Determination of Safety Factors in Design Based on Performance

S. E. MAGNUSSON, H. FRANTZICH, B. KARLSSON and S. SÄRDQVIST
Department of Fire Safety Engineering
Lund University
PO Box 118
S-221 00 Lund, Sweden

ABSTRACT

A pilot study on a very limited scale has been carried out in the application of the FOSM-methodology for fire safety engineering design. The methodology is based on using the mean and standard deviation from distributions in a first order approximation of a limit or failure equation. In this way it is possible to establish an expression for the safety index β both for linear and non-linear limit equations. This paper exemplifies the methodology for a practical scenario deriving the safety index β for a small shopping centre.

KEYWORDS: Fire safety, probabilistic, safety index, FOSM, evacuation

CONCEPT OF PERFORMANCE

Standards may be either performance or prescriptive in nature. A performance standard expresses the desired attributes of the product or process in question and any scheme that results in achievement of these attributes meets the standard. This approach promotes economy and innovation since it focuses on the users' requirements. A prescriptive standard is a specific definition of an acceptable process or product. The desired attributes are often unstated; the process or product may meet the standard but may not meet the users' needs. Prescriptive standards have the advantage that they are objective. It is relatively straightforward to evaluate and determine whether or not the standard is complied with. However, prescriptive standards do not assure satisfaction of the users' needs, let alone in an optimal fashion. Performance standards have the disadvantage that it is conceptually more difficult to evaluate whether they are complied with.

The total fire safety system could be divided into a number of subsystems or design modules DM1 ... DM6. This system is based on the concepts described in recently developed performance-based design systems in Japan [1], Australia [2], US [3] and Canada [4] and the system under development in UK [5]. A more comprehensive and detailed description of the designmodules and interactions between them is given in the quoted references, especially reference 5. The following defines the different design modules:

DM1 procedures for deriving design values (characteristic values), safety factors, site specific evaluation factors, etc
DM2 calculation of fire growth in room of fire origin
DM3 calculation of spread of smoke to other compartments
DM4 calculation of spread of fire (flames) to other compartments
DM5 calculation of times to detection and activation of active systems
DM6 calculation of evacuation times.

In the integrated whole building approach, results from all these subsystems would be combined to describe primarily evacuation safety.

The objective of this paper is to describe ongoing Swedish work on design methods based on calculation and to introduce some concepts and methodologies in the area of reliability-based design. Some introductive and simplified calculations will be shown; more to illustrate the methodology then to derive practically applicable values.

CURRENT SWEDISH RESEARCH ACTIVITIES

In Sweden, a new building code was adopted in january 1994. The fire safety objectives are expressed in performance terms as far as possible but without acceptance criteria and accepted practical solutions. These will be provided by a guidance document with the following structure for each specific design situation:

- functional requirements as expressed by the new building code,
- clarification and explanation of the functional requirements,
- calculation methods and equations plus description,
- examples (calculations + accepted solutions),
- uncertainties,
- suggestions for design solutions above the minimum requirements,
- relevant literature.

Current research activities on this subject at the department of Fire Safety Engineering can be divided into three activities:

1. The production of the preliminary guidance document mentioned above. Probabilistic aspects such as reliability and safety factors will be treated in an ad hoc and non-quantitative way.
2. In parallel with 1) a study of the fundamental problems in reliability-based building fire safety design.
3. Techniques of model evaluation and model validation.

This paper will cover only some preliminary and introductive calculations linked to project 2.

THE SUPPLY-DEMAND R-S RELIABILITY-BASED FORMAT

The term reliability is here defined as the probabilistic measure of assurance of performance. The further discussion necessitates introduction of some of the concepts used in assessment of reliability and design based on reliability. The description will be strongly condensed and incomplete and for further information the reader is referred to standard textbook such as the one by Ang-Tang [6].

For many fire safety engineering components or subsystems the performance may be reformulated in the following way. Let the random variables R and S be defined

R = supply capacity
S = demand requirement

The objective of the reliability analysis is to ensure the event R>S expressed in terms of the probability P(R>S). If the probability distributions of R and S are known and if R and S are statistically independent, probability of failure p_F may be calculated by

$$p_F = \int_0^\infty F_R(s) f_S(s) ds \qquad (1a)$$

where F and f denote the cumulative distribution and frequency functions.

If R and S are normal random variable the distributions of the safety margin M

$$M = R\text{-}S \qquad (2)$$

is also normal = $N(\mu_M, \sigma_M)$

The parameter $(M - \mu_M)/\sigma_M$ is N(0,1) and

$$p_F = F_M(0) = \phi\left(-\frac{\mu_M}{\sigma_M}\right) = 1 - \phi\left(\frac{\mu_M}{\sigma_M}\right) \qquad \text{or} \qquad p_F = 1 - \phi\ (\beta) \qquad (1b)$$

with Φ = cumulative probability function of a standard normal variate. The quantity $\beta = \mu_M/\sigma_M$, which determines reliability $p_s = 1 - p_F$, is often called reliability or safety index β. By definition, β is the safety margin expressed in units of σ_M.

The methodology was developed in the late 1960's (see reference 6) and has since then been systematically improved and extended in application.

Examples of application can be found in structural engineering, civil engineering, hydraulics, environmental engineering, etc. Possibly the first systematic work on the approach in the fire engineering area is a doctoral thesis from 1974, reference 7.

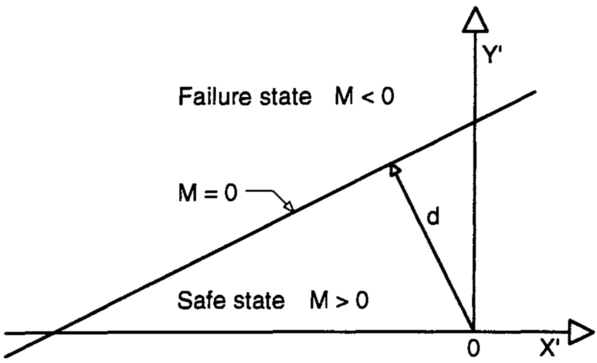

FIGURE 1 Space of reduced variates X' and Y'.

THE FIRST ORDER SECOND MOMENT (FOSM) METHODOLOGY

In a reduced variate system β can be interpreted as the distance from the origin to a failure line (limit state). Redefine the safety margin M = X - Y and introduce the reduced variates

$$X' = \frac{X-\mu_X}{\sigma_X} \qquad (3a) \qquad\qquad Y' = \frac{Y-\mu_Y}{\sigma_Y} \qquad (3b)$$

The limit state equation M = 0 then becomes

$$\sigma_X X_1' - \sigma_Y Y' + \mu_X - \mu_Y = 0 \qquad (4)$$

and it follows from linear algebra that (see figure 1)

$$\beta = d = \frac{\mu_X - \mu_Y}{\sqrt{\sigma_X^2 + \sigma_Y^2}} \qquad (5)$$

If X and Y are normal random variables then $p_s = 1 - p_f = \Phi(d)$ as before.

For other statistical distribution of X and Y the probability of failure has to be calculated by integration, which in most cases requires use of Monte Carlo simulation techniques. It is obviously a very attractive idea to base design on the concept of the safety index β, that is on mean values and variances, and disregard the actual statistical distribution of X and Y [8]. In reference 8 the following arguments can be found:

An approach based on means and variances may be all that is justified when one appreciates:

(1) That data and physical arguments are often insufficient to establish the full proability law of variable; (2) that most engineering analyses include an important component of real, but difficult to measure, professional uncertainty; and (3) that the final output, namely the decision or design parameters, is often not sensitive to moments higher than the mean and variance.

In practice X and Y may be functions of several basic random variables or design parameters. A performance or state function g may be formulated

$$g\,(\mathbf{X}) = g\,(X_1, X_2, \ldots X_n) \tag{6}$$

where $\mathbf{X} = (X_1, X_2, \ldots X_n)$ is a vector of basic state or design variables. $g\,(\mathbf{X}) = 0$ defines a limit state of the system and a n-dimensional failure surface. Based on a first order approximation (Taylor expansion) of the function $g\,(\mathbf{X})$, procedures are available to find the most probable point of failure $x^* = (x_1^*, x_2^*, \ldots x_n^*)$ and the corresponding safety index β.

The point $x'^* = (x_1'^*, x_2'^*, \ldots x_n'^*)$ denotes the point on the failure surface with minimum distance β to the origin of a reduced variate system, see figure 2 [6]. It can show that the reliability index $\beta = \mu_g/\sigma_g$ as before. In a first order approximation

$$\mu_g \approx -\sum_{i=1}^{n} x_i'^* \left(\frac{\partial g}{\partial X_i'}\right)_* \tag{7a}$$

$$\sigma_g^2 \approx \sum_{i=1}^{n} \left(\frac{\partial g}{\partial X_i'}\right)_*^2 \tag{7b}$$

Accordingly, β is given by

$$\beta = \frac{-\sum_i x_i'^* \left(\frac{\partial g}{\partial X_i'}\right)_*}{\sqrt{\sum_i \left(\frac{\partial g}{\partial X_i'}\right)_*^2}} \tag{8}$$

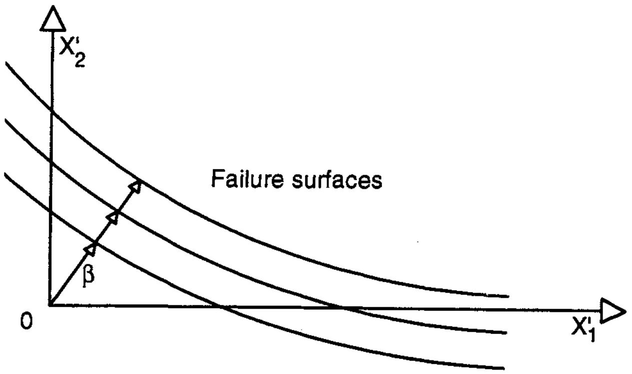

FIGURE 2 Designs corresponding to different failure surfaces.

with X' denoting a reduced variate and the derivative evaluated at $x = x^*$, which is unknown and has to be determined in the calculation prodecure.

For a linear performance equation

$g(\mathbf{X}) = a_o + \Sigma a_i X_i$

the value of β is given explicitly by

$$\beta = \frac{a_o + \sum_i a_i \mu_{xi}}{\sqrt{\sum_i (a_i \sigma_{xi})^2}} \tag{9}$$

and the value of β may be computed directly. For the general non-linear performance equation $g(\mathbf{X}) = 0$ the point of failure $\mathbf{X}^*$ will have to be determined by iteration (the Rackwitz procedure) or by constrained non-linear optimization.

In this case it is necessary to make an integration of the joint probability density functions to obtain the probability of safety. As this is a nonattractive solution an iteration process using the same technics as for the linear case is often used in determining the safety factor. The distance to the tangent plane pertinent to the failure surface at the point ($x_1'^*$, $x_2'^*$ ... $x_n'^*$) is used as an approximation, making it possible to evaluate the safety index as in the linear case.

This approximation will either be on the safe or unsafe side depending on how the actual failure surface looks, figure 3 showing the two-variable-case. The term first-order, second moment is implied from the use of a linearized, first order expansion and the first two statistical moments.

The problem is that the point x_i^* is not known which makes the iteration process necessary.

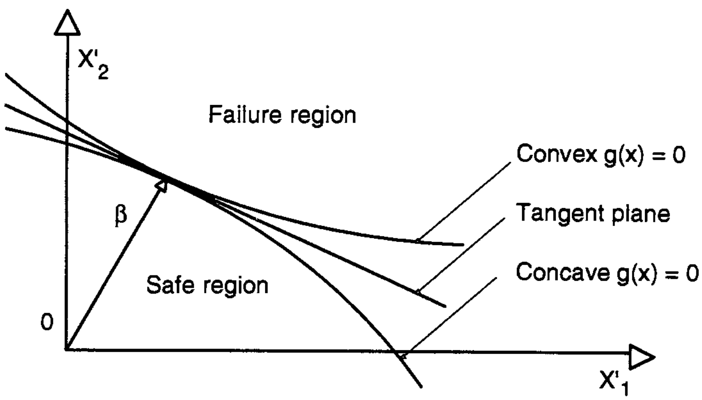

FIGURE 3 Tangent plane to g(**X**) = 0 at x'*.

The most probable failure point in the reduced variable space is

$$x_i'^* = - \alpha_i^* \cdot \beta \tag{10a}$$

where α_i^* is the direction cosines in the x_i' direction

$$\alpha_i^* = \frac{\left(\frac{\partial g}{\partial X_i'}\right)_*}{\sqrt{\sum_i \left(\frac{\partial g}{\partial X_i'}\right)_*^2}} \tag{11}$$

The derivates are evaluated at ($x_1'^*$, $x_2'^*$ ... $x_n'^*$) which gives

$$x_i^* = \sigma_{X_i} x_i'^* + \mu_{X_i} = \mu_{X_i} - \alpha_i^* \sigma_{X_i} \beta \tag{10b}$$

If this expression is put into the limit equation and solved for $g(\mathbf{X}) = 0$ then the β value has been obtained.

Rackwitz has suggested the following simple numerical algorithm which is outlined in reference 6.

1. Assume initial values of x_i^* for i = 1, 2, 3 ... n.
2. Caclulate $x_i'^* = \frac{x_i^* - \mu_{X_i}}{\sigma_{X_i}}$
3. Evaluate $\left(\frac{\partial g}{\partial X_i'}\right)_*$ and α_i^* at x_i^*
4. Calculate $x_i^* = \mu_{X_i} - \alpha_i^* \sigma_{X_i} \beta$
5. Substitute x_i^* in $g(x_1^*, x_2^* ... x_n^*) = 0$ and solve for β
6. Use β to improve the values of $x_i'^* = - \alpha_i \beta$
7. Repeat steps 3 to 6 until convergence of β is obtained.

In the case where the distributions of X_i are nonnormal, it is necessary to transform X_i into equivalent normal distributions. Techniques for this process is available.

EVALUATION OF β FOR A SIMPLIFIED EXAMPLE

To illustrate the methodology we choose a much simplified example, evacuation times for a small shopping centre, 40 x 20 x 2.4 m (L x W x H). The limit equation is

$$g = S - D - R - E \geq 0 \qquad [12]$$

where

S = time for smoke filling to 1.6 m above floor
D = detection time for smoke detectors
R = reaction- and decisionmaking time prior to evacuation
E = evacuation time

S, D, R and E are regarded as stochastic quantities and described by the following:

S Time to smoke filling to 1.6 m above floor is calculated for qt^2-fires with the growth rate q lognormally distributed with a mean of 0.025 kW/s^2 and a standard deviation of 0.030 kW/s^2. Smoke filling times are calculated using reference 9 which gives the analytical expression

$$S = 272\ q^{-0.2} \tag{13}$$

D The detection time is calculated for a qt^2-fire using the same distribution of q as for the smoke filling time. The calculations are performed using the DETACT-model [10] with a distance from the centre of the fire to a detector of 5 m. A similar regression technique as for smoke filling gives the analytical expression

$$D = 17.3\ q^{-0.49} \tag{14}$$

R In general, reaction time is difficult to estimate. An investigation [11] among fire officers in Sweden has been carried out during the summer of '93 in which the fire officers were asked to give their opinion on how long the reaction time could be. This is done for varoius types of buildings and with various types of alarm signals. This investigation shows that the assumed reaction time could be estimated with a lognormal distribution with a mean of 130 seconds and a standard deviation of 120 seconds for this type of building and with an evacuation alarm giving a recorded message to the people telling them to evacuate as soon as possible.

E The evacuation time is calculated with the simple equation $N/(W \cdot f)$ where N is the number of people in the building, W is the total doorwidth and f is the specific flow of people through the door. N is estimated with a lognormal distribution with a mean of 70 persons and a standard deviation of 30 persons. The doorwidth is 2.4 m and the specific flow is 1.0 person/m · s.

Some of the assumptions are rather uncertain but assumed acceptable in this exampel. The reason for choosing lognormal distribution is that this distribution is often found to provide a good representation for physical quantities that are constrained to being non-negative, that are positively showed and with an order-of-magnitude uncertainties.

The resulting probability of failure p_f has been calculated by two methods
- analytical, FOSM and
- Monte Carlo simulation.

The FOSM Methodology

As all the varying parameters; q, R and N are nonnormally distributed and the methodology demands them to be normally distributed, a transformation of the parameters has to be performed. In this case, where the parameters are lognormally distributed, the procedure is rather simple due to the close relationship between lognormal and normal distributions. After the transformation, the parameter X_i will be described with mean $\mu_{X_i}^N$ and standard deviation $\sigma_{X_i}^N$ where N indicates equivalent normal distribution.

The parameters describing the lognormal distribution are ζ and λ where

$$\zeta i_{x_i} = \sqrt{\ln(1+\delta_{x_i})} \qquad (15a) \qquad\qquad \delta_{x_i} = \frac{s_{x_i}}{m_{x_i}} \qquad (15b)$$

$$\lambda_{x_i} = \ln m_{x_i} - \frac{1}{2}\ln(1+\delta_{x_i}) \qquad (15c)$$

which gives the lognormal parameters

	ζ	λ
q	0.94	-4.1
R	0.81	4.5
N	0.41	4.2

The equivalent normal distribution parameters μ_{Xi}^{N} and σ_{Xi}^{N} are given as

$$\mu_{Xi}^{N} = x_i^* \cdot \zeta \qquad (16a) \qquad\qquad \sigma_{Xi}^{N} = x_i^*\,(1 - \ln x_i^* + \lambda_{Xi}) \qquad (16b)$$

x_i^* represent the first guess of the failure point on the tangent plane. As this point is not known the mean values are chosen as the first guess.

$\mu_q^N = 0.0147$ $\mu_R^N = 82.2$ $\mu_N^N = 66.6$

$\sigma_q^N = 0.0235$ $\sigma_R^N = 105.3$ $\sigma_N^N = 28.7$

The limit equation is, as stated earlier,

$$g(q, R, N) = 272\,q^{-0.2} - 17.3\,q^{-0.49} - R - \frac{N}{Wf} = 0$$

and in the reduced variate space

$$g(q', R', N') = 272\,(\mu_q^N + q'\sigma_q^N]^{-0.2} - 17.3(\mu_q^N + q'\sigma_q^N)^{-0.49} - (\mu_R^N + R'\sigma_R^N) - \left(\frac{\mu_N^N + N'\sigma_N^N}{W\cdot f}\right) = 0$$

The partial derivates are as follows at the assumed failure point x_i*

$$\frac{\partial g}{\partial q'} = -58.4 \qquad \frac{\partial g}{\partial R'} = -105.3 \qquad \frac{\partial g}{\partial N'} = -12.0$$

The direction cosines can then be calculated according to eq (11) as

$\alpha_q^* = -0.482$ $\alpha_R^* = -0.870$ $\alpha_N^* = -0.099$

The following equations can then be set up

$q^* = \mu_q^N - \alpha_q^* \sigma_q^N \beta = 0.0147 + 0.0113 \beta$

$R^* = \mu_R^N - \alpha_R^* \sigma_R^N \beta = 82.2 + 91.6 \beta$

$N^* = \mu_N^N - \alpha_N^* \sigma_N^N \beta = 66.6 + 2.84 \beta$

The limit state equation then becomes

$$g(q^*, R^*, N^*) = 272(0.0147 + 0.0113\ \beta)^{-0.2} - 17.3(0.0147 + 0.0113\ \beta)^{-0.49} - (82.2 + 91.6\ \beta) - \left(\frac{66.6 + 2.84\ \beta}{2.4 \cdot 1}\right)$$

From this β can be solved numerically and a better value of the failure point can be calculated. This process continues until convergence of β is reached.

After a number of iterations β will reach the value of 2.0. The probability of failure can then be calculated as

$P_f = 1 - \Phi(\beta)$

which is approximately 2.3 %.

The above calculations were carried out by hand and for a simplified example. Commercial software is available to handle more complex and realistic calculations.

Monte Carlo Simulation

The same scenario was also simulated using the Monte Carlo technique but with the original distribution of q, R and N. The number of samples in the simulation was 10 000. The simulated probability density function PDF S-D-R-E is shown in figure 4. The mean value of S-D-R-E which is the margin of safety is 328 seconds and the standard deviation is 135 seconds which gives a β-value of 2.4. It should be noted that the distribution of the result is not normally distributed.

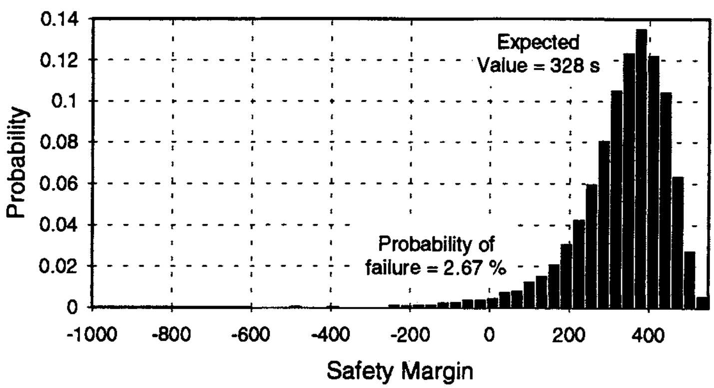

FIGURE 4 PDF of the safety margin.

The probability of failure

$F_{S\text{-}D\text{-}R\text{-}E}(0) = 2.67\ \%$

which is almost the same result as for the analytical calculation. The p_f-value is also a result of the numerical simulation.

IMPORTANCE OF CHANGES IN THE INPUT DATA

The input statistical parameters were chosen subjectively and mainly based on expert opinion. It follows that it would be of value to investigate how P_f and β vary with changes in the characterization of the input parameters. Such a sensitivity analysis has not been carried out in this work and the results must therefore be seen to be preliminary.

It should be noted that a proper sensitivity analysis is essential for any future work of this kind and that the final distribution parameters should be based on comprehencive statistical investigations.

CONCLUSIONS

A pilot study on a very limited scale has been carried out on the application of the FOSM-methodology to fire safety engineering design. It has been demonstrated that the methodology makes possible an ordered and structured quantitative evaluation of the safety levels inherent in component fire safety systems. In a practical design situation, much of the input data has to be derived by use of subjective judgement. To be of use for regulatory purposes, input data must be standardized according to building classification unless reliable statistical data are available. This is a weakness, though, which is characteristic for the general area of fire safety design based on calculation and which is not specific for the FOSM-methodology.

This report is the first of a series from a project dealing with performance and the calculation of safety levels in building fire safety design.

REFERENCES

1. Wakamatsu, T., Development of Deisgn System for Building Fire Safety, Fire Safety Science, Proceedings of the Second International Conference, Hemisphere Publishing, New York, NY, 881-896 pp, 1989.

2. Microeconomic Reform: Fire Regulation, National Building Fire Safety Systems Project of the Building Regulation Review Task Force, May 1991.

3. Bukowski, R.W., Clarke, F.B., Hall Jr., J.R. and Stiefel, S.W., Fire Risk Assessment Method: Description of Methodology, available from NFPRF, Quincy, MA 02269, 1990.

4. Hadjisophocleous, G.V., Yung, D., A Fire Risk and Protection Cost Assessment Model for Highrise Apartment Buildings, Fire Hazard and Fire Risk Assessment, ASTM STP 1150, Marcelo M Hirschler, Ed., American Society for Testing and Materials, Philadelphia, 1992, 224-233 pp.

5. Outline - UK Code of Practice on the Application of Fire Engineering Principles to Fire Safety of Buildings, ISO TC92/SC4/WG1 N21, July 1992.

6. Ang, A. H-S., Tang, W.H., Probability concepts in Engineering Planning and Design, Volume II, John Wiley & Sons, 1984.

7. Magnusson, S.E., Probabilistic Analysis of Fire Exposed Steel Structures, Bulletin 27, Div. of Structural Mechanics and Concrete Construction, Lund Inst. Techn., 1974.

8. Cornell, C.A., First order analysis of model and parameter uncertainty, Proc. Int. Symp. on Uncertainties i Hydrol. Water Resour. Systems, University of Arizona, vol 2 1245-1275, 1972.

9. Klote, J.H., Milke, J.A., Design of Smoke Management Systems. ASHRAE, Atlanta 1992.

10. Bukowski, R.W., Peacock, R.D., Jones, W.W., Forney, C.L., Technical Reference Guide for the HAZARD I Fire Hazard Assessment Method. NIST, Gaithersburg 1989.

11. Frantzich, H., Reaction times prior to evacuation for shops, restaurants and public dance-halls. Estimates done by fire officers. Not published.