten the control time (T_4) as well, thus reducing the fire duration T and the damage to an appreciable extent. An automatic detection system is designed to achieve this saving. In addition the call time (T_2) can be reduced by linking the detector directly to the local fire brigade. The average attendance time (T_3) of a fire brigade can be reduced by establishing more fire stations or relocating some of the existing stations. The control time (T_4) can be reduced by adopting an efficient fire fighting strategy. Other time periods can be introduced into the model in equation (2) by considering the time for the commencement of first-aid fire fighting methods such as extinguishers, the time of operation of sprinklers and the time when the structural fire protection (walls, floors and ceilings) will be expected to play its part. Also, as noted by Labes(2), the basic time-growth curves may be affected by events such as roof venting, the collapse of a floor or ceiling or any other change that alters the compartmentation. These events can occur strictly as a result of fire development.

The five time periods, T_1 to T_5 mentioned above are consistent with the information given in the reports (FDR1) on fires furnished by the fire brigades in the United Kingdom. In this report the fire brigade is required to estimate for each fire the detection time according to the following classification.

(i) discovered at ignition
(ii) discovered under 5 minutes after ignition
(iii) discovered between 5 and 30 minutes after ignition
(iv) discovered more than 30 minutes after ignition.

In the pilot investigation[3] on the economic value of automatic fire detectors classes (ii) to (iv) were considered as a single group. In a subsequent detailed investigation[4] average values of 2, 17 and 45 minutes were used for the three classes (ii), (iii) and (iv) respectively. In this later study, the expanded model

$$A(T) = A(o) \exp \Sigma_i \beta_i T_i \tag{3}$$

was employed to allow for different growth rates during different periods; β_i is the rate of fire growth during the i^{th} period of duration T_i and $T = \Sigma_i T_i$.

For statistical estimation equation (2) may be turned into the simple regression model.

$$\text{Log } A(T) = \text{Log } A(o) + \beta T \tag{4}$$

and equation (3) into the multiple regression model

$$\text{Log } A(T) = \text{Log } A(o) + \Sigma_i \beta_i T_i \tag{5}$$

The i^{th} period may be expected to make a contribution of $\beta_i T_i$ towards Log A(T).

The three periods T_1, T_2 and T_3 before the commencement of fire fighting by the brigade are conceptually independent but are all correlated with the control time T_4. These interactions with T_4 and the ranges of values for the four periods should be taken into consideration in a detailed investigation. The additive model in equation (5) should be regarded as a simple approximation.

The rate at which fire grows can be expected to vary depending on the part of the building where fire starts. Three major areas of fire origin can be identified for an industrial building - production area, storage area and 'other' areas. The rate would also depend on the presence or absence of first-aid fire fighting before the arrival of the fire brigade. All these factors were taken into consideration in the detailed study[4] (mentioned earlier). Since this study was on the economic value of automatic fire detectors, total damage including smoke and water damage was used for the variable A(T).

APPLICATION

Using data furnished by the fire brigades in the United Kingdom an investigation of growth rates of fires in buildings without sprinklers was carried out by Fire Research Station in collaboration with Swedish Fire Protection Association. The presence or absence of first-aid fire fighting was not included as a factor affecting rate of fire growth. The growth parameter was obtained for area damaged by direct burning as mentioned in equation (2). The analysis was based on data on fires which occurred during 1979 and 1980. The number of fires (Sample sizes) ranged from 78 for the storage area of clothing, footwear, leather and fur to 1103 for production area of metal manufacture.

The results are given in Table 1 for eight groups of industrial building where β_A and β_B are the growth rates for the following two periods

t_A = time of ignition to the time of fire brigade arrival at the scene of fire $(T_1 + T_2 + T_3)$.

t_B = arrival time to the time when the fire is brought under control by the brigade (T_4)

The model employed in this analysis may be written as

$$A(T) = A(o) \exp \left[\beta_A t_A + \beta_B t_B\right] \tag{6}$$

where A(o) is the area initially ignited. The growth parameters β_A and β_B were highly significant (1 per cent level) or significant (5 per cent level) in almost all cases.

Conceptually, A(T) = 0 at time T = 0. However, the best engineering application might be made by using the values of parameters giving the best fit for equation (6) rather than values of a modified curve passing through the origin. For any industrial group and area of fire origin the A(o) given in Table 1 is an average value based on all fires in the sample analysed. For a particular building and area in this group, if considered necessary, a distribution of values for A(o) can be obtained by carrying out a fire load survey. The average value or any other parameter of this distribution can be used for 'design' purposes.

The growth parameters given in Table 1 are also expected or average values based on samples of fires analysed. It is difficult to identify the characteristics of the 'average building' to which these results would apply. Information required for this purpose is not available from the fire brigade reports. However, using deterministic models, the growth rates in Table 1 can be modified to take into account room geometry, fuel loading, arrangement of objects, ventilation and other such factors affecting the rates.

DOUBLING TIME

'Doubling time' is the parameter generally used for characterising rate of fire growth. This is the time for fire to double in size and is a constant for the exponential model of fire growth. For example, if it takes 5 minutes for the area damaged to increase from 20 m^2 to 40 m^2 it will also take 5 minutes for the damage to increase from 40 m^2 to 80 m^2 amd 80 m^2 to 160 m^2 and so on. For the model in equation (2), the doubling time in terms of total floor area damaged is given by

$$d = (1/\beta) \log_e 2 \tag{7}$$

Using equation (7) the doubling times d_A and d_B corresponding to the growth rates β_A and β_B are given in Table 2.

According to figures in Table 2, d_A varied from 14.44 minutes to 40.77 minutes for the early period and from 9.24 minutes to 53.32 minutes for the later period.

Equation (7) is applicable in terms of horizontal area destroyed. If it is assumed that fire spreads uniformly in all directions (horizontal and vertical) the doubling time in terms of volume destroyed would be approximately

$$d_v = (2/3)\, d \tag{8}$$

TABLE 1 Fire Growth Parameters

Industry	Production			Storage			Other		
	A(0) (Sq.mtrs)	β_A	β_B	A(0) (Sq.mtrs)	β_A	β_B	A(0) (Sq.mtrs)	β_A	β_B
Food, drink, tobacco	0.504	0.020	0.013	0.694	0.017	0.049	0.327	0.042	0.026
Chemicals and allied	0.225	0.038	0.033	0.628	0.048	0.035	0.218	0.027	0.044
Metal manufacture	0.341	0.033	0.026	1.160	0.017	0.045	0.425	0.032	0.041
Mechanical, instrument and electrical engineering	0.248	0.038	0.038	0.619	0.018	0.072	0.225	0.042	0.045
Textiles	0.304	0.047	0.029	1.793	0.037	0.037	0.215	0.032	0.053
Clothing, footwear, leather and fur	0.723	0.038	0.064	1.346	0.025	0.039	0.315	0.028	0.075
Timber, furniture, etc.	0.485	0.046	0.046	0.949	0.037	0.052	0.566	0.030	0.037
Paper, printing and publishing	0.213	0.044	0.052	0.985	0.027	0.044	0.235	0.023	0.060

TABLE 2 Doubling Time (mts)

Industry	Production		Storage		Other	
	t_A	t_B	t_A	t_B	t_A	t_B
Food, drink, tobacco	34.66	53.32	40.77	14.15	16.50	26.66
Chemicals and allied	18.24	21.00	14.44	19.80	25.67	15.75
Metal Manufacture	21.00	26.66	40.77	15.40	21.66	16.91
Mechanical, instrument and electrical engineering	18.24	18.24	38.50	9.63	16.50	15.40
Textiles	14.75	23.90	18.73	18.73	21.66	13.08
Clothing, footwear, leather and fur	18.24	10.83	27.73	17.77	24.76	9.24
Timber, furniture, etc.	15.07	15.07	18.73	13.33	23.10	18.73
Paper, printing and publishing	15.75	13.33	25.67	15.75	30.14	11.55

The doubling time in terms of volume destroyed would be

$$d^{1}_{v} = (\tfrac{1}{2})\, d \qquad (9)$$

if the vertical rate of spread is twice the horizontal rate and

$$d^{11}_{v} = (2/5)\, d \qquad (10)$$

if the vertical rate is three times the horizontal rate.

For some data quoted by Thomas[5] the doubling time, apparently in terms of volume destroyed, ranged from 1.4 minutes to 13.9 minutes. Rasbash[16] has also quoted a doubling time of 10 minutes for fires in rooms with traditional furniture. At the Factory Mutual Laboratories[7] the growth rates of a series of rather spreading fires were measured by continuous weighing of combustible materials. Data from these tests when forced to fit the exponential growth law indicated doubling times ranging from 21 seconds to 4 minutes.

CROSS VALIDATION

The growth rates derived from statistics of real fires (Table 1) attended by fire brigades involve a number of materials and structural elements of buildings. But the growth rates estimated from experimental fires pertain to individual materials and experimental conditions. There is a need to combine the experimental growth rates of different objects and estimate a composite growth rate for a room or a building. Such an exercise will provide a mechanism for cross validating results of experiments and statistical analysis of real fires. A cross-validation, however, is possible only in probabilistic terms. A real fire involving some material may not continue to burn or spread to another material; this depends on the arrangement of materials in the room, ventilation and environmental conditions.

A cross validation of results for the early stage of fire growth can be attempted by comparing experimental and statistical growth rates for different materials or objects. The difference between the expected (average) values of growth rates for the two cases should be tested statistically for its significance. An alternative test would be to judge whether the experimental growth rate falls within the confidence limits of the statistical growth rate. In Table 3, the expected values of statistical growth rates based on fire area together with their confidence limits are given for each area of fire origin and for two materials for which large numbers of observations were available. The probability of exceeding the upper limit or falling short of the lower limit is 0.025. In three cases only one material with sufficient data was available. The doubling times were calculated according to equations (7) to (10). The figures in Table 3 relate to the early period (t_A) and buildings without sprinklers. Fires during the years 1978 to 1980 provided the necessary data. This pilot exercise for four groups of industrial buildings can be extended to other materials and groups of industrial and commercial buildings.

In problems related to fire safety the expected value of the growth rate may not be the appropriate parameter. There is a fifty per cent chance that the growth rate in a real fire will be greater than the expected value. There is only a 2.5 per cent chance that the rate in a fire would exceed the upper confidence limit. For determining fire protection requirements for a building one could choose a growth rate corresponding to an acceptable level of risk (probability).

TABLE 3 Material ignited first - fire growth rate and doubling time

Industry/Area	Material	Parameter	β	d (mts)	d_v (mts)	d_v^1 (mts)	d_v^{11} (mts)
<u>Timber, furniture,</u> Production	Dust powder, flour	(a)	0.052	13.33	8.89	6.67	5.33
		(b)	0.069	10.05	6.70	5.02	4.02
		(c)	0.035	19,80	13.20	9.90	7.92
	Raw materials	(a)	0.038	18.24	12.16	9.12	7.30
		(b)	0.069	10.05	6.70	5.02	4.02
		(c)	0.006	115.53	77.02	57.76	46.21
Storage	Dust powder, flour	(a)	0.017	40.77	27.18	20.39	16.31
		(b)	0.057	12.16	8.11	6.08	4.86
		(c)	-	-	-	-	-
	Raw materials	(a)	0.150	4.62	3.08	2.31	1.85
		(b)	0.246	2.82	1.88	1.41	1.13
		(c)	0.054	12.84	8.56	6.42	5.13
Other	Dust powder, flour	(a)	0.037	18.73	12.49	9.37	7.49
		(b)	0.064	10.83	7.22	5.42	4.33
		(c)	0.010	69.32	46.21	34.66	27.73
<u>Textiles</u> Production	Textiles	(a)	0.045	15.40	10.27	7.70	6.16
		(b)	0.068	10.19	6.80	5.10	4.08
		(c)	0.022	31.51	21.01	15.75	12.60
	Raw materials	(a)	0.095	7.30	4.86	3.65	2.92
		(b)	0.183	3.79	2.53	1.89	1.52
		(c)	0.007	99.02	66.01	49.51	39.61
Storage	Packaging	(a)	0.010	69.32	46.21	34.66	27.73
		(b)	0.105	6.60	4.40	3.30	2.64
		(c)	-	-	-	-	-
Other	Textiles	(a)	0.007	99.02	66.01	49.51	39.61
		(b)	0.064	10.83	7.22	5.42	4.33
		(c)	-	-	-	-	-
	Electrical	(a)	0.094	7.37	4.92	3.69	2.95
		(b)	0.145	4.78	3.19	2.39	1.91
		(c)	0.043	16.12	10.75	8.06	6.45

(a) Expected value; (b) Upper confidence limit; (c) Lower confidence limit; - negative value (inadmissible)

TABLE 3 Material ignited first – fire growth rate and doubling time (cont'd)

Industry/Area	Material	Parameter	β	d (mts)	d_v (mts)	d_v^{1} (mts)	d_v^{11} (mts)
Chemical							
Production	Raw materials	(a)	0.052	13.33	8.89	6.67	5.33
		(b)	0.079	8.77	5.85	4.39	3.51
		(c)	0.025	27.73	18.48	13.86	11.09
	Electrical	(a)	0.071	9.76	6.51	4.88	3.91
		(b)	0.111	6.25	4.16	3.12	2.50
		(c)	0.031	22.36	14.91	11.18	8.94
Storage	Packaging	(a)	0.019	36.48	24.32	18.24	14.59
		(b)	0.050	13.86	9.24	6.93	5.55
		(c)	–	–	–	–	–
	Raw materials	(a)	0.031	22.36	14.91	11.18	8.94
		(b)	0.073	9.50	6.33	4.75	3.80
		(c)	–	–	–	–	–
Other	Electrical	(a)	0.023	30.14	20.09	15.07	12.06
		(b)	0.060	11.55	7.70	5.78	4.62
		(c)	–	–	–	–	–
	Raw materials	(a)	0.039	17.77	11.85	8.89	7.11
		(b)	0.074	9.37	6.24	4.68	3.75
		(c)	0.004	173.29	115.53	86.64	69.32
Paper							
Production	Dust powder, flour	(a)	0.076	9.12	6.08	4.56	3.65
		(b)	0.152	4.56	3.04	2.28	1.82
		(c)	–	–	–	–	–
	Raw materials	(a)	0.062	11.18	7.45	5.59	4.47
		(b)	0.121	5.73	3.82	2.86	2.29
		(c)	0.003	231.05	154.03	115.53	92.42
Storage	Raw materials	(a)	0.066	10.50	7.00	5.25	4.20
		(b)	0.174	3.98	2.66	1.99	1.59
		(c)	–	–	–	–	–
Other	Electrical	(a)	0.041	16.91	11.27	8.45	6.76
		(b)	0.080	8.66	5.78	4.33	3.47
		(c)	0.002	346.57	231.05	173.29	138.63
	Dust powder, flour	(a)	0.032	21.66	14.44	10.83	8.66
		(b)	0.110	6.30	4.20	3.15	2.52
		(c)	–	–	–	–	–

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