Egress Complexity of a Building

H. A. DONEGAN, A. J. POLLOCK and I. R. TAYLOR
Department of Mathematics, University of Ulster, Jordonstown,
Newtownabbey, County Antrim BT37 OQB, Northern Ireland, UK

ABSTRACT

Any multi-compartment building may be regarded as a complex object or structure. The present Fire Safety literature does not yet offer a model to compare two or more of these structures with respect to EGRESS COMPLEXITY. This paper formalises an approach to resolve this problem and outlines the calculation of appropriate values using a defined algorithm. An ENTROPY measure is generated for network representations of buildings to reflect complexity. The calculation procedure is illustrated for buildings with single floors, multi-storeys and multiple exits. The results are compared with EVACNET+.

KEYWORDS: Building Complexity, Entropy, Networks, Fire Safety, Egress.

INTRODUCTION

Over recent years the fire safety community through the objective of life safety has made a significant contribution to the well established research niche of People & Fire. In fact there is a strong intersection between this area and the area devoted to Statistics & Risk. Distinguished authors like Alvord [1], Beck [2], Fahy [3], Hall & Sekizawa [4], Kendik [5], Kisko & Francis [6], Levin [7], Stahl [8] and Takahashi [9] *inter alia* have produced a variety of egress modelling strategies, some of which include sophisticated behavioural and tracking characteristics. Despite the various distinct aims of these models, the literature has not addressed the more fundamental and potentially comparable notion of egress complexity for habitable buildings.

It is well known that buildings in general offer their visiting occupants a challenge with regard to evacuation. Indeed, large and complicated buildings offer established residents a similar challenge when some form of threat by fire or impending catastrophe is announced. The strength of challenge is related to the information which an occupant has in respect of available egress routes. Pigott [10] in his paper on Fire Intelligence emphasises that evacuees will choose their most familiar route - in many cases alternative routes are uncertain. The movement of visitors in new surroundings represents an uncertain environment and only by moving around the area is new information absorbed until e.g. an exit is found. Attneave [11] characterises this in stating that information is gained only about matters in which there is some degree of ignorance or uncertainty and defines information as that which reduces uncertainty. It is therefore hypothesised that the degree of egress complexity is equivalent to the degree of information required to describe the total egress system. The desire to address this issue and in consequence provide a metric invariant strategy for the optimal location of new building features such as a particular compartment, stairwell or exit is the motivation for the present research by the authors and is coupled with the philosophy of Pauls [12] who advises simplicity in all access and movement routes. Such a premise is fundamental to the reduction of complexity.

Every building has a latent complexity measure consequent upon the design of the compartmentation. This is a static measure which is readily derivable from the plans of a building upon inspection and with the assistance of some algorithm. Such a measure has at least two fundamental advantages over the outcomes from the various available egress models. It is scenario independent and is an index of primary egress design without recourse to the dynamic occupancy - much in the same manner that static balance is primary to good machine design prior to the consideration of dynamic balancing. Essentially the measure performs as an egress characteristic which can be assessed at a design approval stage or as an aid to refurbishment considerations.

Kostreva et al [13] claim that mathematical modelling of egress from a building on fire falls into one of two categories, viz. the descriptive which focuses on the progress of occupants over a given time span or the overall broad-based viewpoint which determines globally optimal trajectories of egress. However in this paper, although an appeal is made to the sentiment of the latter, the strategy is more general. It invokes classical information theory and is primarily based on the notion of Shannon entropy [14] which is adapted in this case to encompass egress uncertainty about the building and to provide a measure of complexity prior to any evacuation. This is achieved through the implementation of an algorithm which analyses a topologically equivalent nodal network of the building however simple or complex. The method relies on well established techniques in the area of Concept Learning Systems (CLS) within the field of Artificial Intelligence. A CLS is a mechanism for creating a concept corresponding to some dichotomy of a sample of objects which have been categorised by an *a priori* rule. In particular Quinlan's ID3 algorithm [15] explicitly prioritises rule structures in the design of expert systems. Jackson [16] has applied entropy ideas to such Concept Learning Systems.

BACKGROUND TO THE STRATEGY

The cornerstone of this research is information theory and its relation to Shannon entropy. The concept of entropy arose in classical thermodynamics and the term was suggested by Clausius in 1854 as a measure of disorganisation of a physical system. Its statistical representation emerged from the theory of ideal gases developed by Boltzmann in 1896. The name entropy was adopted by Shannon in 1948 for his measure of uncertainty presumably because of its similar format to the Boltzmann description. The measure is formulated in probability terms and expressed by the function:

$$H\,(p(x) \mid x \in X) = -\sum p(x) \log_2 p(x) \qquad \text{(where summation is over } x \in x\text{)} \qquad (1)$$

where $(p(x) \mid x \in X)$ is a probability distribution on a finite set X. The codomain of H is clearly $[0,\infty[$. Intuitively if a set X of alternatives with probabilities p(x) for each x in X is considered then it is certain that exactly one of them must occur as the outcome of some experiment. When x occurs the information content of this fact is $-\log_2 p(x)$ bits. Before the occurrence of this particular x, the information content was not known. The expected information content is therefore the weighted arithmetic mean:

$$-\sum p(x) \log_2 p(x) \qquad \text{(where summation is over } x \in x\text{)} \qquad (2)$$

which is precisely the Shannon entropy, given that $\sum p(x) = 1$.

APPLICATION TO EGRESS COMPLEXITY

In the context of egress the main concept is: *Acquiring Knowledge with respect to Egress.* This can be measured with respect to a single attribute viz. the arcs which connect nodes when the building plans have been networked. Knowledge is gained through a positive movement along an arc between two nodes - this is equivalent to one information step on which one packet of knowledge is acquired and is referred to as a positive instance. Conversely knowledge is not gained if an arc is backtracked - this is referred to as a negative instance. On this basis the probabilities, of acquiring and not acquiring knowledge of egress from any node to a specified exit via all other nodes on the same floor level, are given respectively by

$$\overset{+}{p} = \frac{\overset{+}{n}}{\overset{+}{n}+\overset{-}{n}} \quad \text{and} \quad \overset{-}{p} = \frac{\overset{-}{n}}{\overset{+}{n}+\overset{-}{n}} \qquad (3)$$

where n^+ is the number of positive instances and n^- the number of negative instances. Hence the total entropy with respect to the set of instances is given by

$$H = -(p^+) \log_2 p^+ - (p^-) \log_2 p^- \qquad (4)$$

There is no loss of generality in adopting Quinlan's [15] format viz.

$$H = -(n^+) \log_2 p^+ - (n^-) \log_2 p^- \qquad (5)$$

and referring to this as nodal complexity. The overall complexity for a given floor is optionally taken as the mean nodal complexity or the maximal nodal complexity. Further research is continuing on this aspect.

STEPS TO GENERATE A NETWORK FOR A BUILDING

Preliminary assumptions are:

1. Evacuees have no previous knowledge of the building;
2. Each evacuee is treated as the only occupant in the building, so that the influence of other occupants is ignored;
3. The choice of exit from any compartment with more than one doorway is equally likely;
4. There is no signage;
5. Evacuees do not panic;
6. Evacuees are able-bodied;
7. All networks are trees i.e. graphs with no loop structures;
8. A backtrack path represents one positive and one negative instance;
9. A forward path represents one positive instance;
10. Each evacuee has a path memory.

DEFINITION: A compartment is a room, stairwell or area that may be occupied.

The steps to generate the network, given that f represents the floor number, and i, j & k represent the node numbers, are:

1. From the floor plan of a building consider each compartment as a node;
2. Label the non-stairwell nodes as N_{fi};
3. For each stairwell point create a node and label as SW_{fj};
4. For each node that leads to an exit, place a new node in the direction of the exit and label EX_{fk};
5. If a node can access another node directly, then draw an arc between the two nodes. Repeat this step until all appropriate connections are made.

The process is demonstrated in figures 3 and 4 of this paper.

THE ALGORITHM

For different building layouts the algorithms are as specified below. Illustrations follow in the later sections.

I. Single Floor Single Exit

This is the most basic model. The steps are:

1. Select a non-exit node on the network;
2. For all arcs on the path that leads from this node directly to the exit, draw single headed arrows '$\rightarrow$' in the direction of the exit;

3. On all arcs that are remaining draw the double headed arrow ' $\leftrightarrow$ ';
4. Count the number of $\leftrightarrow$'s which is the value of n^-;
5. Count the number of $\rightarrow$'s. Add this number to the value of n^- and the total is n^+;
6. Substitute the values for n^+ and n^- into equation (5) to obtain a nodal entropy value;
7. Repeat steps 1 to 6 for all other non-exit nodes;
8. Sum all entropy values and divide by the number of non-exit nodes;
9. The result is the average entropy value for each node in the floor plan, or the overall complexity value.

II. Multi-Storey Single Stairwell

In this model it is assumed that the stairwell is a separate compartment.

For all non-ground level floors:

1. Treat the SW node as an EX node;
2. Repeat steps 1 to 7 in I;
3. Sum all entropy values and divide by the number of non-exit and non-stairwell nodes.

This gives the complexity value for a non-ground level floor.

For the ground level floor:

4. If the stairwell, SW is part of an exit, treat as an exit node, EX, while if the stairwell is not part of an exit, treat as an ordinary node;
5. Repeat steps 2 & 3 in II.

This gives the complexity value for the ground floor.

For the purpose of this illustration, the overall complexity of a multi-storey building is represented by a vector where each component represents the complexity of the corresponding floor.

III. Single Storey Multiple Exits

This is similar to I. The only difference is step 2, replaced by

2. On the path of arcs that lead from this node directly to the relatively *furthest* (with respect to information packets) exit, draw single headed arrows $\rightarrow$ in the direction of the *exit*.

IV. Multi-Storey Multiple Stairwells

This model is can be treated as a combination of the algorithms in II and III.

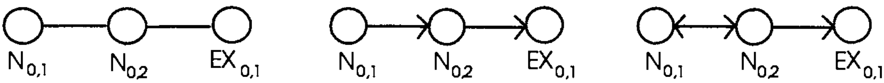
SINGLE FLOOR SINGLE EXIT EXAMPLE

Consider a single node with one exit point, as in figure 1a below. The corresponding directive flow diagram is shown in figure 1b, which represents one forward path towards the exit, as there are no alternative paths and no backtrack paths. This is represented by the instances: $n^+ = 1$, $n^- = 0$. Substituting these into equation (5) above yield an entropy value of H equal

a: Node diagram **b: Flow diagram**

FIGURE 1. Floor plan with one compartment

to 0. This represents a structure with no egress complexity, implying that an evacuee only needs to follow a path of one information step to egress. Figure 2a represents a floor plan with an extra compartment, compared with figure 1.



a: Node diagram **b: Flow from $N_{0,1}$** **c: Flow from $N_{0,2}$**

FIGURE 2. Floor plan with two compartments.

Because there is more than one node, it is necessary to view the complexity of egress from each node and calculate the average. The directional flow diagrams for Nodes $N_{0,1}$ and Node $N_{0,2}$ are shown respectively in figures 2b and 2c. Figure 2b is similar to figure 1a above as an evacuee starting from $N_{0,1}$ has only one choice of an initial direction to travel. As the evacuee arrives at $N_{0,2}$ from $N_{0,1}$ it is natural to assume that the exit is the next choice as a destination. This assumption is based on the fact that the evacuee has travelled along the path from $N_{0,1}$ to $N_{0,2}$ and has learned that there are no other paths to follow from node $N_{0,1}$. This result is represented as: $n^+ = 2$, $n^- = 0$ (A). Figure 2c represents a evacuee starting at the node $N_{0,2}$. Two alternative paths can be selected at this node. Consider the case where the evacuee travels from $N_{0,2}$ to $N_{0,1}$, back to $N_{0,2}$ and then to the exit node. In this situation, more complex than direct exit the evacuee has discovered two paths in the network, one which is travelled twice. This is known as *backtrack*. The result is: $n^+ = 2$, $n^- = 1$ (B). The respective entropy calculations for (A) and (B) yield $H = 0$ and $H = 2.75$. Taking an average of these results gives an overall entropy value of $H = 1.38$.

This illustrates that relatively simple networks can generate non-zero complexity values. If figure 2 is expanded to three nodes and then in succession up to 10, the resulting complexity rises and can be represented by the linear law

$$\mathbb{C} = 1.288k - 1.152 \qquad (6).$$

where $\mathbb{C}$ is the complexity and k is the number of nodes.

MULTI-STOREY SINGLE STAIRWELL EXAMPLE

Consider the floor plan in Figure 3 below:

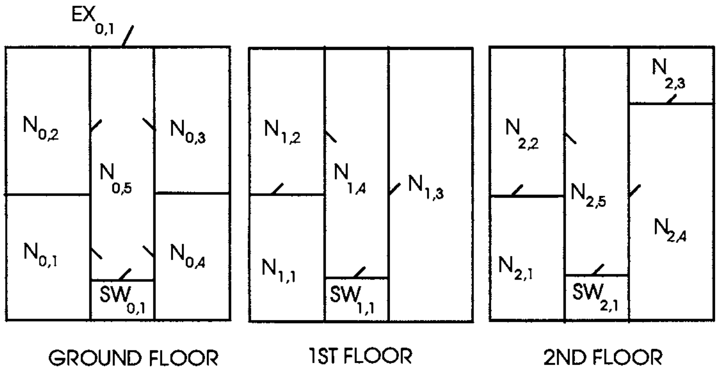


FIGURE 3. Floor Plan for a Three Storey Building

This is transformed to the network in figure 4 below.

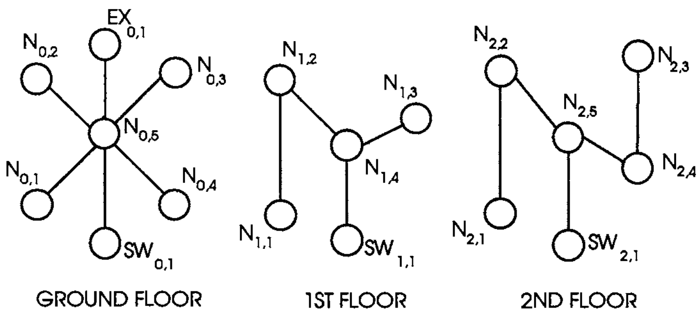


FIGURE 4. Network for Three Storey Building

2nd Floor: The Table 1 below shows that nodes $N_{2,1}$ and $N_{2,3}$ have the same value for complexity as is also true for nodes $N_{2,2}$ and $N_{2,4}$. Node $N_{2,5}$ is effectively the hall or landing node and has the highest value. This reflects the amount of choice available at this node. Considering node $N_{2,5}$ as a starting point, the evacuee must make a decision which may result in two paths leading towards dead ends each requiring backtrack or one path leading directly to an exit. This model examines each node from the worst case point of view and assigns an equal weighting to the choice of path. Generally nodes with more choices increase the complexity. This problem can be resolved if adequate signage is available or if the evacuee is

TABLE 1. Multi-Storey Single Stairwell Results

Floor	Starting Point	Instances n_i^+& n_i^-	Entropy	Floor Average
2nd	$N_{2,1}$	$n_i^+ = 5, n_i^- = 2$	6.04	36.28 / 5 = 7.26
	$N_{2,2}$	$n_i^+ = 5, n_i^- = 3$	7.64	
	$N_{2,3}$	$n_i^+ = 5, n_i^- = 2$	6.04	
	$N_{2,4}$	$n_i^+ = 5, n_i^- = 3$	7.64	
	$N_{2,5}$	$n_i^+ = 5, n_i^- = 4$	8.92	
1st	$N_{1,1}$	$n_i^+ = 4, n_i^- = 1$	3.61	21.53 / 4 = 5.38
	$N_{1,2}$	$n_i^+ = 4, n_i^- = 2$	5.51	
	$N_{1,3}$	$n_i^+ = 4, n_i^- = 2$	5.51	
	$N_{1,4}$	$n_i^+ = 4, n_i^- = 3$	6.90	
Ground	$N_{0,1}$	$n_i^+ = 6, n_i^- = 4$	9.71	59.48 / 6 = 9.91
	$N_{0,2}$	$n_i^+ = 6, n_i^- = 4$	9.71	
	$N_{0,3}$	$n_i^+ = 6, n_i^- = 4$	9.71	
	$N_{0,4}$	$n_i^+ = 6, n_i^- = 4$	9.71	
	$N_{0,5}$	$n_i^+ = 6, n_i^- = 5$	10.93	
	$SW_{0,1}$	$n_i^+ = 6, n_i^- = 4$	9.71	

aware of the escape routes. Furthermore, the complexity can be reduced by introducing multiple exits, as discussed later.

1st Floor: This floor is topologically equivalent to the second floor except that one node has been removed. The results for the 1st floor show that addition of a node at $N_{2,3}$ increases the complexity by 35%. The building in this example is not large with respect to the overall number of nodes, so the addition of a node increases the complexity significantly. If a series of nodes $N_{1,5}$ to $N_{1,n}$ is added to $N_{1,3}$ as shown in Figure 5, the effect of each node on the complexity decreases as the network increases in size.

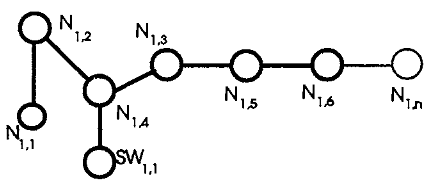


FIGURE 5.- 1st Floor with New Series of Nodes from $N_{1,3}$.

Ground Floor: The ground floor is not directly comparable with the other floors but demonstrates the effect of a stairwell. The stairwell is considered as a node and the resultant network has 6 nodes and one exit. Again the number of nodes has increased and so has the complexity. Evacuees entering the stairwell from $N_{0,5}$ recognise that it is not a valid escape route and then return to node $N_{0,5}$. It is possible to modify the model and not treat the stairwell as an extra node. This is the case if the stairwell is not in a separate compartment. Stairwells may also be modelled as a starting node but not as a possible backtrack destination.

The overall complexity for the building may be expressed as the row vector [9.91, 5.38, 7.26], where the first component is the ground floor complexity and succeeding components are for upper floors.

MULTI-EXIT EXAMPLE

The above models may be extended to consider networks with multiple exits as in figure 6 below.

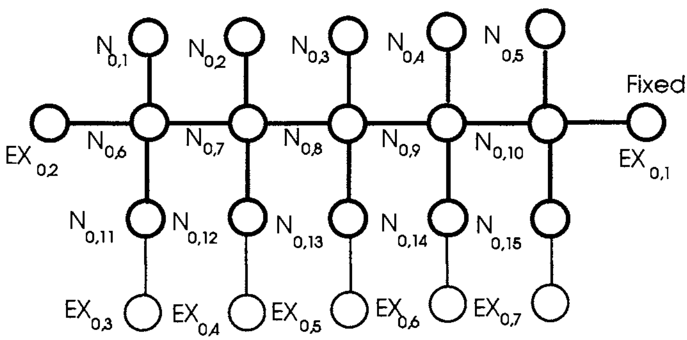


FIGURE 6.- Floor Plan With Choices for a Possible Secondary Exit.

In Table 2 the complexity value for figure 6 with only one exit (the fixed stairwell), is compared with the addition of a second exit at nodes $EX_{0,2}$...$EX_{0,7}$ in turn. For a single exit only, at $EX_{0,1}$ there are 16 nodes and 15 arcs with a corresponding entropy of 25.88.

TABLE 2. Comparison of Complexity Values for each Secondary Exit

NODES						
Position of 2nd Exit	$EX_{0.2}$	$EX_{0.3}$	$EX_{0.4}$	$EX_{0.5}$	$EX_{0.6}$	$EX_{0.7}$
Result	24.41	23.61	24.41	24.93	25.42	24.69
% Improved	5.68	8.77	5.68	3.67	1.78	4.60

The addition of an exit has a pronounced effect in reducing the complexity. The results indicate that from the point of view of complexity it is best to locate the second exit at node $EX_{0,3}$ with less benefit from each of $EX_{0,2}$ and $EX_{0,4}$... $EX_{0,7}$.

However it is unnecessary to add an additional exit to a building with zero complexity. Also, if an additional stairwell is added to a building, the ground floor complexity will increase as this is the same as adding a new node to this floor.

COMPARISON WITH EVACNET+

EVACNET+ is *A Computer Program to Determine Optimal Building Evacuation Plans* [6] with respect to time. This program uses a network of arcs and nodes to represent a building, permits the user to enter occupants, assign priorities to the nodes and allocate capacities to both arcs and nodes. Various results are output from EVACNET+, but three in particular are valuable for the purpose of comparison with the present model:

1. Time Periods to Evacuate the Building (E1).
2. Average Number of periods for an occupant to evacuate (E2).
3. Average number of evacuees per time period (E3).

The different floor plans in figure 6 above were created in EVACNET+ and the results for E1 to E3 were noted. In table 3 below these values are compared to the results from Table 2. The percentage improvement with respect to E1 is denoted by % Imp E1. Similarly % Imp E2 and % Imp E3 represent the improvements with respect to E2 and E3.

E1, E2 & E3 vary for each of the secondary exits. A decrease in E1 and E2 or an increase in E3 represents an improvement in the building performance as measured by EVACNET+.

TABLE 3 Comparison of EVACNET+ Results with Complexity Values

2nd Exit	Entropy Result	% Imp	E1	E2	E3	% Imp E1	% Imp E2	% Imp E3
None	25.88	N/A	12	7.3	1.3	N/A	N/A	N/A
$EX_{0,2}$	24.41	5.68	8	4.9	1.9	33	33	46
$EX_{0,3}$	23.61	8.77	8	5.5	1.9	33	25	46
$EX_{0,4}$	24.41	5.68	8	5.5	1.9	33	25	46
$EX_{0,5}$	24.93	3.67	10	5.9	1.5	17	19	15
$EX_{0,6}$	25.42	1.78	12	7.1	1.3	0	3	0
$EX_{0,7}$	24.69	4.60	12	7.2	1.3	0	1	0

There are two main differences between the EVACNET+ output and the complexity output as illustrated in table 3. First, the best location for the second exit is at $EX_{0,3}$ in the complexity model while it is at $EX_{0,2}$ using the EVACNET+ figures. Secondly, the models also differ in the improvement to be gained by putting a second exit at $EX_{0,7}$.

EVACNET+ is calculating the optimal evacuation plans for each of the secondary exit models. It does not consider that an evacuee may backtrack. Instead the results are based on evacuees moving directly to exits. It represents either the situation where evacuees are familiar with the building or that in which adequate signage exists. The complexity model assumes the contrary and presupposes that the evacuee has covered every path in the building before egress, i.e. on leaving, the evacuee has learned enough information to describe the building network. The differences in the models represent the different behaviour of visitors compared to residents.

The complexity value reflects the number of information steps an evacuee must absorb from any selected start node in covering the entire network and ending at the exit with the least amount of association with the start node. The association for the network, a concept introduced by Kernohan et al [17], is measured with respect to the number of arcs crossed to reach an exit from a given start node. This also explains the differences in the results in table 3.

Another point to note is differences in the degree of each node, where the degree of a node is the number of arcs attached to the node. The network with the second exit at $EX_{0,2}$ has 5 nodes of degree 4 and 12 nodes of degree 1, whereas the network with $EX_{0,3}$ has 4 nodes of degree 4, 1 node of degree 3, 1 node of degree 2 and 11 nodes of degree 1. Nodes with a larger number of possible paths to follow are generally more complex. The network in figure 6 with $EX_{0,2}$ as a second exit includes an extra node of degree 4, explaining why the network's entropy value is larger than that for $EX_{0,3}$.

CONCLUSIONS

This procedure may be used to compare the complexity of different floors within a building or to compare two different buildings. Furthermore, the effect of adding stairwells, compartments or exits may be investigated, together with the best location for such features. The model may be extended to cater for:

1. stairwells not in separate compartments;
2. locked doors;
3. disabled occupants;
4. occupancy weightings;
5. buildings with more than route to an exit, i.e. the network contains at least one loop.

In giving comparable information to evacuation programs such as EVACNET+ the model puts the emphasis on the needs of visitors, unfamilar with the building rather than on the needs of longer term occupants. This aspect of evaluation is therefore a response to the considerations proposed by Sime [18] when he refers to Environmental Design Evaluation in the context of "physically and psychologically complex settings".

REFERENCES

1. Alvord, D.M., "The Escape and Rescue Model: A Simulation Model for the Emergency Evacuation of Board and Care Homes", NBS, NBS-GCR-83-453, 1983.
2. Beck, V.R., "A Cost Effective Decision Making Model for Building Fire Safety and Protection", Fire Safety Journal, 12, 121-138, 1987.
3. Fahy, R., "EXIT89: An Evacuation Model For High-Rise Buildings", Fire Safety Science - Proc 3rd Inter Symposium, 815-823, 1991.
4. Hall, J.R. & Sekizawa, A., "Fire Risk Analysis: General Conceptual Framework for Describing Models", Fire Technology, 27:1, 33-53, 1991.
5. Kendik, E., "Methods of Design for Means of Egress, Towards a Quantitative Comparison of National Code Requirements", Fire Safety Science - Proc 1st Inter Symposium, 497-510, 1986.
6. Kisko, T.M. & Francis, R.L., "Evacnet+: A Computer Program to Determine Optimal Building Evacuation Plans", Dept of Industrial and Systems Engineering, University of Florida, 1984.
7. Levin, B.M., "ExitT - A Simulation Model for Occupant Decisions and Actions in Residential Fires", Technical Ref. Guide for HAZARD 1, NIST, pp C-1 - C-15, 1989.
8. Stahl, F.I., "BFIRES- II A Behaviour Based Computer Simulation of Emergency Egress During Fires", Fire Technology, 18, 49-65, 1982.
9. Takahashi, K., Tanaka, T. & Kose, S., "An Evacuation Model for Use in Fire Safety Design of Buildings", Fire Safety Science - Proc 2nd Inter Conf, 551-566, 1988.
10. Pigott, B.B., "Fire Intelligence - Key to the Design of Economical Means of Escape", Interflam '93, Proc of 6th Inter Fire Safety Conference, 181-188, 1993.
11. Attneave, F., Applications of Information Theory to Psychology, A summary of Basic Concepts, Methods and Results, pp. 1-12, Holt, Rinehart and Winston Inc., 1959.
12. Pauls, J., "The Movement of People in Buildings and Design Solutions for Means of Egress", Fire Technology, 20: 1, 27-47, 1984.
13. Kostreva, M.M., Malgorzata, M.W., Wiecek, M. & Getachew, T., "Optimisation Models in Fire Egress Analysis for Residential Buildings", Fire Safety Science, Proc 3rd Inter Symposium, 805-814, 1991.
14. Shannon, C.E., "The Mathematical Theory of Communication", The Bell System Technical Journal, 27, 379 - 423, 623-656, 1948.
15. Quinlan, J.R., "Semi-autonomous acquisition of pattern-based knowledge", in Introductory Readings in Expert Systems, pp. 192-207, Gordon & Breach, 1982.
16. Jackson, A.H., "Machine Learning", Expert Systems, 5: 2, 132-150, 1988.
17. Kernohan, D., Rankin, G.D., Wallace, G.D. & Walters, R.J., "PHASE: An Interactive Appraisal Package for Whole Hospital Design", Computer Aided Design, 5: 2, 81-89, 1993.
18. Sime, J.D., "Crowd Safety Management and Communications in Disasters", Management of Safety, Health and Environment (Risk Assessment and Control), Ch 9, pp9.1-9.17. Delft: TopTech Studies, Delft University of Technology, Netherlands, 1992.