Elasto-Plastic-Creep Three-Dimensional Analysis of Steel H-Columns Subjected to High Temperatures

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ABSTRACT

The analysis method, which is intended to precisely grasp the thermal elasto-plastic-creep three-dimensional deformation behavior of steel wide-flange columns at elevated temperatures, is presented. This method is a combined nonlinear finite element procedure based on the finite displacement theory of the thin-walled open cross section, and adopts the mechanical model of structural steel at high temperature proposed by Furumura et al.

Based on some assumptions, two kinds of examples for thermal elasto-plastic-creep three-dimensional deformation behavior of steel wide-flange columns subjected to high temperatures are pursued.

KEYWORDS: finite element method, steel column, elasto-plastic-creep, biaxial bending, lateral torsional buckling, plastic theory, creep theory

INTRODUCTION

In the past, several studies on the in-plane elasto-plastic-creep behavior of steel columns were carried out[1-3]. But, in an actual building framework, the columns are frequently subjected to biaxial bending in addition to axial compression. Therefor it is important to advance the studies on the three-dimensional deformation behavior of steel structures at high temperature to develop the rational fire resistant design of steel buildings.

In this study, the nonlinear three-dimensional analysis method of steel columns is described, and the illustrative examples involving out-of-plane buckling and biaxial bending problems at high temperature are investigated.

METHOD OF ANALYSIS

In order to compute precisely the collapse behavior of steel building in fire environment, it is necessary to consider both the material and the geometrical nonlinearities simultaneously. The analytical procedure for three-dimensional behavior of wide-flange steel columns is developed by means of adopting the mechanical model[5] of structural steel at high temperature for the constitutive relation of the finite element procedure[9], which is based on the beam-theory of thin-walled open cross section.

Analytical Assumptions and Modeling of Structures

For the purpose of the analysis, assumptions have been made as follows:

1) The behavior of steel columns is described by a series of incremental time steps, and the temperature and stress in the steel material are assumed to be constant over each time step. And the change of temperature and stress is assumed to occur at boundaries between the incremental time steps. The incremental creep strain at a time step is considered at the beginning of next time step.
2) Wide-flange steel columns having three-dimensions are treated as beam elements with thin-walled open cross section.
3) The stress field in the steel material is idealized to be the isotropic plane stress condition, in which the normal stress perpendicular to the axial stress σ is assumed to be zero, and the elasto-plastic tangent stiffness matrix in the $\{\sigma, \tau\}$-stress field is evaluated by using the incremental theory (Associated Flow Rule) based on the von-Mises yield criterion.
4) The multiaxial creep strain at high temperature is evaluated by using the state equation theory (Creep Potential Theory) based on the assumption of von-Mises creep potential.
5) As for deformation of each element, deformation in axial direction, bending deformation in biaxial directions, torsional deformation including warping of section and St. Venant's torque are taken into account.
6) Local deformation of cross sections in structural members is disregarded, and deformation due to shear in conjunction with bending of members is considered negligible.
7) Regardless of plastic or elastic condition of cross sections, it is assumed that the sections remain plane as far as the bending in biaxial directions is concerned. A similar assumption is applied to warping function; that is, a function $\omega(y,z)$ is used whatever the conditions of elasticity and plasticity are.
8) The distribution of shear strain caused by St. Venant's torsion is linear through the thickness of the component plate even for a partially yielded cross section.
9) Assuming the existence of the initial strain corresponding to the residual stress at initial condition, the residual stress is taken into account.

Constitutive Relation of Structural Steel at High Temperature

The constitutive relation of structural steel under the combined stress condition is required to the analysis of the three-dimensional deformation behavior of steel columns. In this study, the mechanical behavior under simple test condition is generalized to combined stresses. The mechanical model of structural steel[5] developed by Furumura et al. is illustrated in Fig.1, and the assumption(1) represents the change of temperature and stress in steel to be step-wise.

Fig.1(A) shows that if the temperature is changed from a constant temperature T_{i-1} to another one T_i under the stress point B, the stress-strain curve changes GBC to GEF. And Fig.1(B) shows that if the stress point D is maintained constant for a time increment Δt , the incremental creep strain $\Delta \varepsilon_c$ is caused by following the strain-hardening creep law which uses the strain ε_c in

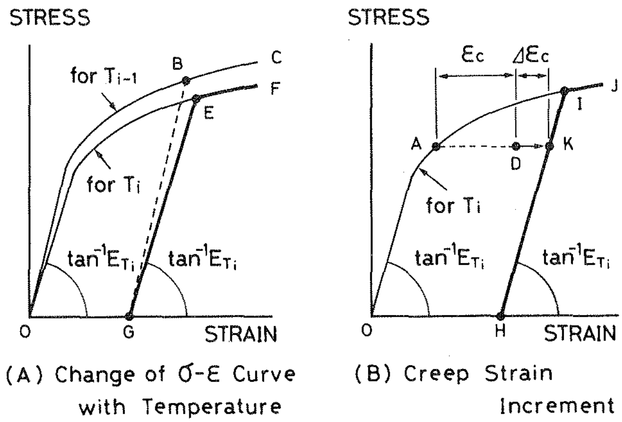


FIGURE 1. Mechanical model of steel material at high temperature

Fig.1(B) as the accumulative creep strain, and the stress follows the curve HKIJ after creep strain increment.

The uniaxial mechanical model shown in Fig.1 is generalized to the plane stress condition as follows.

[Generalization of mechanical model in Fig.1(A)]

At the end of (i-1)-th incremental time step, the elastic strain ε_{ei-1} corresponding to the stress σ_{i-1} must be calculated by the equation $\varepsilon_{ei-1} = \mathbf{D}_{ei-1}^{-1}\sigma_{i-1}$. The elastic matrix $\mathbf{D}_{ei-1}$ must be evaluated by the temperature at the (i-1)-th time step.

In short, the initial elastic strain ε_{ei-1} at the beginning of i-th time step must be obtained beforehand, and the stress σ_i corresponding to the strain $\varepsilon_i = \varepsilon_{ei-1} + \Delta\varepsilon_i$ must be calculated from the initial unstrained point. Herein $\Delta\varepsilon_i$ is the strain increment derived in the iteration process for searching the equilibrium position.

In the elastic range, the incremental stress vector $\Delta\sigma$ is related to the incremental strain vector $\Delta\varepsilon$ as follows:

$$\Delta\sigma = \mathbf{D}_e \cdot \Delta\varepsilon \tag{1}$$

where

$$\Delta\sigma^T = \{\Delta\sigma, \Delta\tau\},\ \Delta\varepsilon^T = \{\Delta\varepsilon, \Delta\gamma\},\ \mathbf{D}_e = \begin{bmatrix} E_T & 0 \\ 0 & \dfrac{E_T}{2(1+\nu)} \end{bmatrix}$$

in which E_T is Young's modulus at temperature T and ν is Poisson's ratio of steel.

In the plastic range, the following stress-strain relation[9] is used taking account of the assumption(3).

$$\Delta\sigma = (\mathbf{D}_e + \mathbf{D}_p) \cdot \Delta\varepsilon \tag{2}$$

where

$$\mathbf{D}_p = -\frac{1}{S}\begin{bmatrix} S_1^2 & S_1S_2 \\ S_1S_2 & S_2^2 \end{bmatrix},\ S_1 = E_T\sigma',\ S_2 = E_T\tau,\ S = 4\bar{\sigma}^2H_T'/9 + (S_1\sigma' + 2S_2\tau),$$

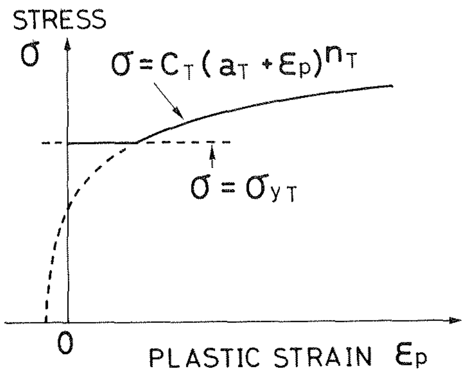


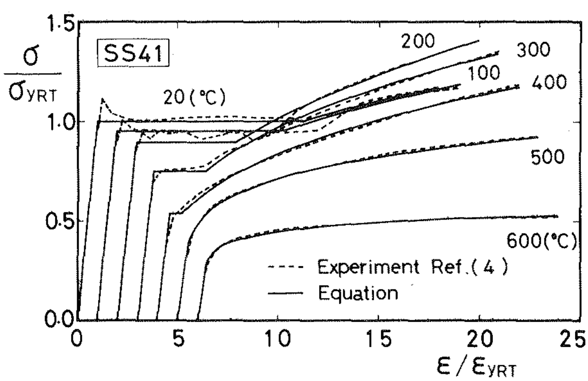
FIGURE 2. Equations for strain-hardening curve of structural steel

TABLE 1. Modulus of elasticity and coefficients of strain-hardening curves

T(°C)	E_T/E_{RT}	σ_{yT}/σ_{yRT}	C_T/σ_{yRT}	a_T/ε_{yRT}	n_T
20	1.000	1.000	0.309	0.000	3.941
100	0.991	0.952	0.305	0.000	3.919
200	0.964	0.897	0.374	0.000	6.139
300	0.919	0.751	0.315	0.000	4.638
400	0.856	0.538	0.304	0.737	3.840
500	0.774	0.000	0.187	0.166	1.931
600	0.675	0.000	0.105	0.026	0.803

T:temperature, RT:room temperature, E:Young's modulus
σ_y:yield strength, $\varepsilon_y = \sigma_y/E$

FIGURE 3. Comparison of predicted and test data of uniaxial stress-strain curves of steel material



σ' is the deviatoric stress and H_T' is strain-hardening rate.

[Generalization of the mechanical model in Fig.1(B)]

Following to the assumption(4), the incremental creep strain vector $\Delta\varepsilon_c$ at the plane stress condition[8] is calculated by the next equation.

$$\Delta\boldsymbol{\varepsilon}_c = \frac{3}{2\bar{\sigma}}\,\boldsymbol{\sigma}'\,\Delta\bar{\varepsilon}_c \qquad (3)$$

in which $\sigma'^T = \{\sigma', 2\tau\}$ and $\Delta\bar{\varepsilon}_c$ is the uniaxial creep strain increment corresponding to the equivalent stress $\bar{\sigma}$. Since the creep strain increment $\Delta\bar{\varepsilon}_c$ is taken into account as the same kind of strain as plastic, this must be added to the equivalent total plastic strain $\bar{\varepsilon}_p$, and the incremental creep strain vector $\Delta\varepsilon_c$ must be subtracted from the elastic strain vector ε_e.

The incremental thermal strain is given by the next equation.

$$\Delta\boldsymbol{\varepsilon}_T = \begin{Bmatrix} \Delta\varepsilon_T \\ 0 \end{Bmatrix} \qquad (4)$$

in which ε_T is the uniaxial thermal strain of steel due to temperature.

The experimental formula for $\sigma - \varepsilon_p$ curve of steel material is approximated directly by the digitalized data[6] of stress-strain curves of steel at high temperature. The yield plateau and the strain hardening part of the curves are expressed as shown in Fig.2, in which $\sigma_{yT}, c_T, a_T, n_T$ are the constants depending on the temperature of steel materials, and ε_p is the plastic strain. The values of these constants are shown in Table 1. The calculated uniaxial $\sigma - \varepsilon$ curves are compared with the experimental ones[6] in Fig.3. For the mechanical properties of SS41 steel at room temperature, σ_{yRT}=244(N/mm^2), E_{RT}=206(kN/mm^2), ν=0.3 are used.

The uniaxial creep strain of steel material at high temperature is evaluated by the following equation[4].

$$\varepsilon_c = 10^{a/T+b} \cdot \sigma^{c/T+d} \cdot t^{eT+f} \qquad (5)$$

in which ε_c (%) is the primary creep strain, T(°K) is absolute temperature, σ(kg/mm^2) is stress, t (minutes) is time and a ~ f are the following constants, $a=-7.45\times10^3$, $b=3.71$, $c=1.78\times10^3$, $d=1.82$, $e=6.47\times10^{-4}$, $f=-1.51\times10^{-1}$.

The thermal strain ε_T is assumed as follows:

$$\varepsilon_T = 5.04\times10^{-9}T^2 + 1.13\times10^{-5}T \qquad (6)$$

in which T is degree Centigrade.

Equilibrium Equation of Finite Elements and Computation Procedure

In order to approximate the curves of a steel column after deformation, the column is divided into some elements shown in Fig.4. Whenever each incremental displacement is computed, local coordinate system for each element is shifted and re-defined so that it passes through the both ends of the element.

The incremental displacement vector $\{\Delta u, \Delta v, \Delta w, \Delta\varphi\}^T$ on the x-axis of the element is expressed by the incremental nodal displacement vector $\Delta_e\mathbf{u}$. Whether or not an element has yielded, incremental displacement in the x-direction(Δu) is expressed as a linear function of x and incremental displacements in the y and z-directions(Δv and Δw) and incremental torsion

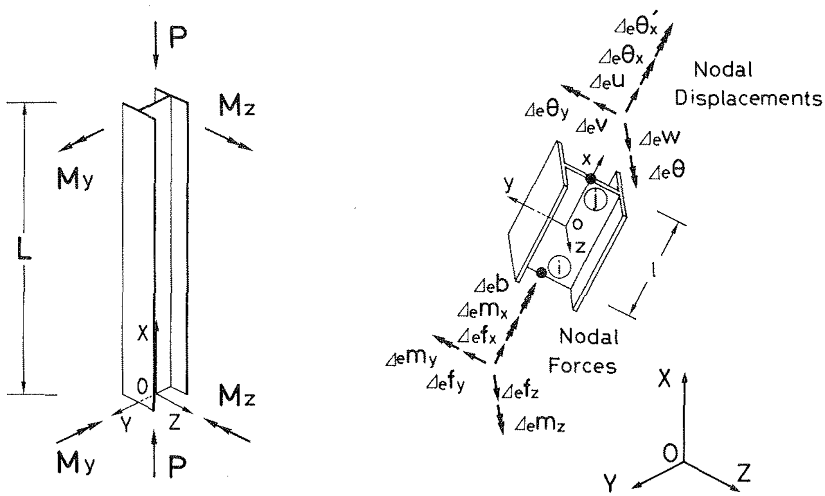


FIGURE 4. Steel column and finite element

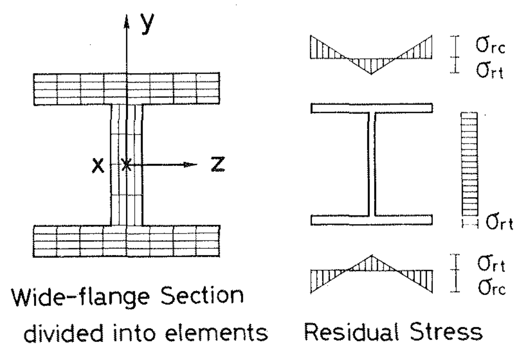


FIGURE 5. Wide flange section of steel column

($\Delta\varphi$) are expressed as cubic functions of x.

On the other hand, the incremental strain within the element can be expressed as follows considering large deformation[9].

$$\Delta\varepsilon = \frac{d\,\Delta u}{dx} - y\frac{d^2\,\Delta v}{dx^2} - z\frac{d^2\,\Delta w}{dx^2} - \omega\frac{d^2\,\Delta\varphi}{dx^2} + \frac{1}{2}\left(\frac{d\,\Delta v}{dx}\right)^2 + \frac{1}{2}\left(\frac{d\,\Delta w}{dx}\right)^2$$
$$+ \frac{1}{2}(y^2 + z^2)\left(\frac{d\,\Delta\varphi}{dx}\right)^2 + z\,\Delta\varphi\frac{d^2\,\Delta v}{dx^2} - y\,\Delta\varphi\frac{d^2\,\Delta w}{dx^2} \qquad (7)$$

$$\Delta\gamma = 2n\frac{d\,\Delta\varphi}{dx} \qquad (8)$$

where ω denotes the warping function, and n is a coordinate which originates at any point on the middle surface contour of the plate.

Applying the principle of virtual work as a usual manner, the incremental equilibrium equation for the beam element is written as follows:

$$(k_{ep} + k_G) \cdot \Delta_e u = (f_{ex} + \Delta f_{ex}) - f_{in} \qquad (9)$$

where k_{ep} is the elasto-plastic stiffness, the matrix k_G is the initial stress stiffness which represents the geometrical nonlinearity caused by finite displacement, the vector (f_{ex} + Δf_{ex}) is the external load, the vector f_{in} is the internal force and the vector $\Delta_e u$ is the incremental nodal displacement.

Based on the assumption(1) about the temperature and stress of the element, the constitutive relation of structural steel under $\{\sigma, \tau\}$-stress field, and the incremental equilibrium equation(9), the distribution of strain and stress in the steel column is calculated for each time interval during the heating process. The equation(9) is reformulated and resolved repeatedly by means of a step by step method and a iteration method. Stress and strain conditions within the elements are examined at both ends of elements, not regarding the intermediate parts, and the stress and strain conditions of the inside were linearly interpolated from the stress distribution at the both ends.

RESULTS AND DISCUSSIONS

The steel H-column in Fig.4 with the cross section (H-200x200x8x12) was used for the example. Both ends of this member are simply supported, and restrained against rotation and warping. The slenderness ratios of the column are L/r_z =30 and L/r_y =51, and the magnitude of axial load is $P=0.3P_{yRT}$.
The computation was carried out for the half part of the column, and divided into 10 longitudinal beam elements. The cross section of beam element is divided into 180 segments (10 divisions along the width or the height and 6 layers along the thickness for both flange and web plates). The magnitude of residual stress shown in Fig.5 is σ_{rc}/σ_{yRT} = 0.3 .

The parameter $\overline{m}_{yz}$ was defined to express the combination of bending moments M_y and M_z as follows:

$$\overline{m}_{yz} = (M_y/M_{pc,y})/(M_z/M_{pc,z}) \qquad (10)$$

where $M_{pc,y}$, $M_{pc,z}$ are the reduced plastic moments about y,z-axis considering the effect of axial force.

Next two kinds of analysis were carried out.
(Example 1) Computation of loads versus deformations curves at constant high temperatures.
(Example 2) Computation of the thermal elasto-plastic-creep deformation behavior of steel columns under the constant loads P,M_z ,M_y at increasing steel temperature.
In Example 1, the creep strain at high temperature is not included, but in Example 2, all analytical models are computed by the elasto-plastic analysis and elasto-plastic-creep analysis in order to investigate the effect of creep strain on the steel column behavior.

In Example 1, six kinds of cases $\overline{m}_{yz}$ =0.0,0.25,0.5,1.0,2.0,∞ are computed. In the case of $\overline{m}_{yz}$ =0.0, the columns were assumed to have the initial crookendness and rotation of sinusoidal half-wave with the central value of L/1000 and L/(500H), where H is the height of cross section.

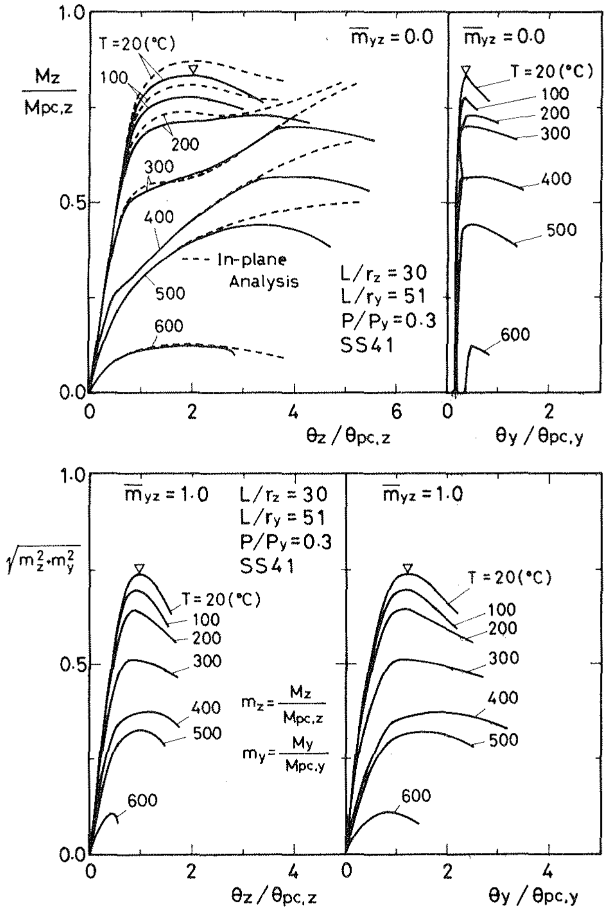


FIGURE 6. Moment versus end-rotation of wide-flange steel column

The computations are accomplished by the displacement control method, in which the target axial load P is loaded at the first step, and at after the second step, the resulting end bending moments M_y , M_z of the column are computed by specifying the value of the end rotation θ_y , θ_z under the condition of keeping the axial load constant.

Fig.6 shows the relationship of bending moments to rotations at the ends of columns for $\overline{m}_{yz}$ = 0.0,1.0. Fig.7 shows the interaction curves of the maximum moment. The dotted line in Fig.6 indicates the results of the in-plane analysis for $\overline{m}_{yz}$ =0.0. As can be seen in Fig.6, even at the temperature 300° C~500° C, the maximum bending moment are affected by the lateral torsional buckling behavior. The case of $\overline{m}_{yz}$ = 1.0 is a example of steel columns subjected to biaxial bending moments acting in two perpendicular direction. As can be seen in Fig.7 , the maximum bending moment $\sqrt{(M_z/M_{pc,z})^2+(M_y/M_{pc,y})^2}$ in this analysis become lower with larger parameter $\overline{m}_{yz}$ and higher temperature.

In Example 2, two kinds of cases $\overline{m}_{yz}$ =0.0,1.0 with three kinds of bending

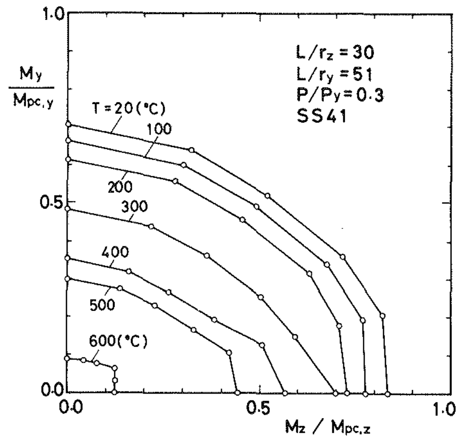


FIGURE 7. Interaction curves for maximum moment of wide-flange column at high temperatures

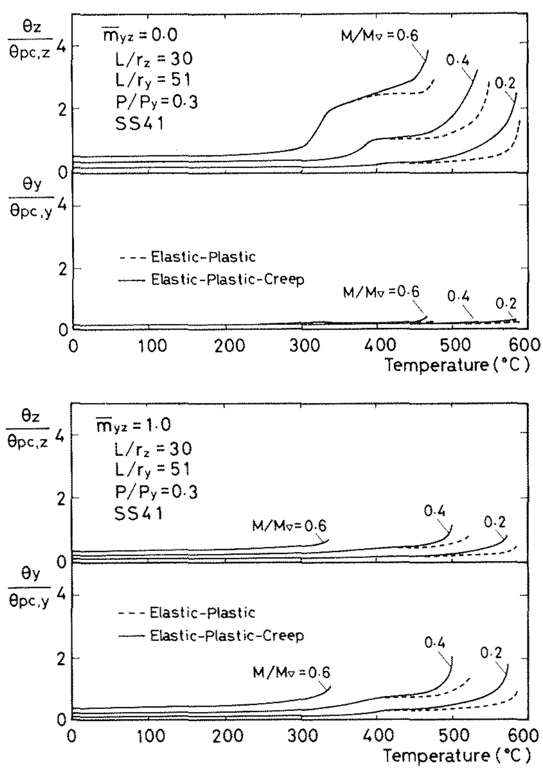


FIGURE 8. End-rotation versus temperature of steel columns

moment loads M/M_∇ are used, and the temperature of steel columns is assumed to change linearly from 0° C to 600° C during 120 minutes. In this case M_∇ is the maximum moment at room temperature shown in Fig.6 and M is the constant applied load about y and z-axis. The computations are accomplished by loading the target axial load P and bending moment loads M_z ,M_y at the first step, and the temperature of models is increased under the condition of keeping external loads constant after the second step.

Fig.8 shows the temperature histories of rotation θ_z , θ_y at the ends of the column. It is observed that the deformation of columns are developed gradually at higher temperature, and finally the collapse behavior of steel columns takes place. From the numerical results, the column is not greatly affected by creep strain until the steel temperature reaches 450° C. The effect of creep strain in those cases become significant over 450° C, and the deformation of columns are made two or three times larger due to the effect of creep strain. It is also observed that the collapse temperature depends upon the magnitude of external loads, the greater the external loads M_z ,M_y , the earlier the buckling of the column takes place. And due to the creep strain, the collapse temperature of steel columns are reduced about 20° C.

CONCLUSIONS

Including the mechanical model of steel materials at high temperature in the nonlinear finite element procedure of the thin-walled member with open cross sections, the analytical method to pursue the thermal elasto-plastic-creep three-dimensional behavior of steel column is developed.

Analyzing the examples of wide-flange steel columns at high temperature, some aspects being important to understand the space behavior of columns in fire have been found out.

REFERENCES

1. Furumura,F. and Shinohara,Y.,"Inelastic Behavior of Protected Steel Columns in Fire",IABSE 10th Congress,Japan,1976.
2. Y.Anderberg,N.E.Forsén and B.Aasen,"Measured and Predicted Behavior of Steel Beams and Columns in Fire",Fire Safety Science,Proceeding of the First International Symposium,1985.
3. Furumura,F. and Ave,T.,"Creep Buckling of Steel Columns at High Temperatures",Trans. of A.I.J.,No.344,1984.
4. Fujimoto,M.,Furumura,F.,Ave,T. and Shinohara,Y.,"Primary Creep of Structural Steel(SS41) at High Temperatures",Trans. of A.I.J.,No.296,1980.
5. Furumura,F.,Ave,T.,W.J.Kim and Okabe,T.,"Nonlinear Elasto-Plastic Creep Behavior of Structural Steel under Continuously Varying Stress and Temperature",Trans. of A.I.J.,No.353,1985.
6. Furumura,F.,Ave,T.,Okabe,T. and W.J.Kim,"A Uniaxial Stress-strain Formula of Structural Steel at High Temperature and its Application to Thermal Deformation Analysis of Steel Frames",Trans. of A.I.J.,No.363,1986.
7. Yamada,Y.,"Plasticity Visco-Elasiticity",Baifukan 1980.
8. F.K.Odoqvist and J.Hult,"Theory of Creep Strength",Baifukan,1973.
9. Fujimoto,M.,Iwata,M.,Nakatani,F. and Wada,A.,"Nonlinear Three-Dimensional Analysis of Steel Frame Structure",2nd International Conference on Space Structures,1975.