Calculation of Load and Deformation Behavior of Structure Elements Taking into Account 3-Dimensional Heat Flow

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ABSTRACT

This paper describes a numerical method for the calculation of load and deformation behavior of structural elements taking into account 3-Dimensional heat flow. The 3-Dimensional heat flow influences the temperature distribution in the area of beam-to-column, beam-to-slab and slab-to-column connections. Because of this, the temperature distribution in the longitudinal direction is different between 2-D and 3-D calculations. However, the difference in structural behavior is small, when comparing calculation results from 2-D and 3-D analysis with test results of single elements under standard fire conditions.

KEYWORDS: 3-dimensional heat flow, composite structure, computer code, load and deformation behavior, natural fire, standard test

NOTATION LIST

c : specific heat
d t : time interval
d x : incremental length
H : the prior response and thermal history of the structure
K : structural stiffness matrix
l : length of cell
L : length of beam element
Δ P : incremental load vector

q : heat flow in unit time, unit volume (included water evaporation)
Q : heat flow
t : duration of fire exposure
T : temperature
ΔU : incremental deformation vector
U : deformation
y : distance from z-axis centroid line to slice
z : distance from y-axis centroid line to slice
ε_R : total strain
ε_T : thermal strain
ε : stress related strain (total mechanical strain)
θ : rotation
λ : thermal conductivity
ρ : unit mass
ϕ : curvature

Subscripts

a, b : nodal point of beam element
c : concrete
i : current time step
j : iteration step
re : reinforcing steel
st : structural steel
x, y, z : position coordinates

INTRODUCTION

At present time, there are many computer codes for calculating the fire behavior of structural members [1][2][3][4]. Some computer codes take into account temperature distribution in the longitudinal direction [5][6], but without 3-Dimensional heat flow. This contribution investigates the influence of 3-D temperature analysis for the load and deformation behavior of structural elements under standard fire conditions.

THEORETICAL MODEL

General

The program CST-F (Response Analysis for Composite STructures in Fire) has been developed for calculating structural behavior under fire conditions of composite structures, as well as of reinforced concrete and steel structures. The theoretical model consists of two main parts: the thermal part for the analysis of temperature distribution (Member Temperature of Steel and Concrete Composite Structures: MTSCCS) and the structural part for the analysis of mechanical behavior (Fire Behavior of Steel and Concrete Composite Structures: FBSCCS).

Thermal Analysis

The program for the thermal analysis MTSCCS calculates the temperature development and distribution in structural members, based on the finite difference method. The program takes into account a 3-Dimensional heat flow between adjacent cells inside the structural members. For each cell of the mesh, the following basic equation stands:

$$\rho c \, (dT/dt) = \lambda_x (d^2T/dx^2) + \lambda_y (d^2T/dy^2) + \lambda_z (d^2T/dz^2) + q$$

For example, the heat flow equation in the X axis is:

$$Q_{x,\,x+dx} = \lambda_{x,\,x+dx} (T_{x+dx} - T) \, l_y \cdot l_z \, (dt/dx)$$

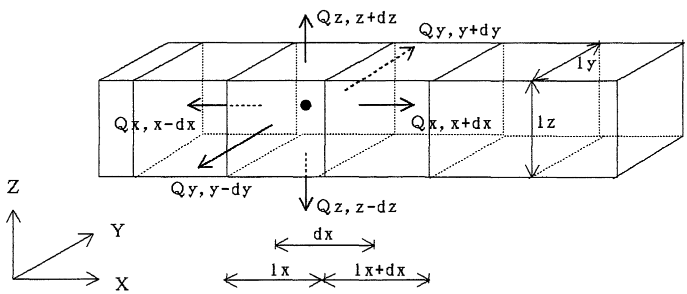

FIGURE 1. Heat flow model in internal cell

Based on the heat balance between adjacent cells (Figure 1.), the increased temperature is calculated. The boundary conditions considered are convection and radiation between the hot gas and the surface temperatures.
Water in the concrete consumes latent heat for evaporation. The temperature of concrete remains at 100℃ while the water evaporates. The specific heat of water is taken into account by a constant value for temperature below 100℃. But transfer of moisture is not considered.

Structural Analysis

The program for the structural analysis FBSCCS uses non-linear direct stiffness method with iterative procedure within a given time step. The method is based on the principle of virtual works, the equilibrium conditions are written in an incremental form using the stiffness matrix of the structure. The basis of the method can be found in [1] and [4]. The load-deformation relationship of the structure is expressed in matrix equation as:

$$K_i \, (U_i, \, T_i, \, H_{i-1}) \, \Delta U_i = \Delta P_i$$

The solution of the equation requires the iterative Newton-Raphson process. (Figure 2.)

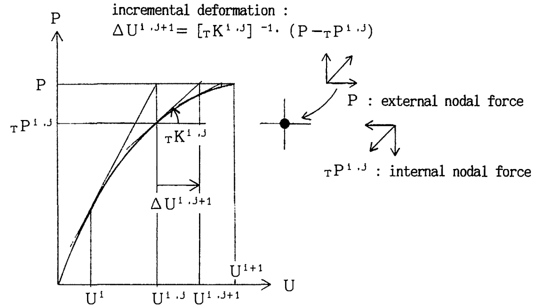

FIGURE 2. Iterative procedure for the solution of the equation

The structure is modeled by a 3-Dimensional beam element with two nodes and six degrees of freedom at each node, which are three translations and three rotations. (Figure 3.) The model takes into account uniform torsion in the element. The shear displacements and the shear energy are not considered. The Bernoulli hypothesis is assumed i.e., plane cross-sections remain plane. The structural member (Column or Beam) is divided in the longitudinal direction into submembers, which are divided into slices with constant temperature distribution during a time interval.

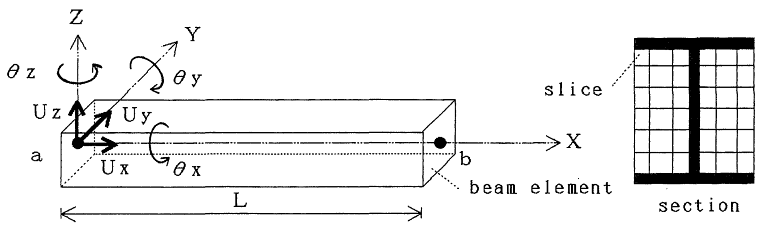

FIGURE 3. 3-dimensional beam element and slices

In each slice, based on the Bernoulli hypothesis, the incremental total strain, $\Delta\varepsilon_{R,\ i}$, is calculated as follows (i-th time step):

$$\Delta\varepsilon_{R,\ i} = \Delta\varepsilon_x + \Delta\phi_y \cdot z - \Delta\phi_z \cdot y$$

$$\Delta\varepsilon_x = (U_{x,b} - U_{x,a}) / L$$
$$\Delta\phi_y = (\theta_{y,b} - \theta_{y,a}) / L$$
$$\Delta\phi_z = (\theta_{z,b} - \theta_{z,a}) / L$$

The incremental stress related strain, $\Delta\varepsilon_i$, is obtained by deducting from the incremental total strain, $\Delta\varepsilon_{R,\ i}$, the incremental thermal strain, $\Delta\varepsilon_{T,\ i}$:

$$\Delta\varepsilon_i = \Delta\varepsilon_{R,\ i} - \Delta\varepsilon_{T,\ i}$$

The incremental creep strain, especially the comparatively large transient creep strain in concrete and some steels [8] is included in the stress-strain relationship. The current total mechanical strain, ε_i, is obtained from the total mechanical strain at last time step, ε_{i-1}, and the incremental stress related strain, $\Delta\varepsilon_i$:

$$\varepsilon_i = \varepsilon_{i-1} + \Delta\varepsilon_i$$

The current level of stress is obtained using the total mechanical strain. At the same time the modulus of elasticity is obtained from stress-strain relationship [7][8]. The integrals of the stiffness matrix and the internal force on the cross section are calculated in a numerical way.

MATERIAL PROPERTIES

Thermal Properties of Steel and Concrete at Elevated Temperatures

Thermal properties of steel and concrete, the specific heat c (J/kg·K), the thermal conductivity λ (W/m·K) and the thermal elongation, are taken from EUROCODE 4 [7].

Unit Mass

The unit mass of steel ρ_{steel} and concrete $\rho_{concrete}$ are taken as constant.

$$\rho_{steel} = 7850 \ (kg/m^3)$$
$$\rho_{concrete} = 2300 \ (kg/m^3)$$

Stress-Strain Relationship of Steel and Concrete at Elevated Temperatures

The stress-strain relationships, which are specified in EUROCODE 4 [7], of structural steel, reinforcing steel and concrete are shown in figures 4, 5 and 6.

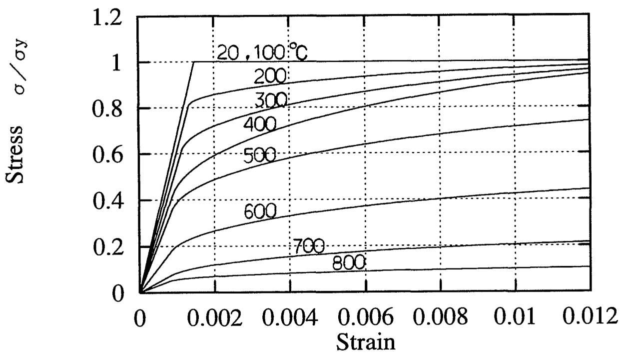

FIGURE 4. Stress-strain relationships of structural steel at elevated temperatures

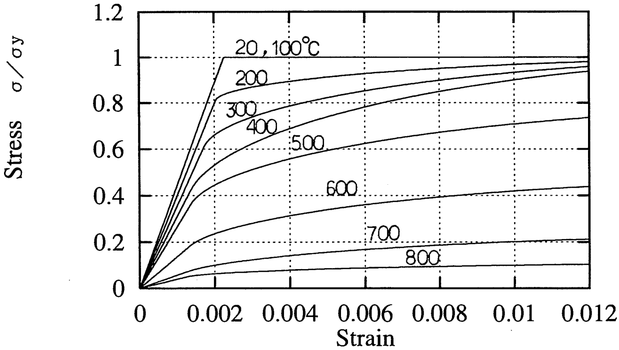

FIGURE 5. Stress-strain relationships of reinforcing steel at elevated temperatures

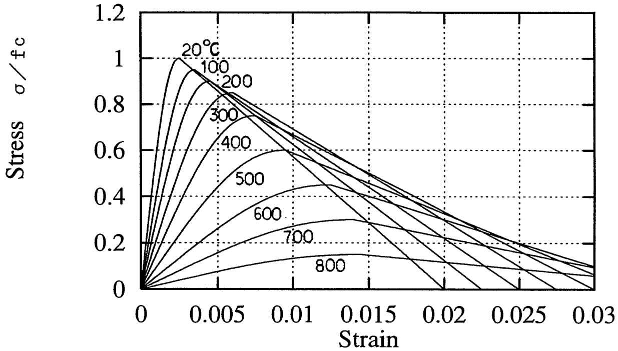

FIGURE 6. Stress-strain relationships of concrete at elevated temperatures

CALCULATION EXAMPLE

A composite column [9] is selected for comparison of test and calculation results.

Member Profile

The cross section of the column, the static loading condition and the eccentricity are shown in figure 7. The increase in horizontal displacement caused by eccentric loading is considered. The finite element mesh which is used in the calculation is shown in figure 8.

Heating Conditions

The gas temperature follows the ISO-834 standard heating curve. The coefficient of convective heat transfer α and the emissivity coefficient of steel $\varepsilon_{m,steel}$ and concrete

$\varepsilon_{m,concrete}$ are taken as follows [7]:

$\alpha = 25$ (W/m^2·K)

$\varepsilon_{m,steel} = 0.625$

$\varepsilon_{m,concrete} = 0.7$

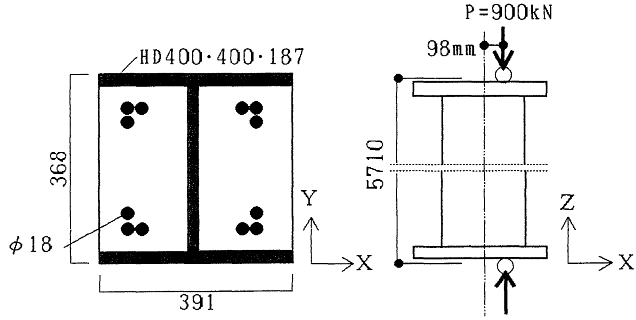

FIGURE 7. The cross section of composite column with static loading

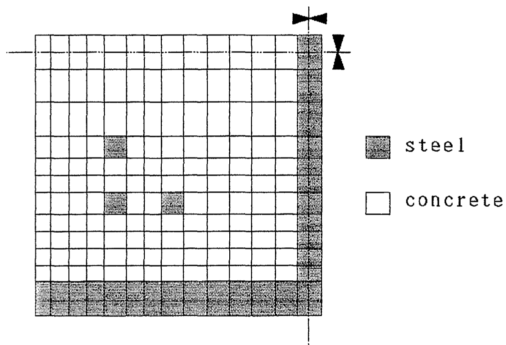

FIGURE 8. Finite element mesh of a quarter of the cross-section

Material Properties

The yield strength of structural and reinforcing steel $\sigma_{y,st}$, $\sigma_{y,re}$ and the compressive strength of concrete f_c are taken as follows [9]:

$\sigma_{y,st} = 310$ (N/mm^2)

$\sigma_{y,re} = 476$ (N/mm^2)

$f_c = 50.2$ (N/mm^2) (on load eccentricity side)

41.4 (N/mm^2)

The water content of concrete is taken as 120 (l/m ^{3}) [9].

Support Conditions

The support conditions of the column are shown in figure 9. The bottom of the column is fixed, except for rotation about Y axis. The top of the column is free in the longitudinal direction. Both ends of the column are not heated, because the loading equipment needs fire protection. The area of heating is shown in figure 10.

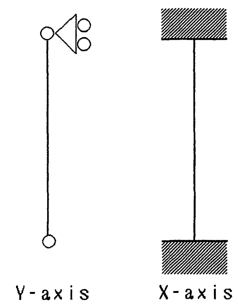

FIGURE 9. Support conditions

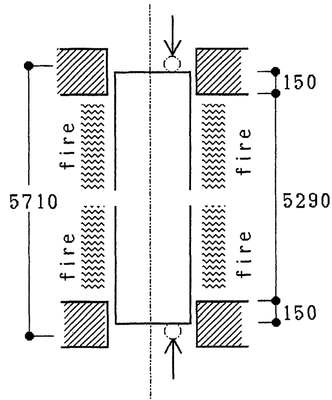

FIGURE 10. The heating area

CALCULATION RESULTS AND COMMENTS

Two calculations were carried out. The first calculation was based on 2-Dimensional and the second calculation on 3-Dimensional heat flow. The first calculation did not consider fire protection at the column ends, i.e., the full length of the column was heated by the ISO-834 standard fire exposure.

Time-Temperature Behavior

Assuming 2-D heat flow, calculated temperature developments in the structural and the reinforcing steels are compared with test results in figures 11 and 12. The test and calculated results agree very well with each other.

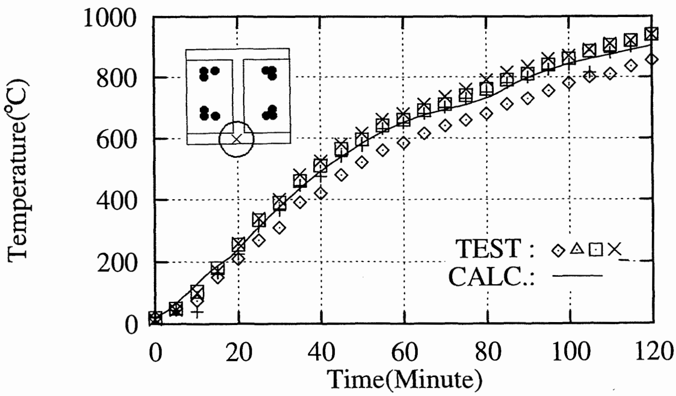

FIGURE 11. Time-temperature curve at structural steel:
comparison between test and calculation results, 2-D heat flow

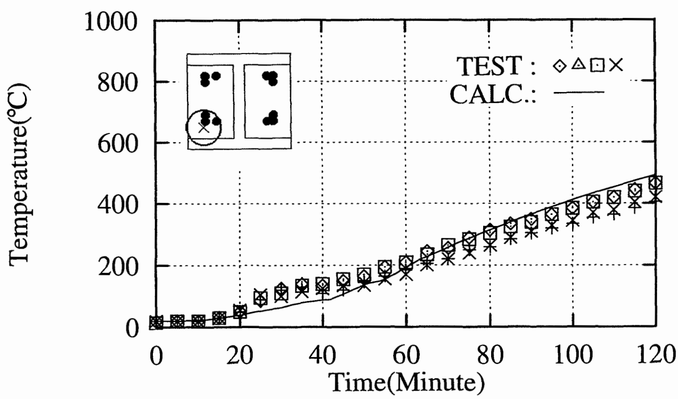

FIGURE 12. Time-temperature curve at reinforcing steel:
comparison between test and calculation results, 2-D heat flow

Figures 13 and 14 show the calculated temperature development at the same measuring points of the cross section, but taking into account 3-Dimensional heat flow. Only the temperature developments at the end of the column are affected by the protected area.

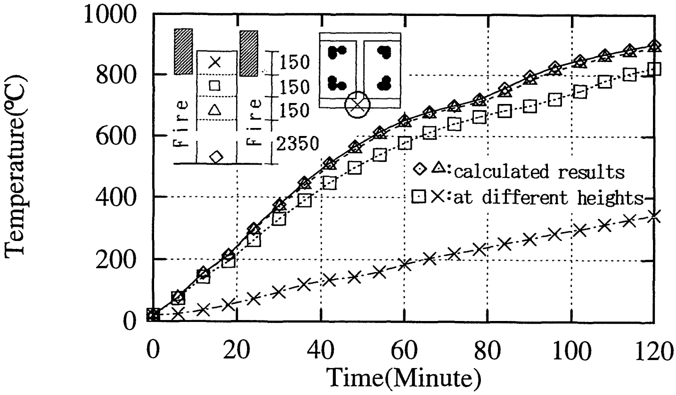

FIGURE 13. Time-temperature curve at structural steel:
temperature developments at different heights (3-D calculation)

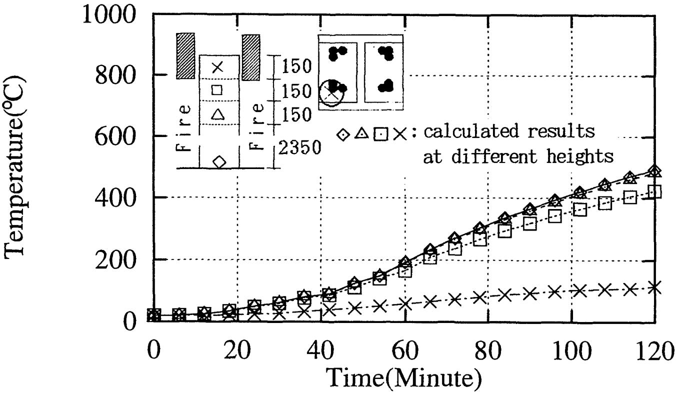

FIGURE 14. Time-temperature curve at reinforcing steel:
temperature developments at different heights (3-D calculation)

Time-Deformation Behavior

The calculated horizontal displacement at the middle length of the column, as well as the calculated vertical displacement at the top of the column are compared with the test results in figures 15 and 16. The differences between the test results and the calculated results are very small. Failure times are also very close. But there are no differences in the load bearing and deformation behavior of the structural element between using 2-D or 3-D heat flow analysis for temperature development.

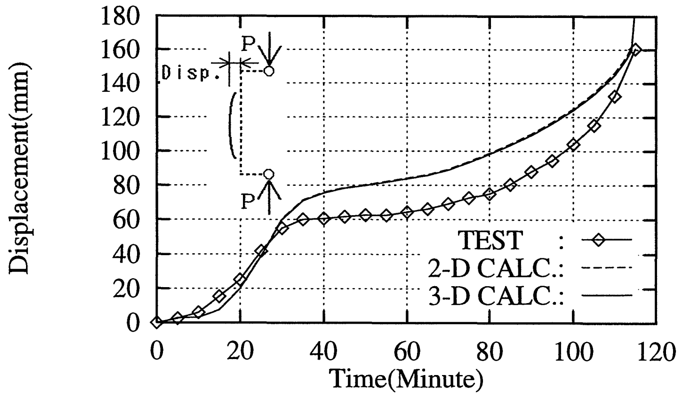

FIGURE 15. Measured and calculated horizontal displacement

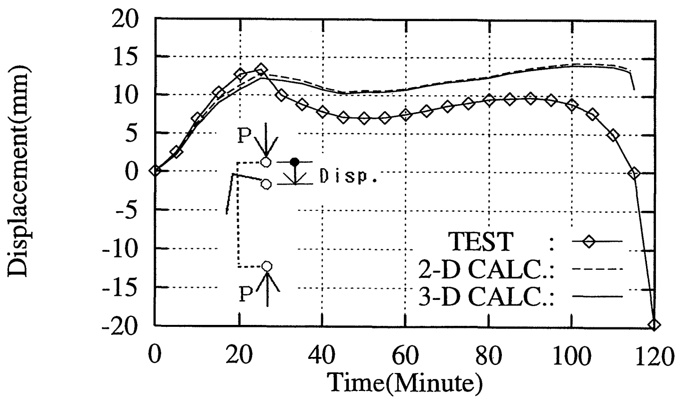

FIGURE 16. Measured and calculated vertical displacement

CONCLUSIONS

The load and deformation behavior of a composite column is calculated taking into account 3-Dimensional heat flow. The test and calculated results were compared. These results suggest that it is not necessary to use 3-D heat flow analysis for a single element under the standard fire condition. Further investigations are required to show whether it is necessary to use 3-D heat flow analysis for the correct simulation of the behavior under natural fire conditions, especially for partially insulated elements or for entire structures, such as that tested at BRE's Cardington Laboratory [10].

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