A Thermal Model for Piloted Ignition of Wood Including Variable Thermophysical Properties

MARC JANSSENS
National Forest Products Association
1250 Connecticut Avenue NW, Suite 200
Washington, DC 20036, USA

ABSTRACT

A simplified thermal model of piloted ignition is formulated. The model equations are then solved numerically for the thermally thick case using a finite difference technique. A systematic analysis of some solutions leads to a functional relationship between ignition time t_{ig} and irradiance $\dot{q}''_e$, suitable for correlation of piloted ignition data. This suggests plotting ignition data in a graph of $(t_{ig})^{-0.547}$ versus $\dot{q}''_e$. The critical irradiance $\dot{q}''_{cr}$ is then found as the intercept with the abscissa of a straight-line fit through the data. An apparent $k\rho c$ can be obtained from the slope of the regression line. Theoretical calculations show that this apparent $k\rho c$ for wood products is evaluated at a temperature approximately halfway between T_∞ and T_{ig}. The suggested correlating procedure is applied to measurements for six oven dry wood species obtained in the Cone Calorimeter.

KEYWORDS: wood, piloted ignition, mathematical model, thermal inertia, Cone Calorimeter

NOMENCLATURE

c	: Heat capacity ($J \cdot kg^{-1} \cdot K^{-1}$)
C	: Constant
C'	: Constant
F	: Characteristic function of time in equation (12)
h	: Convection coefficient ($W \cdot m^{-2} \cdot K^{-1}$)
k	: Thermal conductivity ($W \cdot m^{-1} \cdot K^{-1}$)
n	: Exponent
$\dot{q}$	: Heat flux (kW)
r	: Reflectivity
t	: Time (s)

T : Temperature (K)
x : Space coordinate (m)
Greek
α : Absorptivity
γ : Non-dimensional convection coefficient
ϵ : Emissivity
η : Non-dimensional space coordinate
θ : Non-dimensional temperature
ρ : Density ($kg{\cdot}m^{-3}$)
τ : Non-dimensional time
ψ : Non-dimensional radiation coefficient
φ : Non-dimensional irradiance
Subscripts
a : Average
c : Convective
e : External
dry : Oven dry
ig : At ignition
r : Reference
s : Surface
∞ : Ambient
Superscripts
" : Per unit area

INTRODUCTION

Piloted ignition of wood has been studied extensively over the past 40 to 50 years. The time to piloted ignition, t_{ig}, of a certain material is primarily a function of the incident heat flux. Ignitability at a given heat flux level depends on the thermal properties of the material, in particular the thermal inertia $k\rho c$. In previous work, small samples of wood were usually exposed to the radiant heat flux produced by a gas panel or an electric heater, and t_{ig} was measured as a function of the irradiance, $\dot{q}''_e$. Many investigators correlated such data using a power law of the following form

$$\left(\dot{q}''_e - \dot{q}''_{cr}\right) t_{ig}^n = C, \tag{1}$$

where C is constant for a given material and $\dot{q}''_{cr}$ is the critical irradiance below which piloted ignition under practical conditions no longer occurs. Lawson and Simms suggested $n=2/3$ and correlated C with $k\rho c$ values obtained from the literature [1]. Buschman correlated n, $\dot{q}''_{cr}$, and C with literature values for $k\rho c$ [2]. Magnusson and Sundström suggested an inverse correlating procedure, i.e., a technique to derive an apparent $k\rho c$ from the correlation of piloted ignition data [3]. However, this proposal was not very practical because it required measurement of surface temperature. Quintiere and Harkleroad developed another procedure to obtain an apparent $k\rho c$, without the need for such tedious temperature measurements [4].

The thermal properties used by Simms and Buschman were evaluated at ambient temperature. Both k and c of wood products, however, are strongly dependent on temperature. Intuitively one expects an apparent $k\rho c$ to correspond to a temperature somewhere between T_∞ and the surface temperature at ignition, T_{ig}, perhaps closer to the latter. The results presented below confirm that this is indeed the case.

MATHEMATICAL MODEL OF PILOTED IGNITION

A large number of mathematical models of the piloted ignition problem have been developed with varying degrees of complexity. Some models include gas phase diffusion and mixing [5]. Others consider only the solid phase, but include pyrolysis and other chemical reactions [6]. The model considered in this article is less sophisticated, but still includes many features not addressed by other thermal models. It is based on the following assumptions

- Heat flow in the solid is one-dimensional, i.e., perpendicular to the exposed surface.
- Chemical effects prior to ignition are negligible, i.e., no pyrolysis.
- Convective heat transfer between fuel vapors and the porous solid is negligible.
- Ignition occurs when the surface reaches a given material-dependent temperature. This criterion is acceptable for engineering purposes, as shown experimentally in [7].
- The material is opaque. Although wood is in fact not completely opaque, especially at small wavelengths, it is much less transparent than many other materials.
- Kirchoff's law is valid for the total α, ϵ and r, i.e., $\alpha=\epsilon=1-r$.
- The values for α, ϵ and r are constant between the start of exposure and ignition.
- The heat losses from the surface are partly radiative and partly convective with a constant convection coefficient.
- The specimens behave as a semi-infinite solid. All wood samples tested have a thickness of more than 16 mm and may be considered thermally thick [8].

Under these assumptions, the piloted ignition problem becomes a purely thermal problem. It has the following mathematical form

Energy Conservation

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) = \rho c\frac{\partial T}{\partial t}, \tag{2}$$

Boundary Condition (x=0)

$$-k\frac{\partial T}{\partial x} = \epsilon\dot{q}_e'' - h_c(T_s - T_\infty) - \epsilon\sigma\left(T_s^4 - T_\infty^4\right), \tag{3}$$

Boundary Condition (x→∞)

$$-k\frac{\partial T}{\partial x} = 0, \tag{4}$$

Initial Condition (t=0, x≥0)

$$T = T_\infty. \tag{5}$$

The absorbed part of the critical irradiance, $\dot{q}''_{cr}$, is equal to the heat losses from the surface at ignition because below this irradiance level the surface temperature can never reach T_{ig}, even for $t \rightarrow \infty$. Consequently, a total heat transfer coefficient from the surface at ignition h_{ig} can be defined as

$$\epsilon \dot{q}''_{cr} \equiv h_c(T_{ig} - T_\infty) + \epsilon\sigma\left(T_{ig}^4 - T_\infty^4\right) \equiv h_{ig}(T_{ig} - T_\infty). \tag{6}$$

The objective is to find a solution of (2)-(5) in a form suitable for the correlation of experimental data and to infer an apparent value for $k\rho c$. The latter is a constant representative average over the temperature range between T_∞ and T_{ig}. In order to obtain such a solution, k and c are assumed constant. The following non-dimensional variables can then be defined

$$\theta \equiv \frac{T - T_\infty}{T_{ig} - T_\infty}, \quad \tau \equiv \frac{h_{ig}^2 t}{k\rho c}, \quad \eta \equiv \frac{x h_{ig}}{k}, \quad \varphi \equiv \frac{\dot{q}''_e}{\dot{q}''_{cr}},$$
$$\gamma \equiv \frac{h_c}{h_{ig}}, \quad \psi \equiv \frac{\sigma(T_{ig} - T_\infty)^4}{\dot{q}''_{cr}}, \quad \text{and} \quad \theta_\infty \equiv \frac{T_\infty}{T_{ig} - T_\infty}. \tag{7}$$

The resulting non-dimensional model equations are

Energy Conservation

$$\frac{\partial^2\theta}{\partial\eta^2} = \frac{\partial\theta}{\partial\tau}, \tag{8}$$

Boundary Condition ($\eta=0$)

$$-\frac{\partial\theta}{\partial\eta} = \varphi - \gamma\theta_s - \psi\left((\theta_s + \theta_\infty)^4 - \theta_\infty^4\right), \tag{9}$$

Boundary Condition ($\eta \rightarrow \infty$)

$$-\frac{\partial\theta}{\partial\eta} = 0, \tag{10}$$

Initial Condition ($\tau=0$, $\eta \geq 0$)

$$\theta = 0. \tag{11}$$

SOLUTION WITH LINEARIZED HEAT LOSSES

The solution of (8)-(11) with linearized surface heat losses ($\gamma=1$ and $\psi=0$) can be found in standard textbooks on heat conduction e.g., [9], p.276. At the exposed surface $\eta=0$, this yields a fairly simple expression

$$\theta_s = \varphi\left[1-\exp(\tau)\,\mathrm{erfc}(\sqrt{\tau})\right] = \varphi F(\tau). \tag{12}$$

Equation (12) shows that θ_s/φ is independent of irradiance. The complimentary error function cannot be calculated analytically, but may be obtained numerically. Many investigators have based their correlation of piloted ignition data on equation (12). Unfortunately, due to the presence of the complementary error function, the functional form of (12) is not directly suitable for correlating purposes. Therefore, approximations based on a truncated series expansion have been used instead [1]-[4]. These approximations are only valid over a limited range of τ values. Outside this range, agreement with $F(\tau)$ is poor.

It is remarkable that a number of researchers have been successful at correlating piloted ignition data using a power law of the form of equation (1). If the assumptions are valid that ignition occurs at a given surface temperature and that surface heat losses are approximately linear, (1) indicates that $F(\tau_{ig})$ can be expressed as

$$F(\tau_{ig}) = \frac{1}{\varphi} \approx \frac{1}{1 + C'\tau_{ig}^{-n}}. \tag{13}$$

A non-linear least squares regression fit of a function of the form (13) to the original function $F(\tau)$ in (12) over a range of τ values from 0 to 20[1] gives the following result

$$F(\tau) \approx \frac{1}{1 + 0.73\,\tau^{-0.547}}. \tag{14}$$

The optimum value for n is 0.547 which is very close to 0.5, as suggested by a number of authors (Quintiere [4], Mikkola and Wichman [8], Abu-Zaid [10], Panagiotou and Delichatsios [11]). The functional form (14), although quite accurate (less than 1% error over the range $0<\tau<20$), is remarkably simple. It suggests plotting ignition data as $(t_{ig})^{-0.547}$ versus irradiance $\dot{q}_e''$. The critical flux is then found as the intercept with the abscissa and $k\rho c$ can be calculated from the slope. A question remains whether (14) is still applicable if surface heat losses are non-linear, $\epsilon<1$, and thermal properties vary with temperature.

NON-LINEAR SURFACE HEAT LOSSES AND $\epsilon \leq 1$

A numerical finite difference solution to equations (8)-(11) is used to obtain the non-dimensional time τ_{ig} to reach ignition temperature, T_{ig}, on the surface for ϵ=0.6, 0.7, 0.8, 0.9 and 1. The calculations are performed for T_{ig}=250°C, 350°C, 450°C and 550°C. A power law of the form of (13) is used for a curve fit of the calculated results. Table 1 lists the values for C' and n resulting in the best fit for each of the ignition temperatures considered and ϵ=0.6 as an example. Note that $\dot{q}_{cr}''$ in column 2 is calculated from (6) with ϵ=0.6 and h_c=15 W·m^{-2}·K^{-1} [4]. As in the case of linear heat losses, 0.73 and 0.547 seem

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[1] With equation (7) and the values for $k\rho c$ and h_{ig} of Table 4, the real time corresponding to τ=20 exceeds 24 minutes for all materials tested. Since the test time was always less than 20 minutes, the range of τ chosen for the curve fit is appropriate.

to be very reasonable averages for C' and n respectively. Thus, the conclusion is that the functional relationship (14), valid for linearized surface heat losses, may also be used for the case of non-linear heat losses, and $\epsilon \leq 1$, at least when h_c is of the order of 15 $W \cdot m^{-2} \cdot K^{-1}$ and h_{ig} is used to define τ (see Eq. (7)).

TABLE 1. Values for power law relationship (13) with $\epsilon=0.6$

T_{ig} (°C)	$\dot{q}''_{cr}$ ($kW \cdot m^{-2}$)	C'	n
250	9.6	0.77	0.530
350	16.4	0.74	0.544
450	25.8	0.73	0.551
550	38.9	0.70	0.542

THERMAL PROPERTIES OF WOOD

In all calculations so far, k and c are assumed to be constant. For wood products, as for many other building materials, thermal properties are temperature dependent. Maku found the following (convenient) temperature effect on the thermal conductivity of oven-dry wood [12]

$$k(T) = k_r \frac{T}{T_r}, \tag{15}$$

where k_r is the thermal conductivity at the reference temperature T_r in $W \cdot m^{-1} \cdot K^{-1}$; it can be calculated from MacLean's correlation, for T_r=293 K [13]

$$k_r = 0.0237 + 2 * 10^{-4} * \rho_{dry}. \tag{16}$$

Koch found that the heat capacity of some oven dry wood species is also proportional to the absolute temperature [14]

$$c(T) = c_r \frac{T}{T_r}, \tag{17}$$

where c_r is the heat capacity in $J \cdot kg^{-1} \cdot K^{-1}$ at the reference temperature T_r; c_r is independent of density and equal to 1200 $J \cdot kg^{-1} \cdot K^{-1}$ at T_r=293 K.

SOLUTION WITH TEMPERATURE-DEPENDENT THERMAL PROPERTIES

A similar numerical solution as for (8)-(11) is used to solve equations (2)-(5) with temperature-dependent k and c. Calculations are performed for a material with density ρ_{dry}=500 $kg \cdot m^{-3}$, $\epsilon=1$ and T_{ig} ranging from 250°C to 550°C. Equations (15)-(17) are used to obtain k and c as a function of temperature. Plotted in a graph of $-\log(t_{ig})$ versus $\log(\dot{q}''_e - \dot{q}''_{cr})$, the results at any of the ignition temperatures considered approximately follow a straight line. Therefore, equation (13) is applicable, even if k and c are temperature-dependent. The value for n ranges from 0.539 to 0.561, depending on T_{ig}. Thus, n=0.547 still appears to be a good average and equations of the following form can be used to correlate the calculated results

$$\dot{q}_e'' = \dot{q}_{cr}''\left[1+0.73\left(\frac{k\rho c}{h_{ig}^2 t_{ig}}\right)^{0.547}\right] \quad \text{or} \quad \varphi = 1+0.73\left(\frac{1}{\tau_{ig}}\right)^{0.547}. \tag{18}$$

An apparent $k\rho c$ value may be obtained from the slope of a linear fit according to (18) while $\dot{q}_{cr}''$ follows from the intercept with the abscissa. The results for four values of T_{ig} are presented in Table 2. Again, extrapolated $\dot{q}_{cr}''$ values in column 3 are close to the exact values in column 2. The $k\rho c$ values in column 5 are apparent values, i.e., averages for which the solution of the piloted ignition problem with constant k and c is approximately equal to the solution of the problem with temperature-dependent k and c.

TABLE 2. Correlation according to (18) with temperature-dependent k and c

Exact T_{ig}(°C)	Exact $\dot{q}_{cr}''$(kW·m^{-2})	Extrapolated $\dot{q}_{cr}''$(kW·m^{-2})	$k_r\rho c_r$ (kJ^2·m^{-4}·K^{-2}·s)	$k\rho c$ from slope (kJ^2·m^{-4}·K^{-2}·s)	θ_a
250	7.3	7.4	0.074	0.135	0.45
350	13.1	12.9	0.074	0.189	0.53
450	21.5	21.2	0.074	0.229	0.51
550	33.6	33.2	0.074	0.269	0.50

One would expect these apparent values to be at some temperature between T_r and T_{ig}. This temperature, denoted as T_a, can be found from the ratio of $k\rho c$ to $k_r\rho c_r$, using (15)-(17)

$$\frac{k\rho c}{k_r\rho c_r} = \frac{k_r\frac{T_a}{T_r}\rho c_r\frac{T_a}{T_r}}{k_r\rho c_r} = \left(\frac{T_a}{T_r}\right)^2. \tag{19}$$

Thus, T_a is equal to the square root of the aforementioned ratio multiplied by the reference temperature T_r. This temperature may also be expressed in a non-dimensional form with the definition of θ in (7)

$$\theta_a = \frac{T_a - T_\infty}{T_{ig} - T_\infty}. \tag{20}$$

Column 6 in Table 2 shows θ_a values with T_a calculated from the ratio of $k\rho c$ in column 5 to $k_r\rho c_r$ in column 4. The average θ_a is about 0.5. This means that apparent $k\rho c$ values, as inferred from the correlation of piloted ignition data of wood, are evaluated at a temperature, T_a, approximately halfway between the reference (close to ambient T_∞) and the ignition temperature, T_{ig}. Of course, this remarkable result is obtained for a particular material, i.e., a wood product with oven dry density 500 kg·m^{-3} and relatively simple dependencies on temperature of k and c, characteristic for wood. However, the correlation of experimental data presented below indicates that $\theta_a=0.5$ is equally valid for other densities.

CORRELATION OF SOME EXPERIMENTAL DATA

Materials Tested

The time to piloted ignition was measured for six oven-dry wood species in the Cone Calorimeter [15] over a range of irradiance levels between 15 and 45 $kW \cdot m^{-2}$. Table 3 lists the names of the species tested, the oven-dry density ρ_{dry} and the mean thickness L of the specimens

TABLE 3. Wood species tested

Common name	Botanical name	ρ_{dry} ($kg \cdot m^{-3}$)	L (mm)
Western redcedar	Thuja plicata	330	17.0
Redwood	Sequoia sempervirens	430	19.0
Radiata pine	Pinus radiata	460	17.5
Douglas fir	Pseudotsuga menziesii	465	16.8
Victorian ash	Eucalyptus delegatensis	640	17.2
Blackbutt	Eucalyptus pilularis	810	17.4

The first four species are softwoods, the remaining two are hardwoods. Western redcedar, redwood and Douglas fir are North American species, and the remaining species are Australian.

Correlation of Results

The ignition tests in the Cone Calorimeter were conducted in the vertical orientation. For many specimens, T_{ig} was measured with 5 mil (0.127 mm) thermocouples attached to the surface. Table 4 shows average ignition temperatures and some related quantities.

TABLE 4. T_{ig} and related quantities measured in the Cone Calorimeter

Species	T_{ig} (K)	$\dot{q}''_{cr}$ ($kW \cdot m^{-2}$)	h_{ig} ($W \cdot m^{-2} \cdot K^{-1}$)	$k\rho c$ ($kJ^2 \cdot m^{-4} \cdot K^{-2} \cdot s$)
Western redcedar	627	13.3	34.9	0.087
Redwood	637	14.0	35.9	0.141
Radiata pine	622	12.9	34.6	0.156
Douglas fir	623	13.0	34.6	0.158
Victorian ash	584	10.4	31.5	0.260
Blackbutt	573	9.7	30.6	0.393

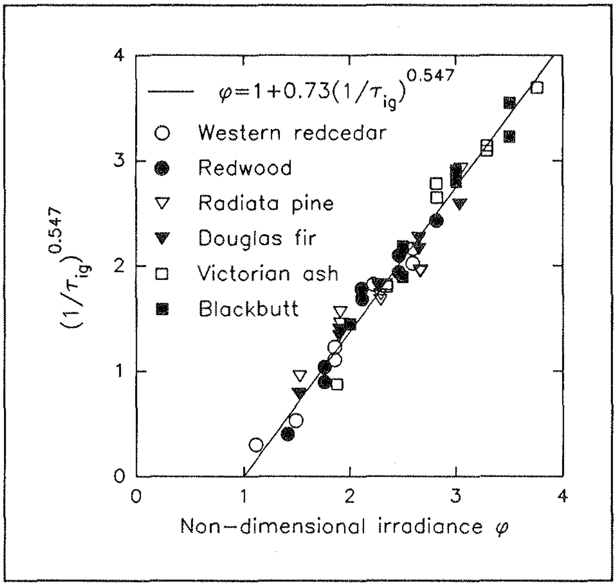

FIGURE 1 Correlation of some experimental data

The quantities $\dot{q}''_{cr}$ and h_{ig} are calculated from (6) using the average measured T_{ig} from column 2 of Table 4. A value of 13.5 $W{\cdot}m^{-2}{\cdot}K^{-1}$ is used for h_c, representative for the Cone Calorimeter in the vertical orientation [16]. Wesson measured absorptivity for a large number of wood species [17]. He found the value for a grey or black radiant heat source to be independent of species and equal to 0.76. However, all his measurements were made at ambient temperature. As wood is heated, its surface darkens and emissivity is close to unity just prior to ignition [18]. Therefore, an average value of ϵ=0.88 is used.

All piloted ignition data can now be correlated using equation (18). The result is shown in Figure 1. The solid line represents (18) in non-dimensional form. The data points are quite close to the line, but there is some inevitable scatter due to slight variances in specimen properties (density) and test conditions.

CONCLUSIONS

Piloted ignition data obtained in the Cone Calorimeter, or similar bench-scale apparatuses, may be correlated with the following formulas

$$\dot{q}''_e = \dot{q}''_{cr}\left[1+0.73\left(\frac{k\rho c}{h_{ig}^2 t_{ig}}\right)^{0.547}\right] \quad \text{or} \quad \varphi = 1+0.73\left(\frac{1}{\tau_{ig}}\right)^{0.547}. \tag{18}$$

The critical irradiance, $\dot{q}''_{cr}$, can be found as the intercept with the abscissa of a straight-line fit through the data in a graph of $(1/t_{ig})^{0.547}$ versus irradiance $\dot{q}''_e$. An apparent $k\rho c$ is obtained from the slope of the line. For wood products, the apparent value corresponds to a temperature halfway between ambient and T_{ig}.

The results obtained so far are for oven dry thermally thick wood species. Further work is needed to determine whether the conclusions presented here can be extended to thin and wet wood specimens and board products such as plywood or particle board.

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