

An Arbitrary Polynomial Chaos-Based Approach to Analyzing the Impacts of Design Parameters on Evacuation Time under Uncertainty

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ABSTRACT

In performance-based design of buildings, much attention is paid to design parameters by fire engineers or experts. However, due to the time-consuming evacuation models, it is computationally prohibitive to adopt the conventional Monte Carlo simulation (MCS) to examine the effects of design parameters on evacuation time under uncertainty. To determine suitable design parameters under uncertainty with the reduced significantly computational cost, an arbitrary polynomial chaos-based method is presented in this paper. Arbitrary polynomial chaos expansion is used to construct surrogate models of evacuation time based on complex evacuation models. Afterwards, simple analytical method can be adapted to calculate the mean, standard deviation of evacuation time and Sobol sensitivity indices based on the arbitrary polynomial chaos coefficients. Moreover, the distribution of evacuation time can be generated by combining Latin hypercube sampling (LHS) with the obtained surrogate model. To demonstrate the proposed method, a hypothetical single-storey fire compartment with two exits is presented as a case in accordance with the Chinese code GB50016-2012, evaluating the impact of exit width on evacuation time under uncertain occupant density and child-occupant load ratio. And results show that the proposed method can achieve the distribution of evacuation time close to that from the MCS while dramatically reducing the number of evacuation simulations. When exit width per 100 persons is designed between 0.1 m and 0.5 m, the uncertainty of evacuation time is severely affected by exit width, which is more significant in smaller exit width. However, exit width has a small effect on Sobol sensitivity indices, the reliability level of a certain safety factor, and safety factor at a certain reliability level.

KEYWORDS: performance-based design, egress, evacuation time, uncertainty analysis, risk assessment, statistics

INTRODUCTION

Occupant evacuation is extremely complex, which is influenced by the physiological, psychological, and sociological aspects of evacuees as well as external environment [1]. Due to the randomness of fire occurrence and the variability of occupants' characteristics, there exist many uncertain factors, such as pre-movement time, occupant density, occupant type, familiarity with exits, and occupant mobility etc. Thus evacuation time is highly uncertain for a certain specific building. Traditionally, prescriptive fire protection design codes adopt safety factors to deal with uncertain factors related to crowd evacuation [2]. However, with the help of prescriptive building fire protection codes, there is still a lack of specific guidance about the selection of safety factors. Meanwhile, it is also difficult to determine the safety performance of buildings and optimal design parameters using prescriptive building codes. In order to address the problems above, it is necessary to employ probabilistic methods to examine the effect of design parameters on evacuation time under uncertainty.

During the last few decades, research has been widely conducted about the effect of design parameters on evacuation time. Based on the Monte Carlo simulation (MCS) of the STEPS and EXIT89 evacuation models, Meacham et al. [3] suggested that significant uncertain parameters for evacuation time may be associated with the geometry of buildings. Fang et al. [4] employed a multi-grid evacuation model to investigate the effect of door width on evacuation time and assumed that door width has a more significant influence on evacuation time in fire situations than normal conditions. Zhao et al. [5] adopted a cellular

automaton evacuation model to conduct the performance-based design of building exits in deterministic scenarios and suggested that the layout of exits should be symmetrical and the distance between exits should be designed using optimization techniques. In order to improve building designs for a certain deterministic scenario with the reduced computational cost, Tavares et al. [6] combined the Building EXODUS evacuation model and design of experiments to obtain the deterministic response surface of evacuation time. Afterwards, numerical optimization methods are applied to determine the optimal design parameter. In current building regulations, exit location is usually determined by the maximum travel distance in a certain specified scenario. In order to find an alternative method to determine suitable exit location, Tavares et al. [7] compared evacuation times from the exit location determined by the relative distance between exits and the maximum travel distance through a case study, and the result is that evacuation time from exit location determined by the relative distance between exits is smaller, which indicates exit location can also be determined by the relative distance between exits. It can be seen that Ref. [6] and [7] focus on the optimization of design parameters with deterministic values of uncertain parameters using the deterministic response surface method and the comparison of deterministic evacuation scenarios, and thus the determination of design parameters under uncertainty are paid attention to in this paper. Furthermore, in order to compute the reliability of evacuating successfully with the low computational cost, Cornelius et al. [8] adopted the adaptive deterministic response surface method to build the surrogate models of fire and evacuation models.

As the discussion above, the impacts of design parameters on evacuation time are mainly studied in the case of assigning deterministic values to uncertain parameters such as occupant density. However, evacuation time is highly uncertain due to the randomness of fire occurrence and human characteristics. Meanwhile, due to the complexity of evacuation models, it is computationally expensive to adopt the conventional MCS to investigate the effects of design parameters on evacuation time under uncertainty. Stochastic response surface methods can be used to deal with the output uncertainty caused by the uncertainty in input parameters with the low computational cost [9]. And the arbitrary polynomial chaos expansion proposed by Witteveen et al. [10] is one of the stochastic response surface methods, which can handle arbitrary distributions of input parameters only with limited statistical moments. Thus, in order to take the reduced significantly computational cost to analyze the effects of design parameters on evacuation time under arbitrary uncertainty, the arbitrary polynomial chaos-based method is proposed here. Based on results of computationally expensive evacuation models, arbitrary polynomial chaos expansion can be used to construct surrogate models of evacuation time with uncertain input parameters. And then, the mean and standard deviation of evacuation time as well as Sobol sensitivity indices can be calculated analytically based on the arbitrary polynomial chaos coefficients. Furthermore, Latin Hypercube sampling (LHS) can be applied to uncertain parameters to generate input samples. Afterwards, the surrogate model of evacuation time is performed on these input sample points to achieve the distribution of evacuation time.

ARBITRARY POLYNOMIAL CHAOS-BASED APPROACH

The procedure for the arbitrary polynomial chaos-based approach, which is used to investigate the influence of design parameters on evacuation time under uncertainty with the reduced significantly computational cost, is shown in Fig. 1.

Evacuation Models and Input Parameters

Evacuation time can be calculated by simple calculations or complex computer evacuation models [11]. Simple calculations assume a number of simplifications in the evacuation process, which are based on observations and evacuation experiments. In order to obtain more accurate results, complex computer evacuation models are developed. According to different classification methods, computer evacuation models can fall into microscopic and macroscopic models; discrete and continuous models; deterministic and stochastic models; rule-based and force-based models; high and low fidelity models [12]. Zheng et al. [13] reviewed seven methodological approaches for crowd evacuation and suggested that different modeling approaches should be combined together to simulate occupant evacuation. Ronchi et al. [14] analyzed the advantages and limitations of computer evacuation models and indicated that computer evacuation models should be based on reasonable modeling assumptions, embedded sub-algorithms and treatment of model uncertainty. FDS+Evac computer model is an agent-based social force model, which is also a stochastic evacuation model. Moreover, FDS+Evac results have already been compared with some

other evacuation models and simulation results of evacuation time are quite similar [15]. Thus, in this paper, the FDS+Evac model is chosen to predict necessary evacuation time samples, which can be used to construct the surrogate model of evacuation time.

Input parameters of evacuation models can be classified into two types: design parameters and uncertain parameters. For the performance-based fire protection design of buildings, design parameters are generally related to the building geometry, which contain exit width, exit location the number of exits and so on. It is expensive to change these design parameters once buildings are built. Thus, design parameters should be chosen prudently by fire protection engineers or experts. Generally the ranges of design parameters can be assumed according to the building codes, building geometry and occupancy type. Due to a high degree of uncertainty in fire occurrence, it is difficult to determine initial values of some parameters associated with crowd evacuation such as occupant density, occupant type and so on, which are considered as uncertain parameters due to the incomplete knowledge. Generally, the distributions of uncertain parameters can be assumed based on the literature, observations, codes and recommendations by experts etc. [16]. Through the analysis above, it can be seen that evacuation time is determined by design parameters and influenced by uncertain parameters, whose uncertainty should be quantified.

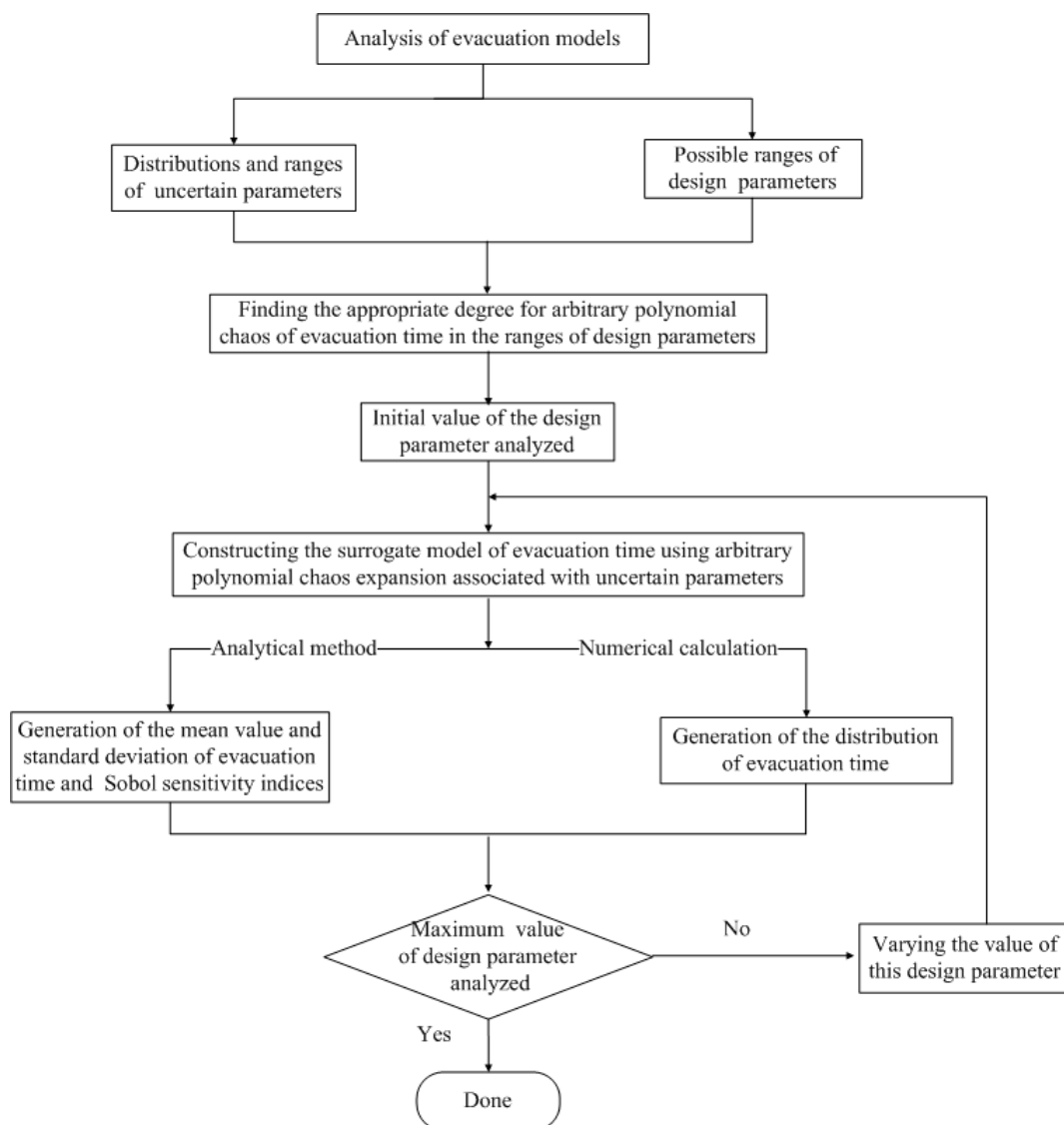


Fig. 1. Procedure for impacts of design parameters on evacuation time under arbitrary uncertainty.

Arbitrary Polynomial Chaos Expansion

Arbitrary polynomial chaos is the combination of polynomial chaos expansion and Gram–Schmidt orthogonalization [10], which can be used to address the output uncertainty affected by uncertain parameters with arbitrary distributions, such as truncated standard distributions and histogram distributions etc. In order to quantify the uncertainty of evacuation time, the predictive model of evacuation time associated with independent uncertain input parameters can be expressed as follows.

$$T_e = F(\mathbf{x}) \quad \mathbf{x} = \{x_1, x_2, \dots, x_n\} \quad (1)$$

Where T_e is evacuation time; n is the number of uncertain parameters; $\mathbf{x} = \{x_1, x_2, \dots, x_n\}$ is the vector set of independent uncertain parameters associated with evacuation time.

According to Wiener [17], the output T_e can be expressed in mathematical series with regard to independent input variables x_1, x_2, \dots, x_n , as shown in Eq. 2.

$$\begin{aligned} T_e(x_1, x_2, \dots, x_n) = & \alpha_0 + \sum_{i=1}^n \alpha_{i1} \psi^1(x_{i1}) + \sum_{i=1}^n \alpha_{i,j1} \psi^2(x_{i1}) + \sum_{1 \leq i < j \leq n} \alpha_{i,j,2} \psi^1(x_{i1}) \psi^1(x_{j2}) + (\sum_{i=1}^n \alpha_{i,j,1,j1} \psi^3(x_{i1}) \\ & + \sum_{1 \leq i < j \leq n} \alpha_{i,j,1,j2} \psi^2(x_{i1}) \psi^1(x_{j2}) + \sum_{1 \leq i < j < k \leq n} \alpha_{i,j,2,j3} \psi^1(x_{i1}) \psi^1(x_{j2}) \psi^1(x_{k3})) + \dots \end{aligned} \quad (2)$$

Where the coefficients α are unknown, which are used to quantify the contribution of individual uncertain parameters or the interactions between/among uncertain parameters to the uncertainty of T_e ; ψ is the optimal orthogonal basis for individual input parameters, whose superscript indicates the degree.

For computational purposes, the Eq. 2 should be truncated with a certain degree d [18], as shown in Eq. 3.

$$\begin{aligned} T_e(x_1, x_2, \dots, x_n) \approx & \alpha_0 + \sum_{i=1}^n \alpha_{i1} \psi^1(x_{i1}) + \sum_{i=1}^n \alpha_{i,j1} \psi^2(x_{i1}) + \sum_{1 \leq i < j \leq n} \alpha_{i,j,2} \psi^1(x_{i1}) \psi^1(x_{j2}) + \dots + \\ & (\sum_{i=1}^n \alpha_{i,j,j,1} \psi^d(x_{i1}) + \sum_{1 \leq i < j < k \leq n} \alpha_{i,j,k} \psi^{d-1}(x_{i1}) \psi^1(x_{j2}) + \dots) \end{aligned} \quad (3)$$

Where the number of unknown coefficients a can be determined by n and d , which is equal to $\frac{(n+d)!}{n!d!}$.

The optimal orthogonal basis ψ can be determined according to the Gram-Schmidt orthogonalization [10]. Based on the Gram-Schmidt orthogonalization, Oladyskin [19] gave the relationship between optimal orthogonal basis and statistical moments of uncertain input parameters, whose distributions can be arbitrary with limited statistical moments. And the main steps to obtain the optimal orthogonal basis are summarized as follows: Firstly, for ease of analytical development, all uncertain parameters x_i are transformed linearly to x'_i with 0 mean value and unit variance, as shown in Eq.(4).

$$x'_i = \frac{x_i - E(x_i)}{\sqrt{\text{Var}(x_i)}} \quad i = 1, 2, \dots, n \quad (4)$$

Where $E(x_i)$ and $\text{Var}(x_i)$ are the mean and variance of x_i .

Secondly, the optimal orthogonal basis $\psi(x'_i)$ can be obtained by solving the Eq. (14) in Ref. [19], which involves the statistical moments of x'_i .

Finally, for more useful properties, the obtained j -th degree optimal orthogonal polynomial $\psi^j(x'_i)$ is normalized as follows.

$$\phi^j(x'_i) = \frac{\psi^j(x'_i)}{\int (\psi^j(x'_i))^2 p(x'_i) dx'} \quad i = 1, 2, \dots, n; j = 1, 2, \dots, d \quad (5)$$

Where $\phi^j(x'_i)$ is the j -th degree normalized orthogonal polynomial for x'_i ; $p(x'_i)$ is the probability density function for x'_i , which is equal to the probability density function of x_i due to the linear transformation between x_i and x'_i .

Determination of Sample Points

Unknown coefficients α can be solved by the regression method [18], which is associated with sample points of the input and output. For a certain uncertain input parameter, the optimal input sample points are the roots of the orthogonal polynomial of one degree higher than that used in the expansion [20]. For the d -th degree expansion with n uncertain input parameters, the number of the optimal input sample points is $(d+1)^n$, which may be much larger than necessary. In order to avoid the unnecessary simulations and assure the accuracy, Isukapalli et al. [9] proposed two methods to select necessary input samples from the set of optimal input samples, i.e. selecting randomly and selecting based on probability distributions. For the former, the input sample size is recommended to be twice as many as the number of unknown coefficients. For the latter, the optimal input samples in high probability regions are given high priorities. In this paper, the latter is adopted to determine the necessary input sample points. Afterwards, the FDS+Evac model is performed on the necessary input sample points to generate the corresponding T_e sample points. Once the input and output sample points are obtained, unknown coefficients α can be acquired by solving Eq. (3) using the singular value decomposition algorithm.

Determination of the Appropriate Degree for the Surrogate Model of Evacuation Time

When the optimal normalized orthogonal basis ϕ and unknown coefficients α are obtained, the d -th degree polynomial representation of T_e can be determined. However, it is necessary to verify the degree for T_e with the purpose of assuring the quality of the results.

In the range of a certain design parameter, the suitable degree for T_e can be determined as follows: firstly, the design parameter is assumed to be distributed uniformly in its range. Afterwards, the d -th and $(d+1)$ -th degree arbitrary polynomial chaos expansions are constructed, whose input parameters are uncertain parameters and the considered design parameter. And the design parameter can be regarded to be distributed uniformly in its design range here. Then, the cumulative distribution functions (CDFs) of T_e can be numerically generated by combining LHS and the d -th and $(d+1)$ -th degree arbitrary chaos expansions. If the difference between two CDFs is smaller than 5%, the $(d+1)$ -th degree is considered to be appropriate for the construction of the surrogate model of T_e in the range of the design parameter. If the difference is significant, the $(d+2)$ -th degree polynomial chaos expansion of T_e should be reconstructed, and the above process should be repeated until two CDF curves are similar.

Here, the coefficient of variation of the root-mean-square deviation, CV (RMSD) is adopted to quantify the difference between two CDF curves of T_e . The larger the value of CV(RMSD) is, the more significant the difference between two CDF curves is, and CV(RMSD) can be expressed as follows.

$$CV(RMSD) = \sqrt{\frac{\sum_{j=1}^m (T'_{e,i} - T''_{e,i})^2}{m}} \bigg/ \frac{\sum_{j=1}^m T''_{e,i}}{m} \quad (6)$$

Where m is the number of points constituting the CDF of T_e ; $T'_{e,i}$ is the $(i/m*100)$ -th percentile point for one CDF curve of T_e ; $T''_{e,i}$ is the $(i/m*100)$ -th percentile point for another CDF curve of T_e based on the arbitrary polynomial chaos expansion of one degree higher than that used to obtain $T'_{e,i}$.

Post-Processing

Once the suitable degree for surrogate models of T_e is determined in the range of a certain design parameter, the surrogate model of T_e associated with uncertain input parameters can be constructed repeatedly at different values of the considered design parameter.

For a certain value of the design parameter, the surrogate model of T_e with one suitable degree d' can be represented with the normalized orthogonal basis ϕ and coefficients α' , as shown in Eq. 7.

$$T_e(x'_1, x'_2, \dots, x'_n) \approx \alpha'_0 + \sum_{i=1}^n \alpha'_{i1} \phi^1(x'_{i1}) + \sum_{i=1}^n \alpha'_{i1,j1} \phi^2(x'_{i1}) + \sum_{1 \leq i < j \leq n} \alpha'_{i1,j2} \phi^1(x'_{i1}) \phi^1(x'_{j2}) + \dots +$$

$$(\sum_{i=1}^n \alpha'_{i1,j1,\dots,j1} \phi^{d'}(x'_{i1}) + \sum_{1 \leq i < j \leq n} \alpha'_{i1,j2,\dots,j2} \phi^{(d'-1)}(x'_{i1}) \phi^1(x'_{j2}) + \dots) \quad (7)$$

Furthermore, due to the orthogonal and normalized properties of ϕ , the mean and variance of T_e , $E(T_e)$ and $\text{Var}(T_e)$ can be analytically calculated as follows.

$$E(T_e) = \alpha'_0$$

$$\text{Var}(T_e) = \sum_{i=1}^n (\alpha'_{i1})^2 + \sum_{i=1}^n (\alpha'_{i1,j1})^2 + \sum_{1 \leq i < j \leq n} (\alpha'_{i1,j2})^2 + \dots + (\sum_{i=1}^n \left(\alpha'_{i1,j1,\dots,j1} \right)^2_{d'}) + \sum_{1 \leq i < j \leq n} \left(\alpha'_{i1,j2,\dots,j2} \right)^2_{(d'-1)} + \dots \quad (8)$$

Based on the definition of Sobol sensitivity indices [21], the main and total sensitivity indices of T_e for uncertain parameter x_i , S_{x_i} and $S_{x_i}^T$ can be expressed as follows.

$$S_{x_i} = \frac{\text{Var}(E(T_e | x_i))}{\text{Var}(T_e)}$$

$$S_{x_i}^T = \frac{\text{Var}(T_e) - \text{Var}(E(T_e | \mathbf{x}_{-i}))}{\text{Var}(T_e)} = \frac{E(\text{Var}(T_e | \mathbf{x}_{-i}))}{\text{Var}(T_e)} \quad (9)$$

Where $E(T_e | x_i)$ is the mean value (expected value) of T_e given the values of x_i , whose variance $\text{Var}(E(T_e | x_i))$ is equal to the variance of $E(T_e | x'_i)$ due to the linear transformation of x_i ; $E(T_e | \mathbf{x}_{-i})$ is the mean value (expected value) of T_e given the values of non- x_i ; $\text{Var}(T_e | \mathbf{x}_{-i})$ is the variance of T_e given the

values of non- x_i , whose mean value (expected value) $E(\text{Var}(T_e | \mathbf{x}_{-i}))$ is equal to the mean value of $\text{Var}(T_e | \mathbf{x}'_{-i})$ due to the linear transformation of x_i .

Due to the orthogonality and normalization of ϕ , the variance of $E(T_e | x'_i)$ and the mean of $\text{Var}(T_e | \mathbf{x}'_{-i})$, $\text{Var}(E(T_e | x'_i))$ and $E(\text{Var}(T_e | \mathbf{x}'_{-i}))$ can be analytically computed, as shown in Eq. (10) and Eq. (11).

$$\text{Var}(E(T_e | x'_i)) = \sum_{i=1}^n (a'_{i1})^2 + \sum_{i=1}^n (a'_{i1,j1})^2 + \dots + \sum_{i=1}^n \left(a'_{\underbrace{i1,j1,\dots,j1}_{d'}} \right)^2 \quad (10)$$

$$\begin{aligned} E(\text{Var}(T_e | \mathbf{x}'_{-i})) = & \sum_{i=1}^n (a'_{i1})^2 + \sum_{i=1}^n (a'_{i1,j1})^2 + \sum_{1 \leq i < j < 2 \leq n} (a'_{i1,j2})^2 + \sum_{1 \leq i < j < i < 2 \leq n} (a'_{i1,j2})^2 + \dots \\ & + \left(\sum_{i=1}^n \left(a'_{\underbrace{i1,j1,\dots,j1}_{d'}} \right)^2 + \sum_{1 \leq i < j < 2 \leq n} \left(a'_{\underbrace{i1,\dots,j1,j2}_{(d'-1)}} \right)^2 + \sum_{1 \leq i < j < i < 2 \leq n} \left(a'_{\underbrace{i1,\dots,j1,j2}_{(d'-1)}} \right)^2 + \dots \right) \end{aligned} \quad (11)$$

Moreover, the CDFs of T_e can be numerically generated by combining surrogate models of T_e and LHS. For different values of the design parameter, T_e at any level of uncertainty of interest can be obtained from the CDF of T_e . Based on the description above, the effect of the design parameter on T_e under arbitrary uncertainty can be examined.

The application of the arbitrary polynomial chaos-based method is demonstrated in the following case study, which is hypothetical due to the difficulty in obtaining the available experimental data of evacuation and scenarios.

CASE STUDY AND ANALYSIS

Case Description

The case considered is a single-room fire compartment of civil buildings, which is enclosed by fire-resistant construction to prevent the spread of fire to adjacent rooms. In order to make the considered fire compartment representative, the fire compartment is set to be 2500 m², which is the maximum area of one fire compartment of civil buildings regulated by the Chinese code GB50016-2012 [22]. Meanwhile, the minimum number of exits is 2 for this considered fire compartment prescribed by GB50016-2012. Given a worse case, 2 exits, denoted by Exit A and Exit B with the same size, are located at the centre of the walls. Additionally, in order to decrease the computational cost of evacuation simulations, obstacles are not taken into consideration in this fire compartment. Meanwhile, given emergency evacuation, occupants will evacuate immediately while notified by fire alarm systems. Thus, the pre-movement times for the crowd are assumed to 0 s here. Moreover, there is a possibility that fire may occur around one exit. Here, it is assumed that Exit B is not available while fire occurs. The plan of this considered fire compartment is shown in Fig.2.

Due to the randomness of fire occurrence, it is difficult to accurately acquire the number and characteristics of evacuees. Thus, two uncertain parameters are considered here: occupant density and child-occupant load ratio. Compared with low density crowd evacuation, there is a higher risk in high density crowd evacuation. Here, the mean value of occupant density is assumed to be 1.0 person/m². Based on the occupant type of "School Children" defined by the SIMULEX evacuation model [23], the mean value of the child-occupant load ratio is assumed to be 0.7 here. Moreover, occupant density and the child-occupant load ratio are assumed to follow normal distributions, which are truncated due to the bounded property of these parameters. Meanwhile, the standard deviations of normal distributions for occupant density and the child-

occupant load ratio are assumed to be 20% and 10% of their respective mean values. Table 1 gives the distributions of occupant density and child-occupant load ratio used in this case.

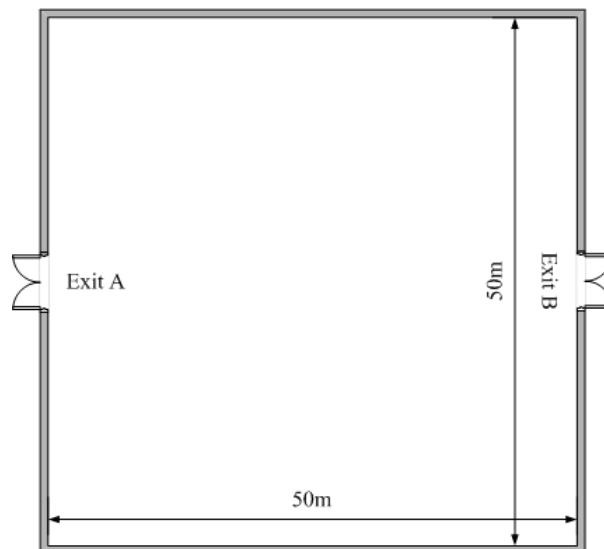


Fig. 2. The plan of the fire compartment considered.

Table 1. Distributions of uncertain parameters considered.

Uncertain parameters	Probability distribution functions
Occupant density, x_1 /(persons/m ²)	Truncated normal distribution: $f(x_1) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(x_1 - u)^2}{2\sigma^2}\right]$ with $u = 1, \sigma = 0.2, 0.8 \leq x_1 \leq 1.2$
Child-Occupant load ratio, x_2	Truncated normal distribution: $f(x_2) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(x_2 - u)^2}{2\sigma^2}\right]$ with $u = 0.7, \sigma = 0.07, 0.56 \leq x_2 \leq 0.84$

In performance-based fire protection design of buildings, exit width is generally paid great attention to by fire engineers or designers. When exit width is too small, people will not evacuate successfully. When exit width is too large, the available area of use will be reduced significantly. Here, the maximum and minimum exit widths are determined based on 0.5 m per 100 people [24] and 0.1 m per 100 persons, respectively. According to the analysis above, the design range of exit width is determined, as shown in Table 2.

Table 2. Design parameter in the case.

Design parameter	Range
Width for Exit A /m	2.5 ^a -12.5 ^b

^a2.5 is calculated by the area of this fire compartment 2500 m² multiplied by occupant density 1.0 person/m² multiplied by 0.001 m/ person.

^b12.5 is calculated by the area of this fire compartment 2500 m² multiplied by occupant density 1.0 person/m² multiplied by 0.005 m/ person that is chosen according to Ref. [24].

According to the proposed method, we should firstly determine the suitable degree for surrogate models of T_e associated with occupant density and child-occupant load ratio in the design range of exit width. Afterwards, surrogate models of T_e can be constructed repeatedly for different values of exit width. Finally,

the analytical and numerical methods are used as post-processing to examine the effect of exit width on the uncertainty of T_e caused by occupant density and child-occupant load ratio.

Results and Discussion

In order to determine the suitable degree for surrogate models of T_e in 2.5-12.5 m exit width, the 2nd, 3rd and 4th degree arbitrary polynomial chaos expansions of T_e are constructed based on 16, 41 and 86 samples respectively, whose input parameters are occupant density, child-occupant load ratio and exit width. Then, the LHS is applied to occupant density, child-occupant load ratio, and exit width to obtain input samples. Afterwards the CDFs of T_e can be obtained by performing the 2nd, 3rd and 4th degree expansions on these input sample points, as shown in Fig.3. For the 2nd and 3rd degree expansions, the value of CV(RMSD) between two CDF curves of T_e is 8.12%. However, for the 3rd and 4th degree expansions, it is 2.78% (smaller than 5%), which suggests that the 4th degree is suitable for the construction of surrogate models of T_e in the design range of exit width. Furthermore, in order to verify the proposed method, the MCS can be applied to the FDS+Evac model. The sample size of the MCS that can be estimated by statistical tolerance limits [25]. For the statistical tolerance limits (99.5%, 99.5%), which indicates that with the confidence level of 99.5%, 99.5% of the samples are in the tolerance limits, the corresponding sample size is estimated around 1000. Thus, the MCS of the FDS+Evac model with 1000 sample size is performed in this case, and the corresponding CDF of T_e is also shown in Fig. 3. And the value of CV(RMSD) between two CDF curves, which are based on the 4th degree expansion of T_e with 86 samples and the MCS with 1000 samples, is 1.86% (smaller than 5%), which suggests that the 4th arbitrary polynomial chaos expansion of T_e is around a 99.5% confidence level. Thus in the range of 2.5-12.5 m exit width, the appropriate degree for surrogate models of T_e is 4 in this case study.

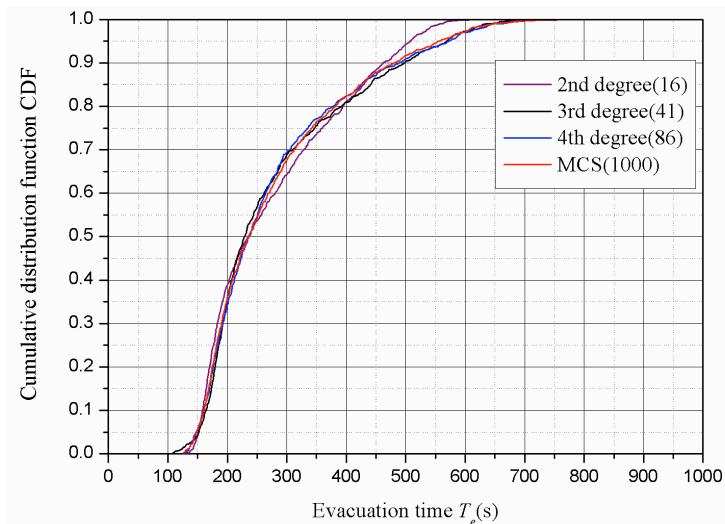


Fig. 3. Cumulative distribution functions (CDFs) of evacuation time T_e based on the 2nd, 3rd and 4th degree arbitrary polynomial chaos expansions and Monte Carlo simulation (MCS).

For a certain fixed value of exit width, 18 samples are required to construct one arbitrary polynomial chaos expansion of the 4th degree for T_e , whose input parameters are occupant density and child-occupant load ratio in this case. To investigate the effect of exit width on evacuation time under uncertainty, the 4th degree expansions of T_e need to be constructed repeatedly for different values of exit width. Afterwards, the corresponding mean value and standard deviation of T_e as well as Sobol total sensitivity indices can be calculated by analytical methods according to Eq.(8) and Eq.(9), as shown in Fig. 4.

From Fig. 4 (a) and (b), it can be seen that the mean and standard deviation of T_e decrease with the increase of exit width. For example, when exit width is 2.5 m, the mean value and standard deviation of T_e are 664.0 s and 71.0 s. However, when exit width is 12.5 m, the mean and standard deviation of T_e are 152.0 s and 15.2 s, which are much smaller than those for 2.5 m exit width. The explanation about the results is that while increasing exit width, the interaction among evacuees around exits will decrease, and congestion and

queuing phenomena around exits will be eased, which also suggests that the effect of occupant density and child-occupant load ratio on T_e will decrease with the increase of exit width. Thus while increasing the exit width, T_e for a certain deterministic evacuation scenario will decrease and the difference in T_e for different scenarios will not be significant due to the decrease in the effects of occupant density and child-occupant load ratio on T_e . From the analysis above, it can be seen that the mean value and standard deviation of T_e for uncertain evacuation scenarios will decrease with the increase of exit width. Moreover, when exit width is large, there are still strong interactions for high density crowd who are far away from exits during the movement process, which suggests that there are still some differences in T_e for different scenarios of high density crowd evacuation. From the analysis above, it can be deduced that when exit width is large enough, the mean value and standard deviation of T_e for high density crowd will almost be unchanged with exit width. Furthermore, the Sobol total sensitivity indices are adopted here to quantify the contributions of occupant density and child-occupant load ratio to the uncertainty of T_e , as shown in Fig. 4 (c) and (d). From Fig. 4 (c) and (d), it can be seen that the contributions of occupant density and child-occupant load ratio to T_e are little affected by exit width. Moreover, compared with the child-occupant load ratio, the contribution of occupant density to the uncertainty of T_e is much larger. Thus, for the high density crowd evacuation, the child-occupant load ratio can be taken as the base value, whose uncertainty can be ignored.

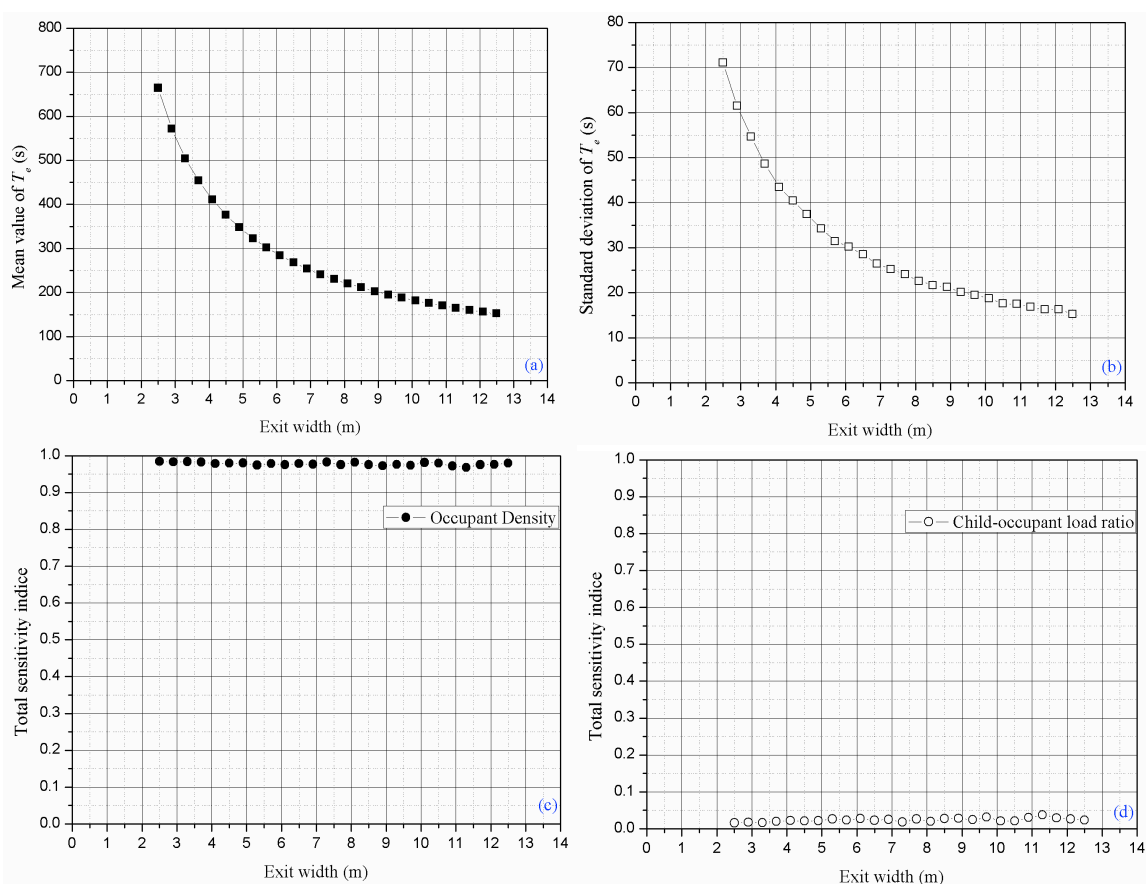


Fig. 4. Mean value (a), and standard deviation (b) of evacuation time T_e as well as Sobol total sensitivity indices for occupant density (c) and child-occupant load ratio (d) versus exit width.

Exit width and the uncertainty of evacuation time are key considerations in performance-based fire safety design. In order to obtain an optimal exit width under uncertainty, the effect of exit width on the distribution of T_e need to be investigated. Considering evacuation times of interest, the base case, 80th percentile, 90th percentile and 99.9th percentile evacuation times are given in Fig.5. Here, the base case evacuation time is calculated from the 4th degree polynomial chaos expansions of T_e , whose uncertain parameters are taken as deterministic average values. The limitations of the RSET (required safety egress

time)/ASET (available safety egress time) approach are widely known [26]. ASET may vary from 60-90 s in ultra-fast fire to 6-9 min in moderately fast growing fire [26]. The proper ASET value shall be the object of study considering the characteristics of each building, as a function of the inflammable material amount, type and distribution. In this case study, the exit width prescribed by the Chinese code GB50016-2012 (around 16.2 m wide for shopping malls) leads to a RSET around 120 s, corresponding to an ultra-fast growth rate fire. This exit may be oversized if there is no enough combustible material in this building. In addition, for a certain acceptable ASET, the optimal exit width at a certain reliability level can be determined from Fig. 5. For example, when the acceptable ASET is 300 s, the optimal exit width at the reliability level of 99.9% should be selected around 7.0 m based on Fig. 5. Furthermore, for a certain exit width, T_e at a certain reliability level can also be obtained from Fig. 5. For example, when exit width is around 10 m, T_e at the reliability probability of 99.9% should be around 218 s in this case. If the corresponding ASET is smaller than 218 s, some measures should be taken, such as improving the detection and alarm system, installing the fire extinction and smoke control systems.

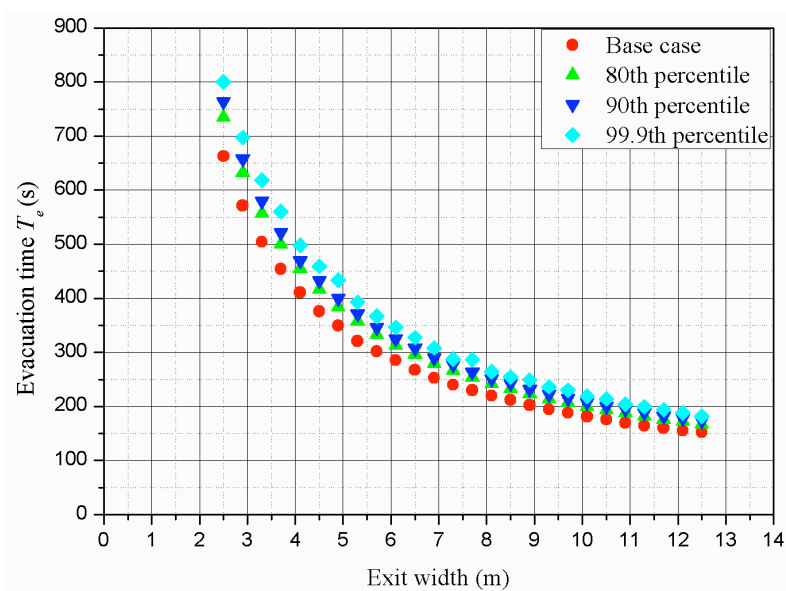


Fig. 5. Evacuation time T_e under uncertainty versus exit width.

Due to the significant computational cost of the MCS for complex evacuation models, the safety factor method is generally adopted to deal with the uncertainty of evacuation time in performance-based fire protection design of buildings. However, the uncertainty of evacuation time is affected by various factors such as occupancy type, exit width, and the characteristics of evacuees, which makes the safety factor random. Thus, the effect of exit width on uncertainty factor [27], which is defined as the ratio of evacuation time under uncertainty to the base case evacuation time, is investigated, as shown in Fig. 6. For a specified exit width, the optimal safety factor at a certain reliability level can be determined from Fig. 6. For example, when exit width is 8.5 m, the optimal safety factor at the reliability level of 99.9% is 1.20. In addition, for a specified exit width, the reliability probability of a certain safety factor can also be determined from Fig. 6. E.g. when exit width is 4.5 m, the reliability probability for the safety factor of 1.15 is 90%. From Fig.6, it can also be seen that in the design range of 0.1 m and 0.5 m per 100 persons, exit width has little effect on the uncertainty factor at a certain reliability level and the reliability level of a certain uncertainty factor, which suggests that the safety factor at a certain reliability level and the reliability level of a certain safety factor are almost independent of exit width.

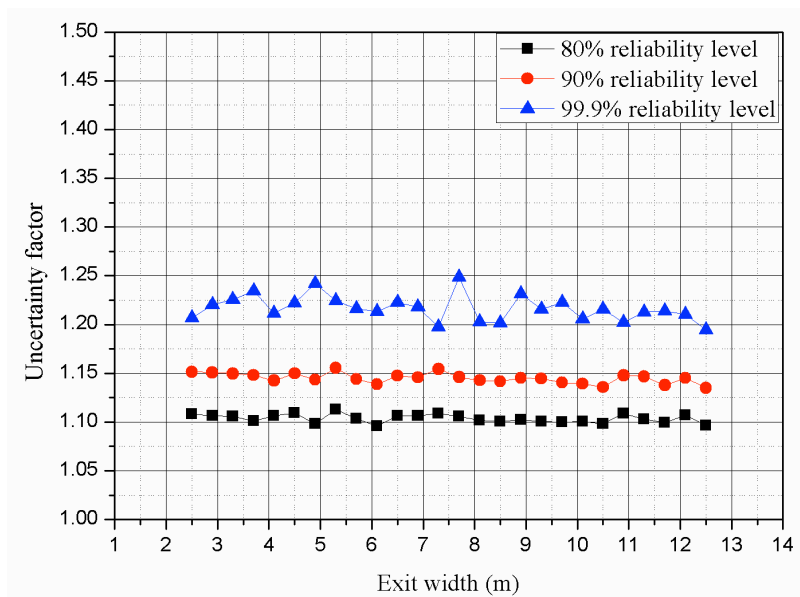


Fig. 6. Uncertainty factor versus exit width.

CONCLUSIONS

In order to conduct a cost-effective performance-based fire protection design of buildings with the reduced computational cost, an arbitrary polynomial chaos-based method is presented to examine the effects of design parameters on evacuation time under arbitrary uncertainty in this work. Based on a hypothetical case study of a single-room fire compartment with two exits, some conclusions can be summarized as follows.

The proposed method is demonstrated by the issue of the effect of exit width on the uncertainty of evacuation time resulting from the uncertainty of occupant density and child-occupant load ratio. Results reveal that the arbitrary polynomial chaos-based method can be used to investigate the impacts of design parameters on evacuation time under arbitrary uncertainty, whose computational cost is reduced significantly compared with the conventional MCS. Besides, the mean and standard deviation of evacuation time as well as Sobol sensitivity indices can be analytically calculated on the basis of the arbitrary polynomial chaos expansion coefficients, which does not need extra evacuation simulations.

From this case, it can be seen that the uncertainty of evacuation time is significantly affected by exit width designed between 0.1 m and 0.5 m per 100 persons, which is more significant with smaller exit width. Moreover, in this considered design range, the contributions of occupant density and child-occupant load ratio to the uncertainty of evacuation time are not severely affected by exit width. Meanwhile, the safety factor at a certain reliability level and the reliability level of a certain safety factor are almost independent of exit width. This method can also be used to optimize other design parameters in performance-based design of buildings, such as exit position.

However, there are still some limitations in this proposed method. First of all, we construct surrogate models of evacuation time based on the simulations of complex evacuation models, which makes the accuracy of the proposed method depend to a considerable degree on the selected evacuation model. Thus, it should be prudent to choose the evacuation model to make constructed surrogate models of evacuation time accurate enough.

Secondly, this proposed method is applied to a hypothetical case, which has not been validated due to the difficulty in obtaining the available evacuation data. In the future, we will focus on designing and carrying out the realistic evacuation experiments to obtain the valuable evacuation data.

Thirdly, for multiple input parameters, it is computationally expensive to adopt the arbitrary polynomial chaos-based method to investigate the effect of the design parameter on evacuation time under uncertainty due to the significant increase of the number of necessary evacuation simulations with the number of input parameters.

Finally, the proposed method only can be used to deal with input parameters that are independent or linearly correlated. And the uncertainty of the output resulting from nonlinear uncertain input parameters need to be addressed in our future work.

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