Numerical Simulation of Fire Resistance Test of a Concrete Slab

KAZUNORI HARADA and TOSHIO TERAI
Department of Architecture
Kyoto University
Kyoto, Japan

ABSTRACT

When concrete is intensely heated during fire, creeping of temperatures in the region of 100°C are often observed. To simulate this phenomenon, a simple mathematical model for heat and moisture transfer in concrete is presented. The model takes into account the conservation of heat, water vapor and liquid water. As to the water contained in concrete, reversible evaporation of adsorbed water in the pores and irreversible thermal decomposition of chemically adsorbed water in cement paste are considered.

These equations are so-called stiff equations and rather difficult to solve. For numerical stability, they are formulated by using integral equations coupled with the diagonally implicit Runge-Kutta method. Numerical solution of the model is compared with the experimental data.

KEY WORDS

Fire Resistance of Concrete, Numerical Simulation, Evaporation of Water, Decomposition of Cement Paste

INTRODUCTION

When predicting the temperature history of concrete in case of fire, Kawagoe/1/,Wakamatsu/2/,Fujii/3/ assumed that liquid water in concrete evaporates, taking latent heat of vaporization at 100°C. In these models, the conservation of water vapor is not included. In spite of this simplification, these models give reasonable results for engineering purposes. Models which take into account the movement of water vapor and liquid water are also presented by many authors(Harmathy/4/, Matsumoto/5/, Huang et al./6/, Sahota et al./7/, Dayan et al./8/). They solve sophisticated equations of simultaneous heat and moisture transfer.

The model we treat here is a simple diffusion model. It consists of a heat transfer equation, conservation of water vapor and liquid water in the pores. As to the water contained in concrete, not only the evaporation of physically adsorbed water, but also thermal decomposition of chemically adsorbed water in cement paste are considered.

DEFINITION OF POROUS SYSTEM

Concrete is a porous material made of cement paste and aggregate. A part of the water is bonded to the cement as chemically adsorbed water,

and the rest is contained in the pores as liquid water and water vapor. Water vapor is mixed with air.

This porous system is shown in Figure 1. The water content is defined as the sum of physically adsorbed water and chemically adsorbed water.

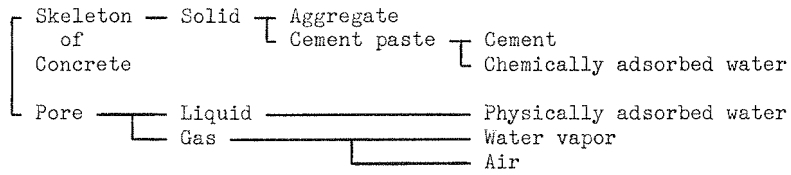


Figure 1. Definition of Porous System

To simplify the heat and mass transfer in this system, the following assumptions are made.

1) Heat flows through the solid matrix only. Heat flux due to convection in the pores is neglected.
2) Water vapor is transferred by diffusion only. Convection due to total pressure gradient is neglected.
3) Liquid water is transferred by diffusion only.
4) Air is purged in the early stage of heating. Therefore, treatment as a binary mixture of water vapor and dry air is omitted.

GOVERNING EQUATIONS

The governing equations for conservation of heat, water vapor and liquid water are

$$\rho c \frac{\partial \theta}{\partial t} = \nabla (\lambda \nabla \theta) - L R_{loc} , \tag{1}$$

$$\rho_0 \frac{\partial v}{\partial t} = \nabla (\rho_0 D_v \nabla v) + R_{loc} , \tag{2}$$

$$\rho_0 \frac{\partial w}{\partial t} = \nabla (\rho_0 D_w \nabla w) - R_{loc} . \tag{3}$$

The source term, the rate of local evaporation, in equations (1),(2) and (3), is defined as

$$R_{loc} = \gamma w (v_{eq} - v) + \rho_0 A_d w \exp(- E_d/RT) . \tag{4}$$

The first term in equation (4) represents reversible physical adsorption and desorption of water. Equilibrium vapor pressure is calculated from Frenkel-Halsey equation/9/,

$$\log(P_{v,eq}/P_{v,sat}) = - C_s/(T w^{ns}) . \tag{5}$$

Saturated vapor pressure of free water $P_{v,sat}$, is assumed to be given by the Clausius-Clapeyron equation

$$P_{v,sat} = A_v \exp(- B_v/T) , \tag{6}$$

with A_v=0.9243E+11, B_v=0.5128E+4. Regarding water vapor as perfect gas, v_{eq} becomes from equations (5) and (6)

$$v_{eq} = \varepsilon_{eff} A_v \exp(-B_v/T - C_s/(T w^{n_s})) / (\rho_0 R_v T) . \quad (7)$$

The second term in equation (4) represents the thermal decomposition of chemically adsorbed water. This takes place at high temperature irreversibly. However, since detailed information on the mechanism of thermal decomposition is not available, we applied empirical Arrhenius equations. Coefficients A_d and E_d can be determined by thermo-gravimetric analysis.

At normal temperature, the second term is negligible in comparison with the first, but at a high temperature (about 100 °C) , the orders of these two terms become comparable, and at a still higher temperature, the second becomes dominant.

Boundary Conditions

Boundary conditions are given in order to fit to the experiments described later(See Figure 3).

Exposed Surface. In the test of the composite slab, the exposed surface is covered with a steel plate , therefore

$$- \lambda \frac{\partial \theta}{\partial n} = h_f (\theta_f - \theta) , \quad \frac{\partial v}{\partial n} = \frac{\partial w}{\partial n} = 0 . \quad (8),(9),(10)$$

Unexposed Surface. At the unexposed surface, the transfer of heat and water vapor to ambient air are considered, therefore

$$- \lambda \frac{\partial \theta}{\partial n} = h_{amb} (\theta_{amb} - \theta) , \quad (11)$$

$$- \rho_0 D_v \frac{\partial v}{\partial n} = h_{v,amb} (v_{amb} - v) , \quad \frac{\partial w}{\partial n} = 0 . \quad (12),(13)$$

In these equations, symbol n denotes inward normal.

NUMERICAL PROCEDURE

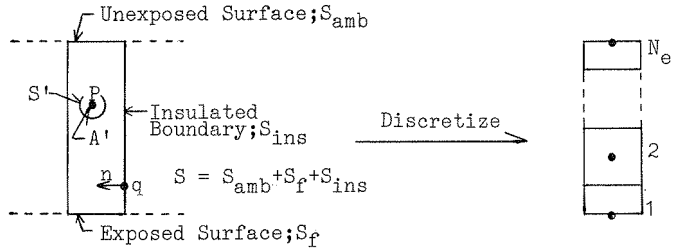
Governing equations (1) to (13) are to be solved numerically. For numerical stability, we transformed to integral equations, and used the implicit Runge-Kutta method. To save space, only the main points for equation (1) will be shown. The same procedure should be applied to equations (2) and (3).

Transformation into an Integral Equation

To be applicable to the two dimensional problems, two dimensional formulations are described as follows. However, the calculations are carried out for the one dimensional case to compare them with the experiments.

The fundamental solution of the two dimensional Laplace equation is

$$G_{pq} = \frac{1}{2\pi} \log(\frac{1}{r_{pq}}) . \quad (14)$$



a) Domain of Analysis b) Discrete System

Figure 2. Domain of Analysis

Here , r_{pq} is the distance from the reference point p to the moving point q. G_{pq} satisfies $\nabla^2 G_{pq}=0$, when $r_{pq}\neq 0$. Rearranging equation (1) gives

$$\nabla^2 \theta = \frac{\rho c}{\lambda}\frac{\partial\theta}{\partial t} - \frac{\nabla\lambda\,\nabla\theta - L R_{loc}}{\lambda} \tag{15}$$

In order to apply Green's formula, consider the domain of analysis A, as shown in Figure 2a). The singular point should be excluded from A by a circular domain A', whose center is p and its radius is r'.

Green's formula for the domain A-A' and its boundary S+S', with inward normal n, is

$$\iint_{A-A'} (\theta\nabla^2 G_{pq} - G_{pq}\nabla^2\theta)dA = -\int_{S+S'} \left(\frac{\partial G_{pq}}{\partial n}\theta - \frac{\partial\theta}{\partial n}G_{pq}\right)dS. \tag{16}$$

Substituting equations (8),(11),(14) and (15) into equation (16), and letting r' tend to zero, gives

$$\iint_A G_{pq}\frac{\rho c}{\lambda}\frac{\partial\theta}{\partial t}dA = \iint_A G_{pq}\frac{\nabla\lambda\,\nabla\theta - L R_{loc}}{\lambda}dA$$

$$+\int_{S_f} G_{pq}\frac{h_f}{\lambda}(\theta_f - \theta)dS + \int_{S_{amb}} G_{pq}\frac{h_{amb}}{\lambda}(\theta_{amb} - \theta)dS$$

$$+\int_S \frac{\partial G_{pq}}{\partial n}\theta\, dS - \alpha\,\theta_p \;. \tag{17}$$

α is unity when point p is in the domain A, 0.5 when p converges to the smooth boundary S.

Space Discretization

Dividing the domain A into rectangular elements as shown in Figure 2b), and assuming that temperature, material properties and rate of local evaporation are constant in each element, equation (17) can be reduced to the system of ordinary differential equations. The same procedure is applied to equations (2) and (3), and we get

$$\left(\frac{\partial u}{\partial t}\right) = f(u,t) \ . \tag{18}$$

Where, symbol u denotes vector of unknown variables

$$u = \{ \theta_1, \theta_2, \ldots, v_1, v_2, \ldots, w_1, w_2 \ldots \}^T \ . \tag{19}$$

Time Integration

Next, equation(18) is integrated with respect to time. Integration is carried out by the Diagonally Implicit Runge-Kutta scheme of two stages(DIRK22)/10/ to save computing time. The scheme is defined with two quadrature points $t_{n,i}(=t_n + \Delta t\ \tau_i)$ and their weights b_i

$$u_{n+1} = u_n + \Delta t \sum_{i=1}^{2} b_i\ f(u_{n,i}, t_{n,i}) \ . \tag{20}$$

In this equation, function f is evaluated by $u_{n,i}$. These values are not known at the beginning of the time step. Thus, the following equations are to be solved to get $u_{n,i}$.

$$u_{n,1} = u_n + \Delta t\ a_{11}\ f(u_{n,1}, t_{n,1}) \tag{21}$$

$$u_{n,2} = u_n + \Delta t \sum_{j=1}^{2} a_{2j}\ f(u_{n,j}, t_{n,j}) \tag{22}$$

The values of $u_{n,i}$ obtained in this way, are used in equation (20) to get u_{n+1}.

Coefficients a_{ij}, b_i, τ_i are

$$\begin{aligned} a_{11} &= 0.29289 , & & & \tau_1 &= 0.29289 , \\ a_{21} &= 0.70711 , & a_{22} &= 0.29289 , & \tau_2 &= 1.0 , \\ b_1 &= 0.70711 , & b_2 &= 0.29289 . \end{aligned} \tag{23}$$

MATERIAL PROPERTIES AND BOUNDARY CONDITION PARAMETERS/11/

Material properties and boundary condition parameters used in the numerical calculations are listed in the Table 1. Three types of furnace temperature histories are used in the calculations as described in the next section.

OUTLINE OF EXPERIMENTAL PROCEDURES/12/

Experiments were carried out in accordance with JIS A 1304, standard fire test procedure/13/. The specimen size is 880x1000x100mm. The specimen's bottom and sides are covered with 1.6mm thick steel plate to simulate a composite floor(deck plate). The specimen is horizontally suspended by wire ropes from a load cell, and its bottom surface is heated. The weight is continuously recorded during the test. Thermocouples are located on the steel plate, on the bottom surface of the concrete, at 25,50,75mm from the bottom of the concrete, and on the top surface(Figure 3).

Experiment No.A. The furnace temperature is regulated to act in accordance with the standard furnace temperature. Heating time was 150 minutes.

MATERIAL PROPERTIES

λ	$= -1.971E-9\ \theta^3 + 4.522E-6\ \theta^2 - 3.898E-3\ \theta + 2.189$	ρ_0	2200.
		ε_0	0.4
c	921.	γ	7.2E+8
D_v	1.94E-5	D_w	5.56E-10 exp(19.2w)
C_s	0.1892	n_s	2.01
A_d	1.08E+11	E_d/R	8331.

BOUNDARY CONDITIONS

$h_f = 4.65+8.59E-9(T_f^4-T_s^4)/(T_f-T_s)$

h_{amb}	23.	θ_{amb}	20.0
$h_{v,amb}$	2.44	v_{amb}	6.2E-6

TIME INCREMENT

Δt 60 Sec

INITIAL VALUE

θ_{ini}	20.0	v_{ini}	0.245E-5
w_{ini}	0.087(No.A), 0.082(No.B),0.077(No.C)		

Table 1. List of Parameter Values used.

Experiment No.B. The furnace temperature is regulated to 90 percent of the standard furnace temperature. Heating time was prolonged to 164 minutes.
Experiment No.C. The furnace temperature is regulated to 110 per cent of the standard furnace temperature. Heating time was 136 minutes.

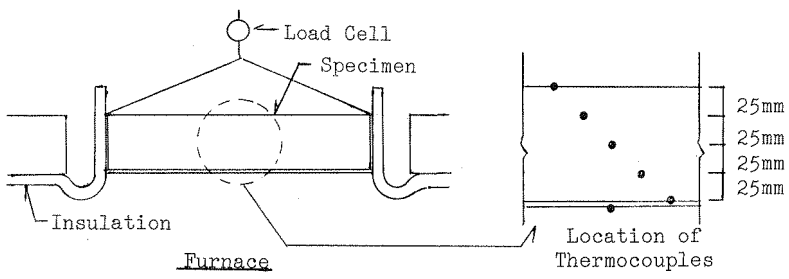


Figure 3. Experimental Set-Up

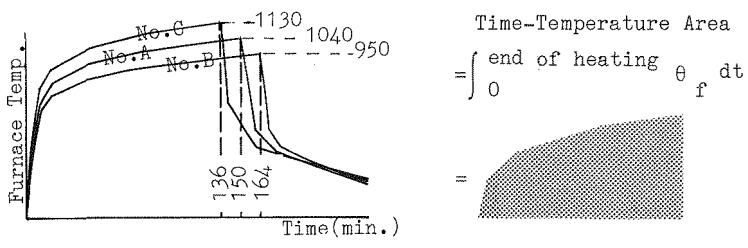


Figure 4. Furnace Temperature Histories.

In these three experiments, the "Time-Temperature area"(Figure 4) is kept constant, but the temperature itself differs ±10%. Current JIS regards these three heating conditions to be within the error tolerance.

The initial water content of the three specimens varied slightly.

Specimen No.		A	B	C
Age of Specimen	(Days)	69	76	82
Initial Water Content	(%)	8.7	8.2	7.7
Heating Rate	(%)	100	90	110
Heating Period	(min.)	150	164	136

Table 2 Condition of Experiments

DISCUSSION

Measured and calculated temperature histories are shown in Figures 5a),6a) and 7a). These figures show that the calculated results are in reasonable agreement with the measured data. The main features are,

1) Maximum temperatures of each point is first reached in experiment No.C, and its value is also highest.

2) Creeping temperature is about 100 to 140°C both in experiments and calculations. These temperatures become higher when the measuring point is nearer to the exposed surface.

In Figures 5b),6b) and 7b), vapor content histories are shown. In

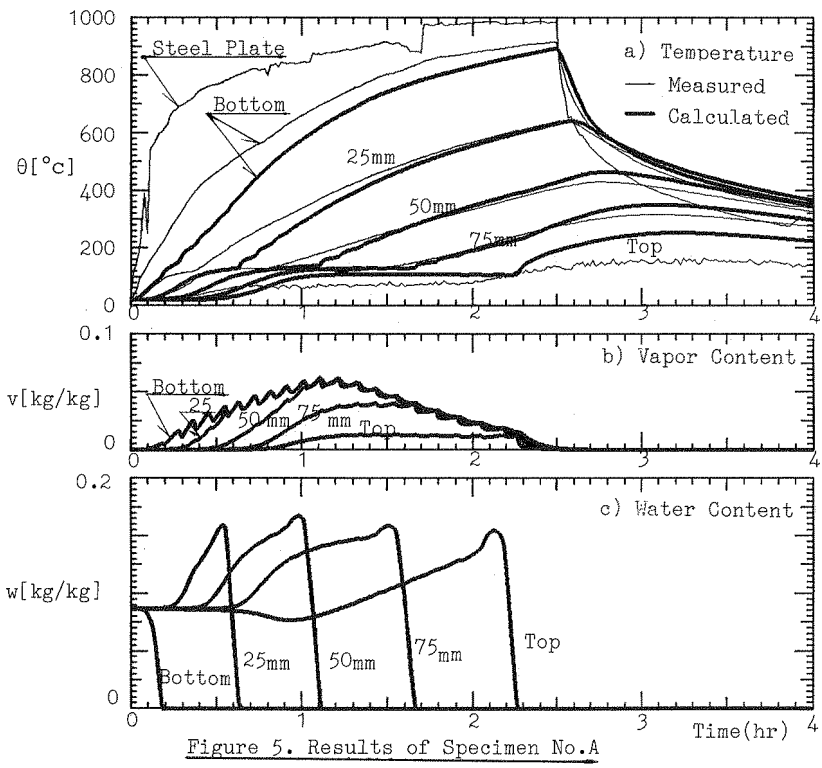


Figure 5. Results of Specimen No.A

these figures, small oscillations appear. They are not intrinsic, but are the effect of discretization. They can be reduced by using small grid sizes and small time increments. However, the computing time then becomes longer. (Computing time was about 80 seconds for each calculation)

In Figures 5c),6c) and 7c), the history of water content is shown. Water content of each point initially rises due to the diffusion of hot water vapor from the zone of exposed surface, but falls as the temperature of the point begins to rise.

The measured and calculated weight loss of each specimen are shown in Figure 8. The weight loss is calculated by the following equation,

$$W_{loss}(t) = \iiint_V \rho_0 \, (w_{ini} - w(t))dV \qquad (24)$$

• To reduce the difference between measured and calculated results, the following factors should be taken into account:

1) The material properties of concrete should be precisely measured.
2) The heat applied to the specimen should be precisely measured. In actual furnace conditions, radiative heat flux plays an important role. A properly estimated heat transfer coefficient is needed.
3) Decomposition, which take place at much higher temperatures, should be precisely measured and taken into account. These reactions also work as a heat sink.

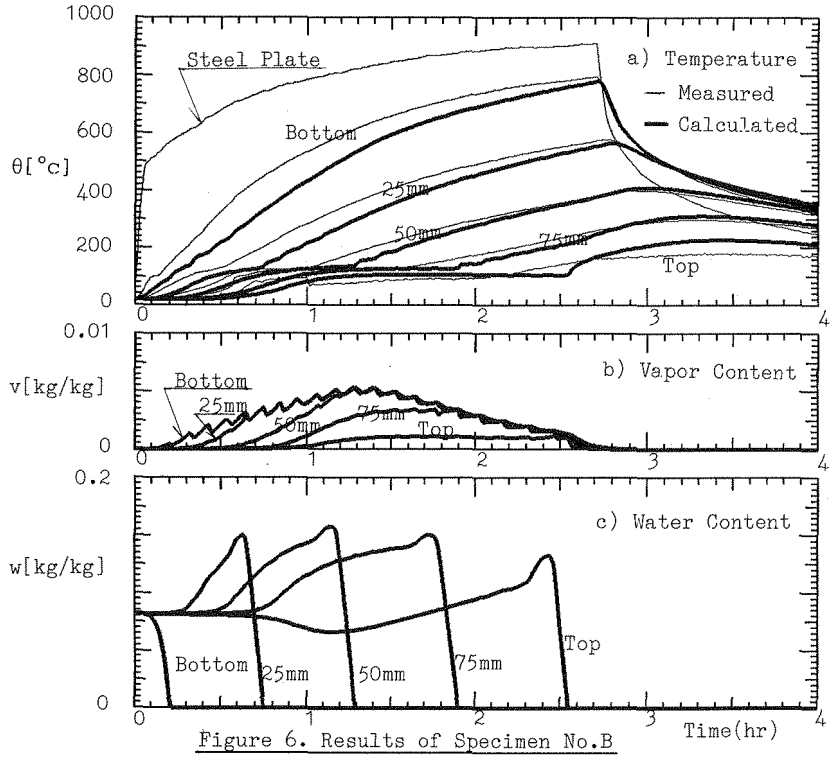


Figure 6. Results of Specimen No.B

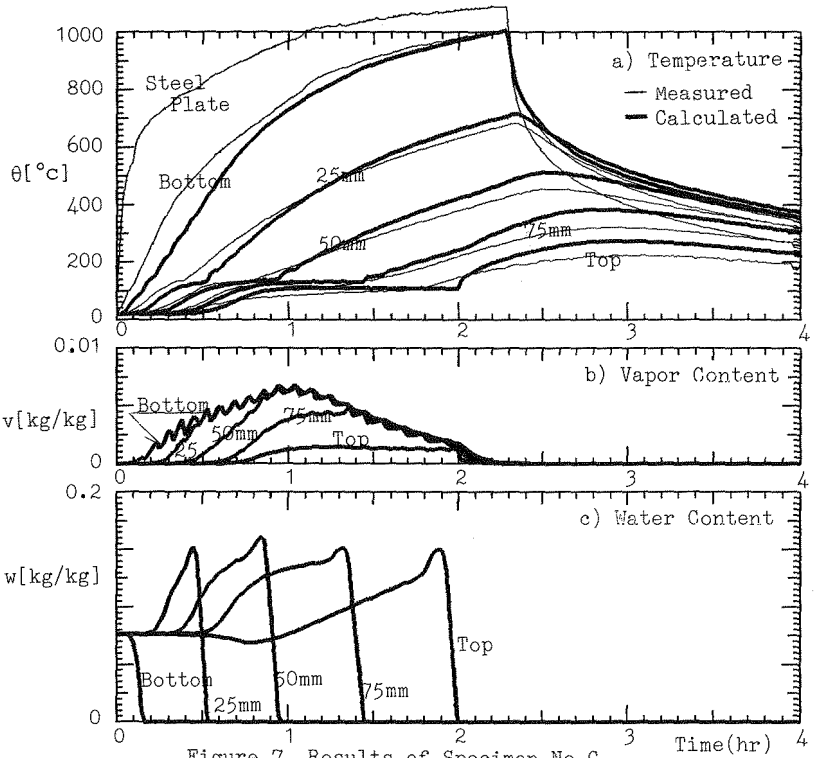


Figure 7. Results of Specimen No.C

SUMMARY

Sample calculations based on the diffusion model are presented. In spite of the simplicity of this model, typical temperature histories of concrete exposed to test fires, agree reasonably well with the experiments.

ACKNOWLEDGMENT

This work is supported by the Grant-in-Aid for Developmental

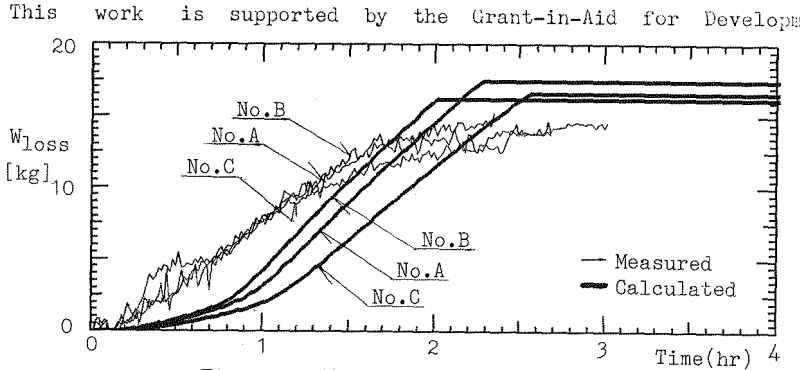


Figure 8. Weight Loss of the Specimens

Scientific Research of Ministry of Education of Japan. Numerical calculation was carried out at the Data Processing Center of Kyoto University.

NOMENCLATURE

Symbols

A_d	Frequency Factor	[1/Sec]	E_d	Energy of Activation	[J/kg]
C_s, n_s	Constants in Equation(5)		c	Heat Capacity	[J/kg K]
D	Diffusion Coefficient	[m^2/Sec]	h	Heat Transfer Coefficient	[W/m^2K]
h_v	Vapor Transfer Coefficient	[kg/m^2Sec]	L	Latent Heat	[J/kg]
T	Absolute Temperature	[K]	v	Vapor Content	[kg/kg]
R_{loc}	Rate of Local Evaporation	[kg/m^3Sec]	R_v	Gas Constant of Water Vapor	[Pa m^3/kgK]
w	Water Content	[kg/kg]	W	Weight	[kg]
γ	Evaporation Coefficient	[kg/m^3Sec]	ε	Void Fraction	[-]
θ	Temperature	[°C]	ρ	Density	[kg/m^3]
λ	Thermal Conductivity	[W/mK]			

Subscripts

0 Dry Concrete, v Water Vapor, w Liquid Water, eq Equilibrium, sat Saturated, eff Effictive, f Furnace, amb Ambient, ini Initial

REFERENCES

/1/ Kawagoe,K., "Calculation of Temperature in Double-layer walls heated from one side",Bull. of Fire Prevention Soc.Japan,13-2,29-35,1965

/2/ Wakamatsu,T., "Heat Flow Analysis of Building Members during Fire Exposure", Trans. Architectural Institute of Japan,109,73-79, and 111,31-36,1965

/3/ Fujii,S.,"The Theoretical Calculation of Temperature of Thermally Protected Steel Column Exposed to the Fire",Bull.of Fire Prevention Soc. Japan,13-1,13-21,1965

/4/ Harmathy,T.Z., "Simultaneous Moisture and Heat Transfer in Porous Systems With Particular Reference to Drying",I&EC Fundamentals,8-1,92-103,1969

/5/ Matsumoto,M.,(in Japanese), Ph.D. dissertation, Faculty of Engineering, Kyoto university,1978

/6/ Huang,C.L.D.,Siang,H.H and Best,C.H, "Heat and Moisture Transfer in Concrete Slabs",Int.J.Heat Mass Transfer,22,257-266,1979

/7/ Sahota,M.S.,and Pagni,P.J., "Heat and Mass Transfer in Porous Media Subjected to Fires",Int.J.Heat Mass Transfer,22,1069-1081,1979

/8/ Dayan,D., "Heat and Mass Transfer within an Intensely Heated Concrete Slab", Int.J.Heat Mass Transfer,25,1461-1467,1982

/9/ Young,D.M and Crowewll,A.D, "Physical adsorption of gases", Butterworth,1962

/10/ Alexander,R,"Diagonally Implicit Runge-Kutta Methods for Stiff O.D.E's",Siam J. Numerical Analysis,14-6,1006-1021,1977
/11/ Ueda,T. and Tokura,T., "An Experimental Study on Thermophysical Properties of Concrete at High Temperatures",Summaries of Technical Papers of Annual Meeting,Series A,Architectural Institute of Japan,761-766,1987
/12/ Tasaka,S.,Yoshida,M. and Harada,K.,"A Study on Temperature History in Concrete of Composite Floor in Fire",ibid,Series A,761-766,1987
/13/ "Method of fire resistance test for structural parts of buildings.",Japanese Industrial Standard,A 1304,1975